## A Nonlinear Dynamics Perspective for the Analysis, Control and Design of Mechanical and Structural Systems

## **Giuseppe Rega**

Department of Structural and Geotechnical Engineering, Sapienza University of Rome via A. Gramsci 53, 00197 Rome, Italy

The achievements occurred in nonlinear dynamics over the last 30 years entail a substantial change of perspective when dealing with vibration problems, since they are now deemed ready to meaningfully affect the analysis, control and design of mechanical and structural systems. This lecture aims at highlighting and discussing the important, yet still overlooked, role that some relevant concepts and tools may play in engineering applications. Upon dwelling on such topical concepts as local and global dynamics, bifurcation and complexity, theoretical and practical stability, recent results obtained by the lecturer and his co-authors for a variety of systems and models of interest in mechanics and structural dynamics are overviewed in terms of modelling, analysis, nonlinear phenomena and their control.

Local and global stability of systems attractors are discussed by also considering the effects of imperfections or small, but finite, dynamic perturbations, along with variations of some control parameters. All of them may arise in technical applications and experiments, and are to be properly considered in the design stage in order to secure the system capability to sustain changes without modifying the desired outcome.

The concept of practical stability implies definitely abandoning the merely local perspective traditionally assumed in the analysis and design of systems and structures, and moving to a global one where the whole dynamical behaviour of the system is considered, even if being actually interested in only a small (but finite) neighbourhood of a given solution. In spite of its conceptual simplicity, this is a paramount enhancement, full of theoretical and practical implications. Among them, the need to analyse the effects of non-classical solutions (stable and unstable manifolds of a given saddle, chaotic attractors, escape, etc.) and to develop new numerical tools able to perform the global analysis of systems with large number of degrees of freedom are mentioned.

Dynamical integrity is a fundamental concept in the analysis and design of dynamical systems. It implies properly defining the system safe basin and measuring its integrity, by also accounting for the possibly non-compact structure of the basin in certain ranges of values of control parameters. The existence of possibly competing dynamic solutions requires investigating their robustness against variations of initial conditions or of control parameters, which is strongly affected by the basin magnitude and integrity.

Erosion profiles allow to evaluate the loss (or increase) of integrity, to be possibly explained in terms of global phenomena occurring in the system. Both robustness and integrity may become residual well before the nominal disappearance of a solution of interest via local bifurcations: it is just this item that makes the concept of practical stability, and the associated global analysis, necessary in applications.

The overall transition from a local stability perspective to a global safety concept has also major implications as regards the feasibility and effectiveness of a control of nonlinear dynamics of engineering systems based just on exploiting some associated global bifurcation. In fact, the control approach and techniques may drastically change when the goal is increasing the overall dynamical integrity of the system, instead of merely controlling (i.e. stabilizing) a single trajectory, as in classical chaos control techniques.

In this lecture, all these issues, which also permit to explain partial discrepancies between experimental and theoretical/numerical results based on merely local analyses, are discussed for a number of systems of interest in applications ranging from macro- to nano-mechanics. For the sake of presentation, archetypal discrete systems and reduced order models of continuous systems are presented separately. The former refer basically to an inverted mechanical pendulum in different configurations, and include an asymmetric planar system, a symmetrically stiffened spatial pendulum, a simplified model of a guyed cantilever tower, and a parametrically excited experimental pendulum. The latter refer to reduced order approximations of infinite-dimensional systems at either a micro-scale (a clamped micro-beam, a non-contact AFM cantilever) or the macro-scale (a post-buckled cylindrical shell, a laminated composite plate, a cantilever beam with closed or open cross-section), and exploit specific system symmetry features or dynamic characteristics under excitations of interest for theoretical purposes or practical applications.

Specific modeling, solution, and phenomenological aspects are discussed for the various systems, paying however proper attention to the common or distinguishing nonlinear dynamics features which are expected to play a meaningful role in the analysis and design for engineering applications.