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THE RIGID-FLEXIBLE TWO LINK ROTATING SYSTEM: COMPARISON BETWEEN ONE FLEXURAL MODE EXPANSION AND TWO FLEXURAL MODES EXPANSION FOR THE FLEXIBLE LINK DISCRETIZATION

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Abstract. *This A rigid-flexible two link rotating system is mathematically modeled using Lagrange's equations. For the flexible link is considered linear curvature and inertia-free boundary conditions. Two different approaches are considered for the flexible link: (a) only the first flexural mode is considered in the flexible link discretization and (b) two flexural modes are considered in the flexible link discretization. The main idea here is to investigate the participation of the second flexural mode in the system dynamics for slow and fast maneuvers. Nonlinearity arises in this problem from the coupling between the variable representing the angular velocity of the rotating axis connected to the flexible link and the variable representing the vibration of the flexible link. Sufficiently large angular velocities are considered in order to the system to undergo sufficiently strong nonlinear behavior. Position/velocity and vibration control using both approaches to the beam-like flexible structure is also investigated here. For position/velocity of both rotating axes and elimination of vibration in the flexible link the nonlinear control technique named State Dependent Riccati Equation (SDRE) is applied. The results for the different mathematical descriptions of the system are compared and discussed.*

1 INTRODUCTION

With the increasing demand for precise high-speed operation and lightweight mechanisms for space missions and the industry, it was no longer adequate to treat certain links in a manipulator as rigid [1]. Expanding this idea, a rigid-flexible two-link manipulator can be lighter and faster than a rigid-rigid one and can cover a wider workspace using less energy. Lightweight mechanical structures are expected to improve the performance of robotic manipulators, which also often have low payload-to-arm mass ratios. However, such manipulators with flexible parts exhibit undesirable vibrations which might limit their performance [2]. The investigation of these systems can also be considered as a starting point for more complicated mathematical models for multi-link flexible or rigid-flexible manipulators.

Several methods have been developed for handling with the design of control algorithms for similar nonlinear system as the one investigated in this work. The State-Dependent Riccati Equation (or simply SDRE) method, developed over the past several years, is one such method. This technique for nonlinear regulator problems has become well-known within the control community. It has been successfully employed in a considerable number of mathematical and real applications [3-5]. A good survey of the SDRE design technique can be found in [6].

A different mathematical model for the rigid-flexible manipulator investigated here considering the modeling of the DC motor actuator and the same nonlinear control technique discussed here was developed in [7].

2 THE GOVERNING EQUATIONS OF MOTION

The geometric model of the system investigated in this work is presented in Figure 1. This system comprises a rigid link connected to a flexible link. Each one of the links is driven by an actuator. In this figure, the inertial axis is represented by XY, the moving axis (attached to the flexible link and rotating with it) is represented by xy, the beam deflection is represented by $\nu(x,t)$ and the angular displacements are given by θ_1 and θ_2.

The governing equations of motion are obtained through the lagrangian formalism. To apply the Lagrange's equations one needs to know the kinetic energy stored in the rigid link and the kinetic and potential (strain) energies stored in the flexible link during its time evolution.

The total kinetic energy, T, of this system is given by:

$$
\begin{aligned}
T = \frac{1}{2}(I_1 + m_1 L_{c1}^2)\dot{\theta}_1^2 + \frac{1}{2}\int_0^{L_2} \rho A_2 \Big\{ & [\dot{\nu} + x(\dot{\theta}_1 + \dot{\theta}_2)]^2 + [\nu\nu\dot{\theta}_1 + \dot{\theta}_2)]^2 + \\
& 2L_1\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)(x\cos\theta_2 - \nu\sin\theta_2) + 2L_1\dot{\nu}\dot{\theta}_1\cos\theta_2 + L_1^2\dot{\theta}_1^2 \Big\} dx
\end{aligned} \tag{1}
$$

The strain energy of this system is given by:

$$
V = \frac{1}{2}\int_0^{L_2} EI_2 \nu''^2 dx \tag{2}
$$

In Eq. (2), E represents the Young's modulus, and I_2 represents the moment of inertia of the cross-section area of this link. The Lagrangian is given by:

$$
L = T - V \tag{3}
$$

One assumes here linear curvature for the flexible link [8,9]. The Lagrangian is discretized considering the expansion given by:

$$
\nu(x,t) = \sum_{i=1}^{n} \Phi_i(x)\, q_i(t) \tag{4}
$$

where n represents the number of modes and $\Phi_i(x)$ each one of the modes. Inertia-free normal modes [10] are considered.

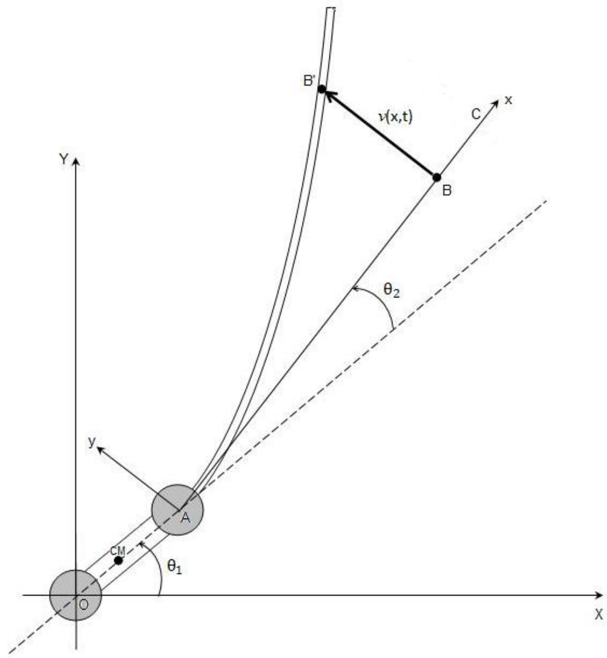

Figure 1. The slewing rigid-flexible beam system.

The discretized Lagrangian for n modes is given by:

$$\begin{aligned} L = {} & \frac{1}{2}\left(I_1 + m_1 L_{c1}^2 + \frac{\rho A_2 L_1^2 L_2}{2} + \frac{\rho A_2 L_2^3}{6} \right)\dot{\theta}_1^2 + \frac{\rho A_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\int_0^{L_2} \Phi_i \Phi_j dx \right)\dot{q}_i \dot{q}_j + \\ & \rho A_2(\dot{\theta}_1 + \dot{\theta}_2)\sum_{i=1}^{n}\left(\int_0^{L_2} x\Phi_i dx \right)\dot{q}_i + \frac{\rho A_2 L_2^3}{3}\dot{\theta}_1\dot{\theta}_2 + \frac{\rho A_2 L_2^3}{6}\dot{\theta}_2^2 + \frac{\rho A_2}{2}(\dot{\theta}_1 + \dot{\theta}_2)^2 \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\int_0^{L_2} \Phi_i \Phi_j dx \right) q_i q_j + \\ & \frac{\rho A_2 L_1 L_2^2}{2}\dot{\theta}_1^2 \cos\theta_2 + \frac{\rho A_2 L_1 L_2^2}{2}\dot{\theta}_1\dot{\theta}_2 \cos\theta_2 - \rho A_2 L_1 \dot{\theta}_1^2 \sin\theta_2 \sum_{i=1}^{n}\left(\int_0^{L_2} \Phi_i \, dx \right) q_i - \\ & \rho A_2 L_1 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 \sum_{i=1}^{n}\left(\int_0^{L_2} \Phi_i \, dx \right) q_i + \rho A_2 L_1 \dot{\theta}_1 \cos\theta_2 \sum_{i=1}^{n}\left(\int_0^{L_2} \Phi_i \, dx \right)\dot{q}_i - \frac{EI_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\int_0^{L_2} \Phi_i'' \Phi_j'' \, dx \right) q_i q_j \end{aligned} \tag{5}$$

The free vibration mode shapes of continuous systems satisfy the orthogonality condition given by $\int_0^L \Phi_1 \Phi_2 dx = 0$ and $\int_0^L \Phi_1'' \Phi_2'' \, dx = 0$.

Substituting Eq. (5) with n=1 and with n=2 into the Lagrange's equations [11,12] results the governing equations of motion for the system depicted in Figure 1. The governing equations of motion for one mode expansion are written as:

$$\begin{aligned} & \ddot{\theta}_1 + 2w_3\eta_1\omega_1^2 q_1 + w_1\left(2v_7\dot{\theta}_1\dot{q}_1 + 2v_7\dot{\theta}_2\dot{q}_1 - 2v_8\dot{\theta}_1\dot{\theta}_2 - v_8\dot{\theta}_2^2\right) + w_2\left(2v_9\dot{\theta}_1\dot{q}_1 + 2v_9\dot{\theta}_2\dot{q}_1 + v_8\dot{\theta}_1^2\right) + \\ & w_3\left(-2v_9\dot{\theta}_1\dot{\theta}_2 - v_9\dot{\theta}_2^2 - v_7\dot{\theta}_1^2\right) = w_1 Q_{\theta_1} + w_2 Q_{\theta_2} + w_3 Q_{q_1} \end{aligned} \tag{6a}$$

$$\ddot{\theta}_2 + 2w_5\eta_1\omega_1^2 q_1 + w_2\left(2v_7\dot{\theta}_1\dot{q}_1 + 2v_7\dot{\theta}_2\dot{q}_1 - 2v_8\dot{\theta}_1\dot{\theta}_2 - v_8\dot{\theta}_2^2\right) + w_4\left(2v_9\dot{\theta}_1\dot{q}_1 + 2v_9\dot{\theta}_2\dot{q}_1 + v_8\dot{\theta}_1^2\right) + w_5\left(-2v_9\dot{\theta}_1\dot{\theta}_2 - v_9\dot{\theta}_2^2 - v_7\dot{\theta}_1^2\right) = w_2 Q_{\theta_1} + w_4 Q_{\theta_2} + w_5 Q_{q_1} \qquad (6b)$$

$$\ddot{q}_1 + 2w_6\eta_1\omega_1^2 q_1 + w_3\left(2v_7\dot{\theta}_1\dot{q}_1 + 2v_7\dot{\theta}_2\dot{q}_1 - 2v_8\dot{\theta}_1\dot{\theta}_2 - v_8\dot{\theta}_2^2\right) + w_5\left(2v_9\dot{\theta}_1\dot{q}_1 + 2v_9\dot{\theta}_2\dot{q}_1 + v_8\dot{\theta}_1^2\right) + w_6\left(-2v_9\dot{\theta}_1\dot{\theta}_2 - v_9\dot{\theta}_2^2 - v_7\dot{\theta}_1^2\right) = w_3 Q_{\theta_1} + w_5 Q_{\theta_2} + w_6 Q_{q_1} \qquad (6c)$$

where:

$v_1 = I_T + 2\eta_1 q_1^2 + 2\eta_6\cos\theta_2 - 2\eta_7 q_1\sin\theta_2$ $\quad v_2 = \eta_5 + 2\eta_1 q_1^2 + \eta_6\cos\theta_2 - \eta_7 q_1\sin\theta_2$ $\quad v_3 = \eta_3 + \eta_7\cos\theta_2$

$v_4 = \eta_5 + 2\eta_1 q_1^2$ $\quad v_5 = \eta_3$ $\quad v_6 = 2\eta_1$

$v_7 = 2\eta_1 q_1 - \eta_7\sin\theta_2$ $\quad v_8 = \eta_6\sin\theta_2 + \eta_7 q_1\cos\theta_2$ $\quad v_9 = 2\eta_1 q_1$

$\alpha_1 = \int_0^{L_2}\Phi_1\,dx$ $\quad \alpha_3 = \int_0^{L_2} x\Phi_1\,dx$ $\quad \gamma_1 = \int_0^{L_2}\Phi_1^2\,dx$ $\quad I_T = I_1 + m_1 L_{c1}^2 + \dfrac{\rho A_2 L_1^2 L_2}{2} + \dfrac{\rho A_2 L_2^3}{6}$ $\quad \eta_1 = \dfrac{\rho A_2\gamma_1}{2}$

$\eta_3 = \rho A_2\alpha_3$ $\quad \eta_5 = \dfrac{\rho A_2 L_2^3}{3}$ $\quad \eta_6 = \dfrac{\rho A_2 L_1 L_2^2}{2}$ $\quad \eta_7 = \rho A_2 L_1\alpha_1$ $\quad \Phi_1''\Phi_1'' = \Phi_1^{iv}\,\Phi_1 = \left(\dfrac{\rho A_2\omega_1^2}{EI_2}\Phi_1\right)\Phi_1$

and the governing equations of motion for two modes expansion are written as:

$$\ddot{\theta}_1 + 2z_6\eta_1\omega_1^2 q_1 + 2z_7\eta_2\omega_2^2 q_2 + z_1\left(-2n_1\dot{\theta}_1\dot{\theta}_2 + n_2\dot{\theta}_1\dot{q}_1 + n_3\dot{\theta}_1\dot{q}_2 + n_4\dot{\theta}_2\dot{q}_1 + n_5\dot{\theta}_2\dot{q}_2 - n_1\dot{\theta}_2^2\right) + z_5\left(2n_8\dot{\theta}_1\dot{q}_1 + 2n_9\dot{\theta}_1\dot{q}_2 + 2n_8\dot{\theta}_2\dot{q}_1 + 2n_9\dot{\theta}_2\dot{q}_2 + n_1\dot{\theta}_1^2\right) - z_6\left(2n_8\dot{\theta}_1\dot{\theta}_2 + n_6\dot{\theta}_1^2 + n_8\dot{\theta}_2^2\right) - z_7\left(2n_9\dot{\theta}_1\dot{\theta}_2 + n_7\dot{\theta}_1^2 + n_9\dot{\theta}_2^2\right) = z_1 Q_{\theta_1} + z_5 Q_{\theta_2} + z_6 Q_{q_1} + z_7 Q_{q_2} \qquad (7a)$$

$$\ddot{\theta}_2 + 2z_8\eta_1\omega_1^2 q_1 + 2z_9\eta_2\omega_2^2 q_2 + z_5\left(-2n_1\dot{\theta}_1\dot{\theta}_2 + n_2\dot{\theta}_1\dot{q}_1 + n_3\dot{\theta}_1\dot{q}_2 + n_4\dot{\theta}_2\dot{q}_1 + n_5\dot{\theta}_2\dot{q}_2 - n_1\dot{\theta}_2^2\right) + z_2\left(2n_8\dot{\theta}_1\dot{q}_1 + 2n_9\dot{\theta}_1\dot{q}_2 + 2n_8\dot{\theta}_2\dot{q}_1 + 2n_9\dot{\theta}_2\dot{q}_2 + n_1\dot{\theta}_1^2\right) - z_8\left(2n_8\dot{\theta}_1\dot{\theta}_2 + n_6\dot{\theta}_1^2 + n_8\dot{\theta}_2^2\right) - z_9\left(2n_9\dot{\theta}_1\dot{\theta}_2 + n_7\dot{\theta}_1^2 + n_9\dot{\theta}_2^2\right) = z_5 Q_{\theta_1} + z_2 Q_{\theta_2} + z_8 Q_{q_1} + z_9 Q_{q_2} \qquad (7b)$$

$$\ddot{q}_1 + 2z_3\eta_1\omega_1^2 q_1 + 2z_{10}\eta_2\omega_2^2 q_2 + z_6\left(-2n_1\dot{\theta}_1\dot{\theta}_2 + n_2\dot{\theta}_1\dot{q}_1 + n_3\dot{\theta}_1\dot{q}_2 + n_4\dot{\theta}_2\dot{q}_1 + n_5\dot{\theta}_2\dot{q}_2 - n_1\dot{\theta}_2^2\right) + z_8\left(2n_8\dot{\theta}_1\dot{q}_1 + 2n_9\dot{\theta}_1\dot{q}_2 + 2n_8\dot{\theta}_2\dot{q}_1 + 2n_9\dot{\theta}_2\dot{q}_2 + n_1\dot{\theta}_1^2\right) - z_3\left(2n_8\dot{\theta}_1\dot{\theta}_2 + n_6\dot{\theta}_1^2 + n_8\dot{\theta}_2^2\right) - z_{10}\left(2n_9\dot{\theta}_1\dot{\theta}_2 + n_7\dot{\theta}_1^2 + n_9\dot{\theta}_2^2\right) = z_6 Q_{\theta_1} + z_8 Q_{\theta_2} + z_3 Q_{q_1} + z_{10} Q_{q_2} \qquad (7c)$$

$$\ddot{q}_2 + 2z_{10}\eta_1\omega_1^2 q_1 + 2z_4\eta_2\omega_2^2 q_2 + z_7\left(-2n_1\dot{\theta}_1\dot{\theta}_2 + n_2\dot{\theta}_1\dot{q}_1 + n_3\dot{\theta}_1\dot{q}_2 + n_4\dot{\theta}_2\dot{q}_1 + n_5\dot{\theta}_2\dot{q}_2 - n_1\dot{\theta}_2^2\right) + z_9\left(2n_8\dot{\theta}_1\dot{q}_1 + 2n_9\dot{\theta}_1\dot{q}_2 + 2n_8\dot{\theta}_2\dot{q}_1 + 2n_9\dot{\theta}_2\dot{q}_2 + n_1\dot{\theta}_1^2\right) - z_{10}\left(2n_8\dot{\theta}_1\dot{\theta}_2 + n_6\dot{\theta}_1^2 + n_8\dot{\theta}_2^2\right) - z_4\left(2n_9\dot{\theta}_1\dot{\theta}_2 + n_7\dot{\theta}_1^2 + n_9\dot{\theta}_2^2\right) = z_7 Q_{\theta_1} + z_9 Q_{\theta_2} + z_{10} Q_{q_1} + z_4 Q_{q_2} \qquad (7d)$$

where:

$m_1 = I_T + 2\eta_1 q_1^2 + 2\eta_2 q_2^2 + 2\eta_6\cos\theta_2 - 2\eta_7 q_1\sin\theta_2 - 2\eta_8 q_2\sin\theta_2$ $\quad m_2 = \eta_5 + 2\eta_1 q_1^2 + 2\eta_2 q_2^2$

$m_3 = 2\eta_1$ $\quad m_4 = 2\eta_2$ $\quad m_5 = \eta_5 + 2\eta_1 q_1^2 + 2\eta_2 q_2^2 + \eta_6\cos\theta_2 - \eta_7 q_1\sin\theta_2 - \eta_8 q_2\sin\theta_2$

$m_6 = \eta_3 + \eta_7\cos\theta_2$ $\quad m_7 = \eta_4 + \eta_8\cos\theta_2$ $\quad m_8 = \eta_3$ $\quad m_9 = \eta_4$

$n_1 = \eta_6\sin\theta_2 + \eta_7 q_1\cos\theta_2 + \eta_8 q_2\cos\theta_2$ $\quad n_2 = 4\eta_1 q_1 - 2\eta_7\sin\theta_2$ $\quad n_3 = 4\eta_2 q_2 - 2\eta_8\sin\theta_2$

$n_4 = 4\eta_1 q_1 - 2\eta_7\sin\theta_2$ $\quad n_5 = 4\eta_2 q_2 - 2\eta_8\sin\theta_2$ $\quad n_6 = 2\eta_1 q_1 - \eta_7\sin\theta_2$ $\quad n_7 = 2\eta_2 q_2 - \eta_8\sin\theta_2$

$n_8 = 2\eta_1 q_1$ $\quad n_9 = 2\eta_2 q_2$ $\quad \alpha_2 = \int_0^{L_2}\Phi_2\,dx$ $\quad \alpha_4 = \int_0^{L_2} x\Phi_2\,dx$ $\quad \gamma_2 = \int_0^{L_2}\Phi_2^2\,dx$ $\quad \eta_2 = \dfrac{\rho A_2\gamma_2}{2}$

$$\eta_4 = \rho A_2 \alpha_4 \qquad \eta_8 = \rho A_2 L_1 \alpha_2 \qquad \Phi_2'' \Phi_2'' = \Phi_2^{iv} \Phi_2 = \left(\frac{\rho A_2 \omega_2^2}{EI_2} \Phi_2 \right) \Phi_2$$

In Eqs. (6) and (7), w_i are the elements of the inverse of the mass matrix for the one mode expansion equations coupled in the second derivatives and z_i are the elements of the inverse of the mass matrix for the two modes expansion equations coupled in the second derivatives.

The state dependent matrices A(x) and B(x) can assume several forms. For the numerical simulations presented here one such form is chosen. Since there is no guarantee that this form is te best one to be chosen, the results presented here are sub-optimal. Another disposition of the elements in matrices A(x) and B(X) for the same weighting matrices R and Q may produce better results or not. However, for systems of this complexity it is impossible to test all possibilities.

3 THE STATE-DEPENDENT RICCATI EQUATION (SDRE) CONTROL

The State-Dependent Riccati Equation (SDRE) approach to nonlinear system control relies on representing a nonlinear system's dynamics with state-dependent coefficient matrices that can be inserted into state-dependent Riccati equations to generate a feedback law [4].

The main idea of this method is to represent the nonlinear system:

$$\dot{x} = f(x) + B(x)u$$

in the form:

$$\dot{x} = A(x)\,x + B(x)u \tag{8}$$

The feedback law is given by:

$$u = -R^{-1}(x)\,B^T(x)\,P(x)\,x \tag{9}$$

where P(x) is obtained from the SDRE:

$$P(x)A(x) + A^T(x)P(x) + Q(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) = 0 \tag{10}$$

In Eqs. (9) and (10), Q(x) and R(x) are design parameters that satisfy the positive definiteness condition Q(x) > 0 and R(x) > 0. Eqs. (6) and (7) can be written in the state space form given by Eq. (8).

4 NUMERICAL RESULTS

The numerical integrator used in this work is the fourth order Runge-Kutta with a time step of 0.001 s.

In the numerical simulations, the SDRE control technique is used to bring the angles to the desired angular positions at zero degrees. At the same time that the controller is acting on the angles it is also used to eliminate the vibration on the flexible link.

Table 1 presents the parameters values used in the numerical simulations.

	Aluminum Beams		Rigid Beam			Flexible Beam		
Parameter	E	ρ	height (beam cross section)	width (beam cross section)	L_1	height (beam cross section)	width (beam cross section)	L_2
Value	$0.7\ 10^{11}$ N/m^2	2700 kg/m^3	0.030 m	0.030 m	0.300 m	0.030	0.001 m	2.000 m

Table 1: Parameters values used in the numerical simulations.

The initial conditions considered here intend to simulate the system in a situation into which there is a weak influence of the nonlinear terms and in a situation into which there is a strong influence of the nonlinear terms. These conditions are respectively presented in Table 2.

Initial Conditions	θ_1	$\dot{\theta}_1$	θ_2	$\dot{\theta}_2$	q_1	$\dot{q}_1$	q_2	$\dot{q}_2$
Case 1	10°	0 rad/s	10°	0 rad/s	0 m	0 m/s	0 m	0 m/s
Case 2	500°	0 rad/s	– 400°	0 rad/s	0 m	0 m/s	0 m	0 m/s

Table 2 – Initial condition assumed in the numerical simulations.

The time behavior of θ_1 is presented in Figure 2 for Case 1 and in Figure 3 for Case 2. The time behavior of θ_2 is presented in Figure 4 for Case 1 and in Figure 5 for Case 2. The time behavior of q_1 is presented in Figure 6 for Case 1 and in Figure 7 for Case 2. The time behavior of q_2 is presented in Figure 8 for Case 1 and in Figure 9 for Case 2. In each figure the one mode expansion model is compared with the two modes expansion model.

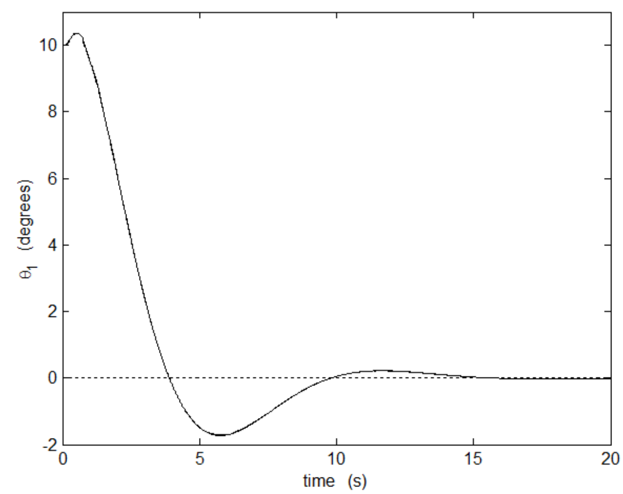

Figure 2 – Time behavior of θ_1 : (- - - -) one mode expansion and (——) two modes expansion – Case 1.

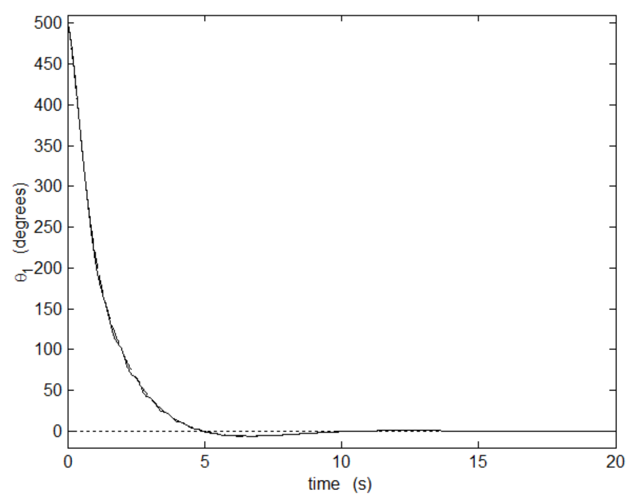

Figure 3 – Time behavior of θ_1 : (- - - -) one mode expansion and (——) two modes expansion – Case 2

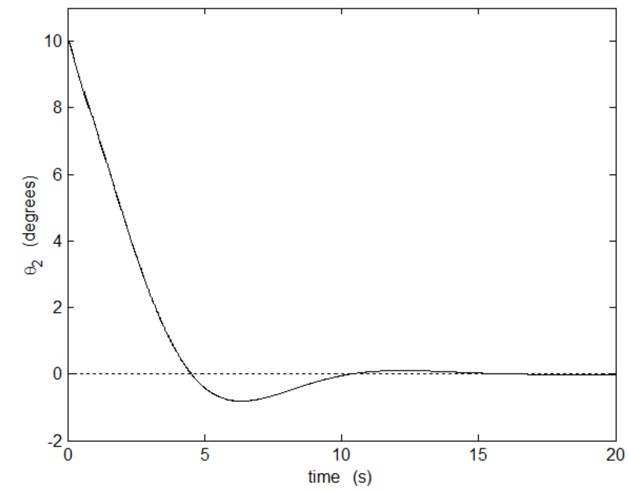

Figure 4 – Time behavior of θ_2 : (- - - -) one mode expansion and (——) two modes expansion – Case 1

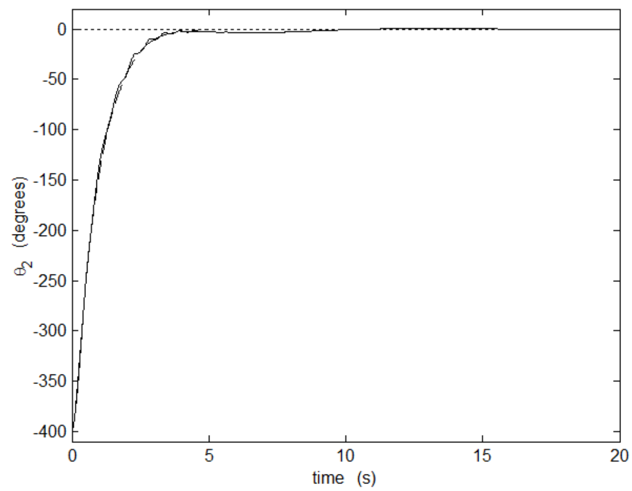

Figure 5 – Time behavior of θ_2 : (- - - -) one mode expansion and (——) two modes expansion – Case 2

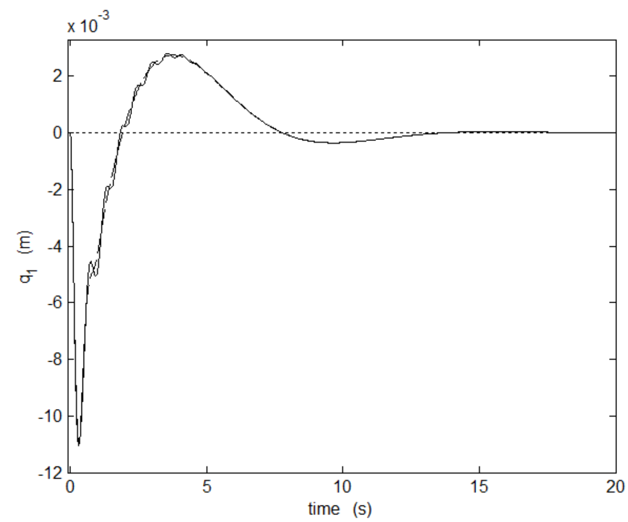

Figure 6 – Time behavior of q_1 : (- - - -) one mode expansion and (——) two modes expansion – Case 1

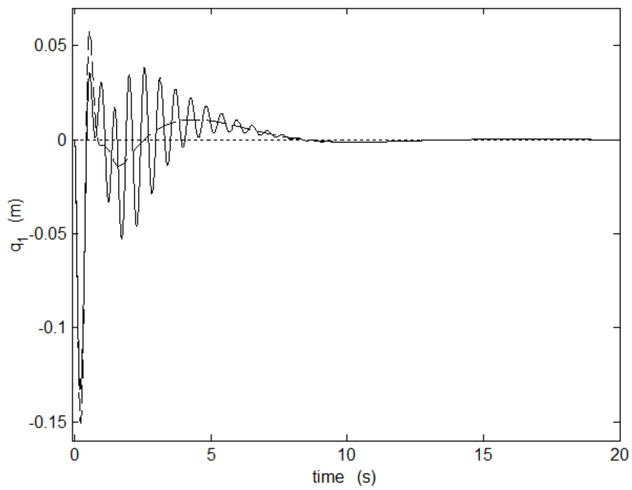

Figure 7 – Time behavior of q_1 : (- - - -) one mode expansion and (——) two modes expansion – Case 2

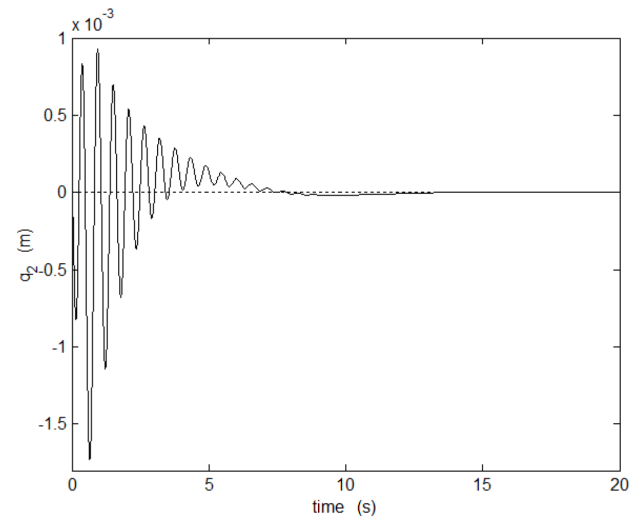

Figure 8 – Time behavior of variable q_2 : (——) two modes expansion – Case 1

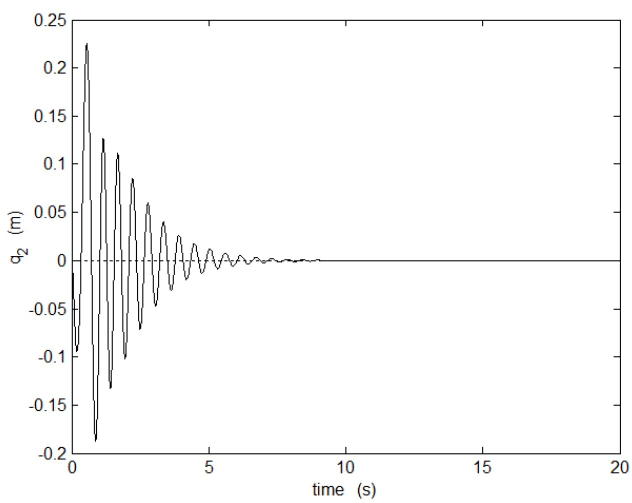

Figure 9 – Time behavior of variable q_2 : (——) two modes expansion – Case 2

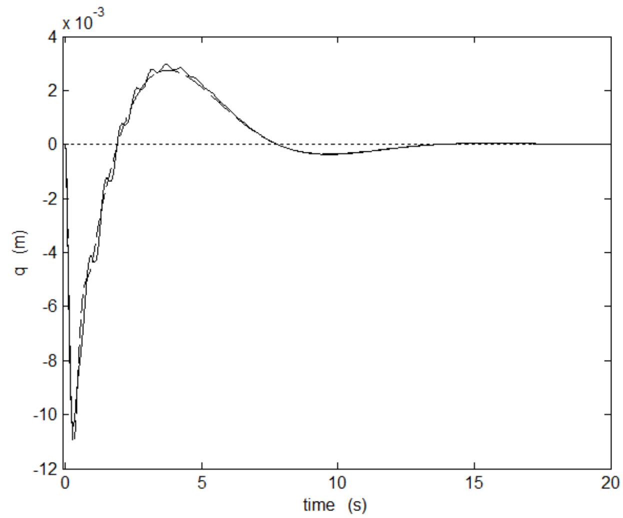

Figure 10 – Time behavior of q : (- - - -) one mode expansion and (——) two modes expansion – Case 1

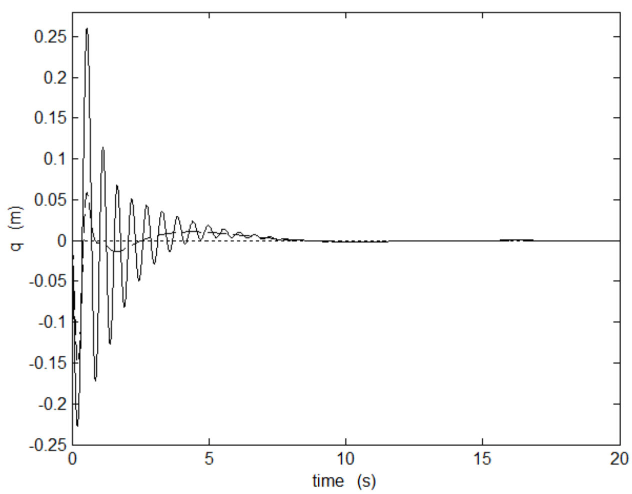

Figure 11 – Time behavior of q : (- - - -) one mode expansion and (——) two modes expansion – Case 2

As depicted in Figures 2 to 9, all the variables are satisfactorily controlled for Case 1 (weakly nonlinear condition) and for Case 2 (strongly nonlinear condition).

In Figures 10 and 11, the time behavior of q(t) is presented comparing the one flexural mode approach ($q = q_1$) with the two flexural modes approach ($q = q_1 + q_2$).

No friction forces or structural damping is considered here. All the amplitudes attenuation illustrated in the figures are due to the nonlinear control actuation only.

Figures 2 to 11 show that the influence of the second mode is relevant in Case 2 when the nonlinear terms in the governing equations of motion play an important role. For Case 2 the amplitude of deflection of the beam at the tip is approximately 20% of its length. This value is the limit for the linear curvature approach for the flexible beam considered here.

5 CONCLUSIONS

The results obtained in this work using the nonlinear control technique named SDRE for the rigid-flexible two link robot manipulator are completely satisfactory. The angular displacements of the slewing axes converge to the desired final values and at the same time the vibration on the beam (first and second modes) is eliminated. The initial conditions are chosen in order that the angular velocities may assume large values. The nonlinear terms in the governing equations have sufficiently high amplitudes to influenciate the system dynamics.

In the numerical simulations it is shown that the influence of the second mode in the system response is relevant when the nonlinear terms in the governing equations of motion have sufficiently high amplitudes. For the weakly nonlinear case this influence is negligible.

The elimination of the overshoot in the time responses of q_1 and q_2 and the inclusion of friction forces and structural damping are the next steps in the improvement of this research.

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