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EXPERIMENTAL AND FINITE ELEMENT MODAL ANALYSIS OF VARIABLE STIFFNESS COMPOSITE LAMINATED PLATES

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Abstract. *When the fibre paths of a fibre reinforced laminate are curvilinear, the stiffness changes in space and the laminate can be designated as a Variable Stiffness Composite Laminate. Experimental analysis is here carried out on a Variable Stiffness Composite Plate with curvilinear fibres, in order to extract the modes of vibration. Free boundary conditions are simulated by hanging the plate so that it stands vertically and the plate is excited with random excitation via an electromagnetic shaker. Acceleration is measured by a light-weight piezoelectric accelerometer; a drive rod and a piezoelectric force transducer connect the shaker to the plate. Frequency response functions are obtained in a pre-defined grid and, from these frequency response functions, natural frequencies, modal dampings and mode shapes are identified. The experimental results can be used for model validation. Here, we compare them with the modes computed by a Finite Element Method approach and quite good agreement is achieved.*

1 INTRODUCTION

Although fibre reinforced composite materials frequently have straight fibres, leading to stiffness that does not vary in the laminate domain, this single direction of the fibres in each layer is not obligatory. With the tow-placement technology available today, reinforcement fibres can be disposed with curvilinear paths [1, 2]. In these composite materials, the stiffness changes in the laminate, and therefore they can be designated as Variable Stiffness Composite Laminates (there are other means of building VSCL, which are not addressed here). VSCL can be designed so that they better withstand specific demands, increasing buckling loads, avoiding failure or changing the vibration frequencies [3-6].

Laminates with curvilinear fibres seem to be progressively more common, with the number of institutions holding Advanced Fibre Placement machines apparently increasing. The practical interest of VSCL is large, particularly in aeronautics, because the expanded design space can lead to weight reductions and to performance improvements. Provided that a decrease in manufacturing costs is achieved, VSCL laminates may well become more common in other applications. There are disadvantages nevertheless; a few of them are related with manufacturing issues, another one is the cost of these laminates. Technological issues to solve include: reducing gaps and overlaps; reducing resin rich areas and fibre discontinuity; avoiding fibre kinking.

Even though there is a growing body of research on VSCL and although thin laminated panels are frequently used in applications where vibrations are of concern, there is a lack of experimental vibration analysis on VSCL. In this paper, results from experimental analysis carried out on a Variable Stiffness Composite Plate, in order to extract the modes of vibration, are shown. The data obtained can be employed for model validation. The modes of vibration obtained experimentally are compared with the ones defined by an *h*-version finite element method, based on Reissner-Mindlin theory. The finite element model assumes that the fibre distribution is ideal – that is, perfectly shifted fibres, no gaps or overlaps –, hence the comparison with the real plate gives us a measure on the effect of these "imperfections" on the natural frequencies and mode shapes of a VSCL plate.

2 PLATE PROPERTIES AND EXPERIMENTAL SET-UP

The plate analysed is a rectangular plate with 10 layers and the following geometric properties: a = 0.4 m, b = 0.3 m, h = 0.00175 m. The thickness is represented by h and, as shown in Figure 1, a represents the length and b the width.

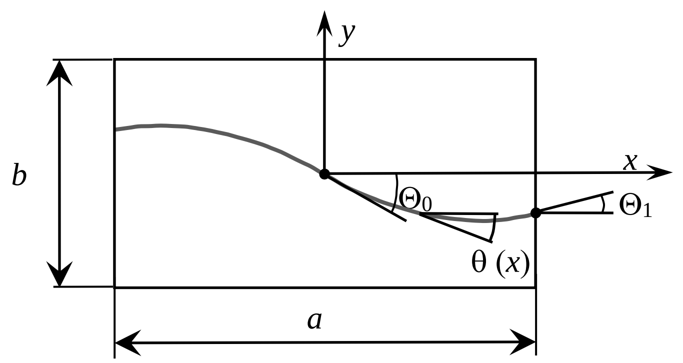


Figure 1: Curved fibre and plate dimensions. In this figure Θ_0 and $\theta(x)$ are negative.

The angle the fibre makes with the x axis changes linearly with x, so that the angle in lamina k is defined as

$$\theta^k(x) = \frac{2\left(\Theta_1^k - \Theta_0^k\right)}{a}|x| + \Theta_0^k \quad (1)$$

Θ_0^k gives the angle between the fibre and the x axis at x=0, and Θ_1^k gives this angle at $x=\pm a/2$. This fibre path is represented by $< \Theta_0^k \,|\, \Theta_1^k >$. The layup of the plate here analysed is the following: (<30|10>, <-30|-10>, <30|10>, <-30|-10>, <90|90>)$_s$.

The material used is Hexply AS4/8552, a high performance material for aerospace structures. Regarding the material properties, the following values were specified by the manufacturer (E_1 and E_2 are average values): longitudinal modulus E_1 = 126.3×10^9 GPa, transverse modulus E_2 = 8.765×10^9 GPa, in-plane shear modulus G_{12} = 4.92×10^9 GPa. The mass density was obtained by weighting the plate and calculating the mass per unit volume; it is approximately ρ= 1600 kg/m^3. As values for the other properties are not available, we assumed values based on their usual (from the literature) relation with the properties available: v= 0.31 (Poisson’s ratio), and G_{13} = 4.92×10^9 GPa, G_{23} = 3.35×10^9 GPa (transverse shear moduli).

Figure 2: Plate hanging.

Free boundary conditions were simulated by hanging the plate so that it stands vertically, as represented in Figure 2. After some tests with a hammer, performed to obtain an idea of the values of the first natural frequencies, it was decided to extract modes with frequencies ranging from 0 to 200 Hz. For that purpose, the plate was excited with random excitation via an electromagnetic shaker, which was also hanged in strings and another aluminium frame. A drive rod and a piezoelectric force transducer, Bruel & Kjaer model 8203, connected the shaker to the plate, so that the force was applied perpendicularly to it. A grid of 9×9 points was defined and the acceleration measured by a light-weight piezoelectric accelerometer, model 27 A11 of Endeveco. The frequency response functions were obtained in this pre-defined grid. An example of the FRFs obtained can be seen in Figure 3, which represents the accelerance that corresponds to an acceleration at the point where the excitation was applied, hence the characteristic anti-resonances. From these eighty-one FRFs, natural frequencies, modal dampings and mode shapes were identified using a modal analysis software.

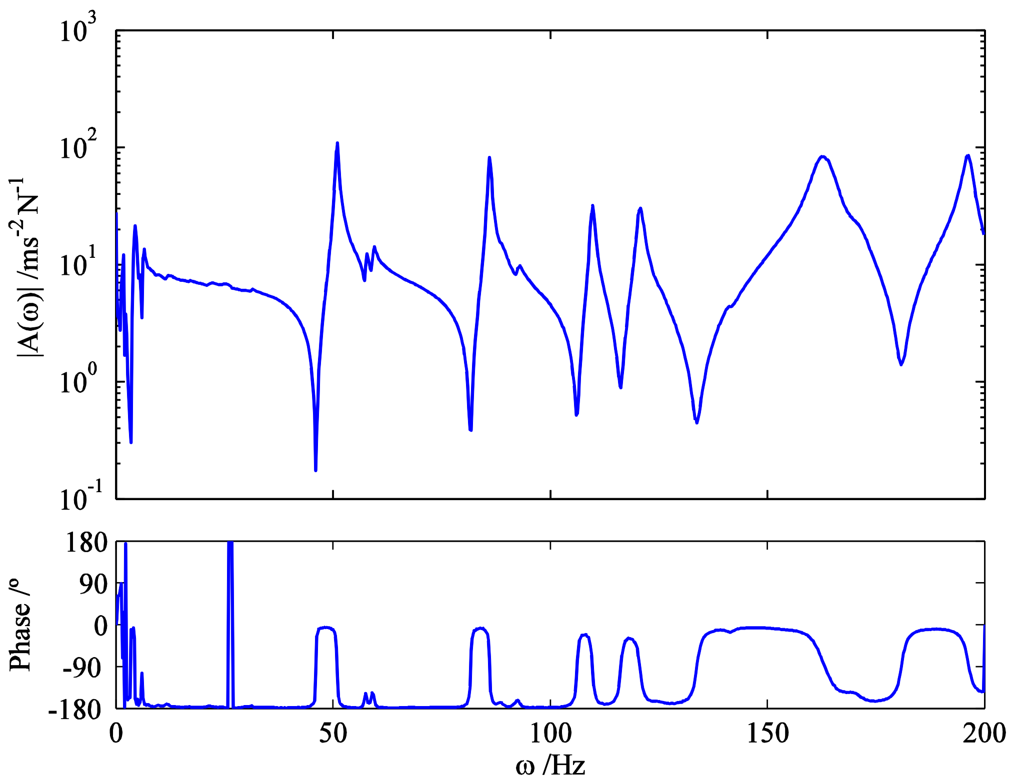


Figure 3: Point accelerance (force and acceleration on the same point).

3 MODES OF VIBRATION

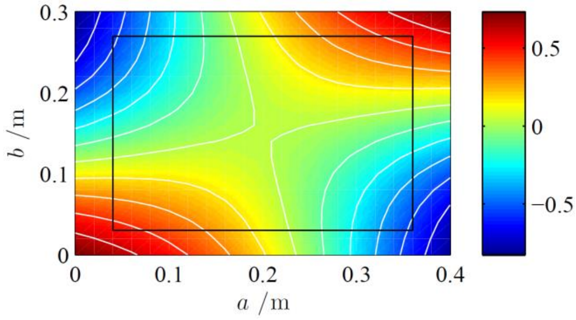
In this section, we compare the modes of vibration obtained experimentally with the ones of the finite element method. The finite element employed is a 4-node isoparametric quadrilateral element, based on an equivalent single layer, Reissner-Mindlin approach, with a shear correction factor equal to 5/6. It has five degrees of freedom per node (three displacement components, and two rotations about the in-plane axes). Selective integration is employed, i.e., full integration is used for bending and membrane terms and reduced integration is used in shear. The value of the fibre orientation is taken into consideration at the integration points.

Table 1 presents the first seven frequencies computed using finite elements alongside the identified natural frequencies. Two finite element meshes were used: 20 per 15 and 40 per 30 elements. The column on the right-hand side of the table contains the relative difference between the numeric values, computed with the 40×30 mesh, and the experimental values.

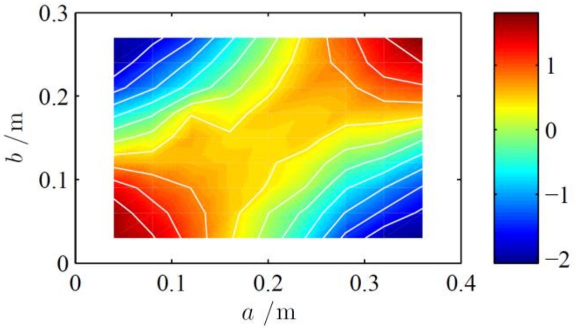
Mode	Frequencies (Hz)			Relative difference (%)
	FEM 20 × 15	FEM 40 × 30	Experimental	
1	47.194	47.126	50.897	-7.4
2	49.658	49.449	57.522	-14
3	84.704	84.279	85.777	-1.7
4	106.63	106.16	109.28	-2.9
5	116.33	115.65	120.14	-3.7
6	162.34	159.40	162.36	-1.8
7	201.23	199.34	195.64	1.9

Table 1: Example of the construction of one table.

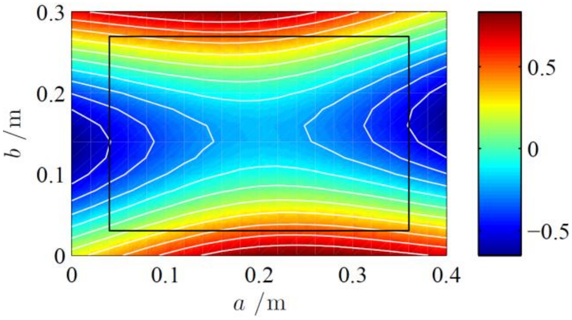
In Figure 4 one can see side by side the mode shapes computed via finite element analysis and the ones identified from experiments.



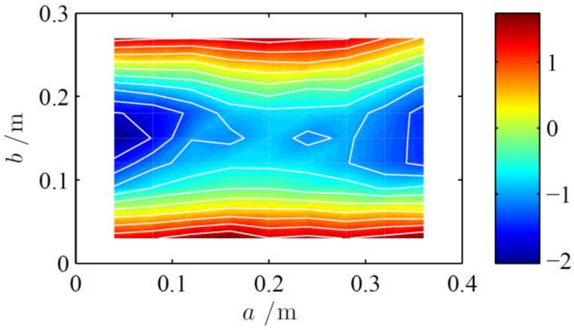
Mode shape 1 - FEM



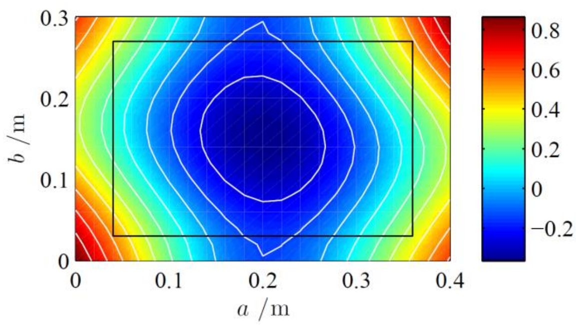
Mode shape 1 - experimental



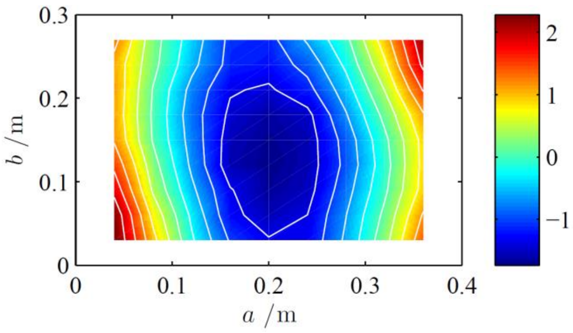
Mode shape 2 - FEM



Mode shape 2 - experimental

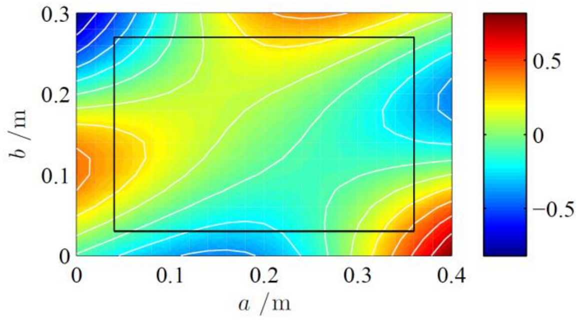


Mode shape 3 - FEM

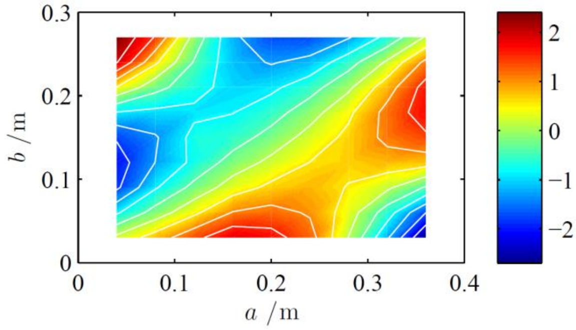


Mode shape 3 - experimental

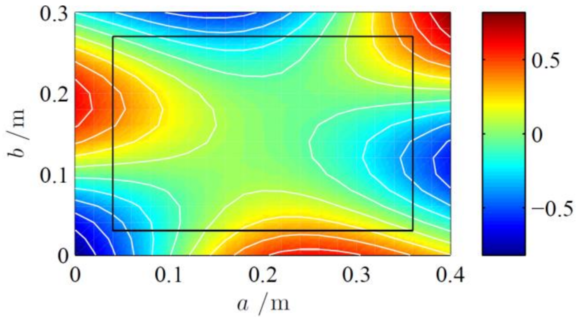
Figure 4 (continues): Numerical and experimental vibration mode shapes.



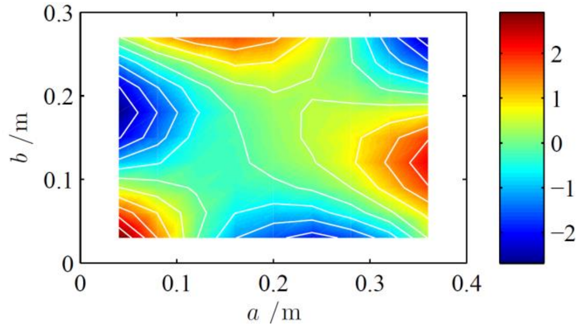
Mode shape 4 - FEM



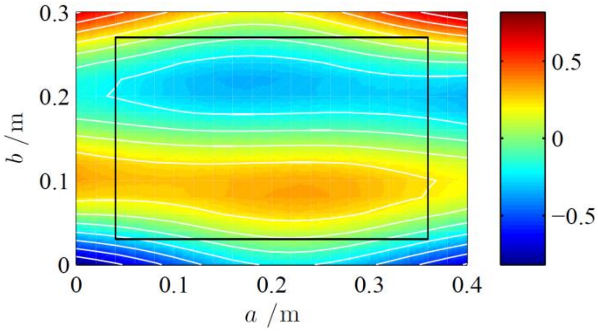
Mode shape 4 - experimental



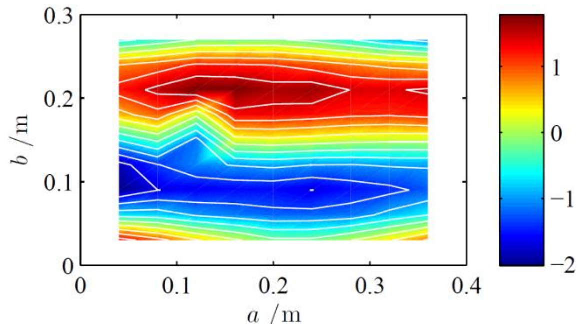
Mode shape 5 - FEM



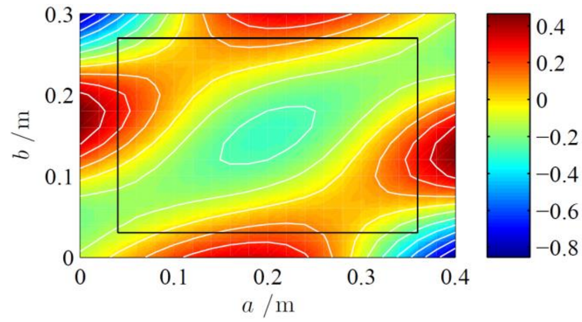
Mode shape 5 - experimental



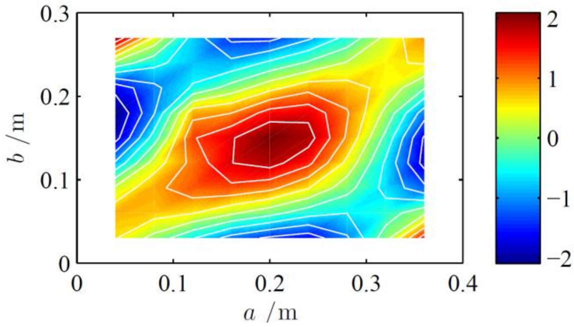
Mode shape 6 - FEM



Mode shape 6 - experimental



Mode shape 7 - FEM



Mode shape 7 - experimental

Figure 4 (conclusion): Numerical and experimental vibration mode shapes.

Most natural frequencies are rather well predicted by the finite element model. The exception is the natural frequency of the second mode, where a difference of 14% between theory and experiments was found. This may be explained by the particularity that the excitation point is not far from a nodal line of this mode. Presently, experiments and identifications are ongoing using another excitation point, in order to verify this. Other reasons justify the differences encountered. One of them regards the material constants: the in-plane elasticity and shear moduli are tabulated, they should be approximately correct, but it was not possible to verify if they are completely accurate; the transverse shear moduli were not given by the supplier and we guessed the values. The second reason is related with the orientation of the fibres. In the model it is assumed that the fibres are perfectly shifted, all obeying the same analytical formula, but in truth this does not occur, as only the central tow is shifted, the other tows are parallel to it. The real plate shows gaps and overlaps that are not in the theoretical model. Finally, there are always errors in finite element discretizations, although the two meshes here compared indicate that these are negligible.

4 CONCLUSION

- Experimental modal analysis was performed on a Variable Stiffness Composite Plate with curvilinear fibres, in order to extract the modes of vibration. Free boundary conditions were simulated by hanging the plate and the modal parameters were extracted from the frequency response functions measured.
- A finite element model was put together for the analysis of the same plate.
- Although the gaps and overlaps, which exist in the real plate, were not taken into account in the model, the model generally provided natural frequencies and mode shapes close to the experimental ones. The only exception is the natural frequency of the second mode. A new identification is ongoing to verify if this is solely a consequence of the actual differences between the real plate and the idealized one.
- The modes shapes and natural frequencies identified via experimental modal analysis can be used by other researchers, to verify their models.

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