11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

PARAMETRIC STUDY OF NONLINEAR BEHAVIOR OF JENKINS ELEMENT USING HARMONIC BALANCE METHOD

Hamed Kashani*[1]

[1]Aerospace Research Institute (ARI)
kashani@ari.ac.ir

Keywords: Jenkins element, Harmonic balance, Dynamic stiffness.

Abstract. *Jenkins element, composed of a linear spring in series with a coulomb slider, is frequently used in modelling of bolted and riveted joints. Stick-slip phenomenon in coulomb slider causes the behaviour of the Jenkins element to be nonlinear. In this paper, first, third and fifth order harmonic balance are employed to study the behaviour of the element. Dynamic stiffness estimated by harmonic balance is investigated when threshold of friction and stiffness of spring vary.*

1 INTRODUCTION

Joints are the main parts of the most structures. They are used to connect substructures to each other to construct the main structure. There are several types of joints e.g. bolted, riveted and adhesive joints. Micro/macro stick-slip is a phenomenon which may occur in joints in which two parts of structure are connected together using normal pressure like two first mentioned types. Sticking or slipping due to dry friction in joints causes the dynamic behaviour of the structure to be nonlinear[1]. Having knowledge about the nonlinear behaviour of joints is required for estimating or controlling the response of structure.

Iwan's model[2] is a common dynamical model for bolted or riveted joints, this model has been shown in Figure 1. This model is composed of a number of Jenkins elements, shown in Figure 2, in parallel arrangement. To identify the joint dynamic characteristics using Iwan's model it is necessary to identify the behaviour of Jenkins element as constructive component of Iwan's model.

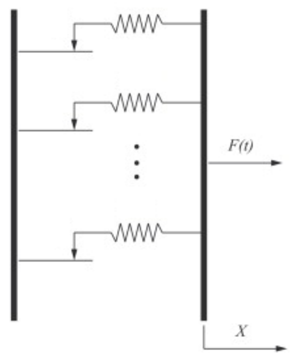

Figure 1: Iwan's model [2].

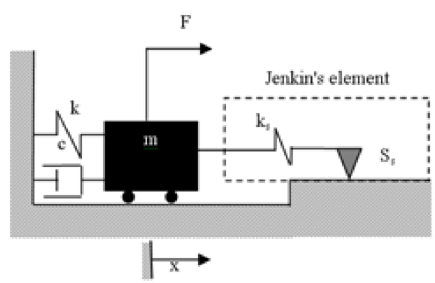

Figure 2: Jenkins element in a SDOF system.

Here, Harmonic Balance Method (HBM) is employed to extract the dynamic stiffness of Jenkins element. First order HBM, used for Jenkins element in[3], third and fifth orders are investigated here. Also, the effect of Jenkins parameters, friction threshold S_s and spring stiffness k_s , on the dynamic stiffness is described.

2 HARMONIC BALANCE ESTIMATION OF DYNAMIC STIFFNESS

To determine the dynamic stiffness of Jenkins element using HBM, it is necessary to know how this element behaves. Force-displacement curve of this element has been presented in Figure 3. As seen in this figure loading (ABC path) and unloading (CDA path) parts of this curve do not coincide to each other. On the other hand bilinear variation of force vs. displacement causes the overall system behaviour become nonlinear.

2.1 First order harmonic balance

In first order HBM it is assumed that response of a nonlinear system to a harmonic excitation is harmonic with the same frequency as excitation. If the force in Figure 2 is a harmonic as $f\,(t) = F\sin\omega t$ then the displacement, according to first order HBM assumption, will be as follows.

$$x\,(t) = X\,\sin(\omega t - \varphi) \tag{1}$$

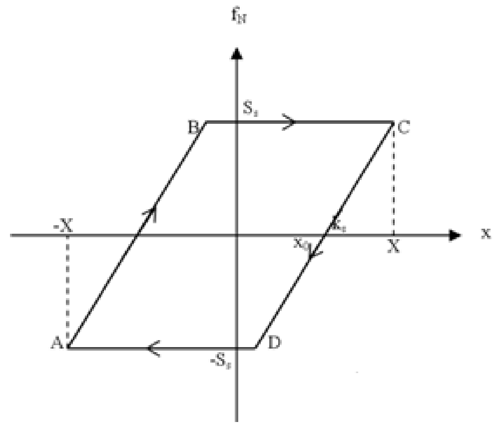

Figure 3: Force-displacement curve of Jenkins element.

Considering harmonic excitation and assuming harmonic response in equation of motion of SDOF system of Figure 2 implies that the contribution of Jenkins element is a harmonic with frequency ω. Therefore,

$$f_j\,(t) \simeq a^*\cos\omega t + b^*\sin\omega t \tag{2}$$

In fact, Eq. (2) is the first harmonic approximation of Fourier expansion of f_j. The coefficients of the expansion can be calculated as Eq.(3).

$$\begin{aligned} a^* &= -\frac{4(2S_s - k_sX)\sqrt{S_s(k_sX - S_s)} + k_s^2X^2(\pi + 2\beta)}{2\pi k_sX}\sin\varphi + \frac{4S_s(k_sX - S_s)}{\pi k_sX}\cos\varphi \\ b^* &= +\frac{4(2S_s - k_sX)\sqrt{S_s(k_sX - S_s)} + k_s^2X^2(\pi + 2\beta)}{2\pi k_sX}\cos\varphi + \frac{4S_s(k_sX - S_s)}{\pi k_sX}\sin\varphi \end{aligned} \tag{3}$$

Where, $\beta = \arcsin(2S_s/k_sX - 1)$. For introducing the parameter β it is necessary to determine the times related to the corners of hysteresis in a cycle of motion. Putting the displacement related to each corner in Eq. (1) and solving for corner times leads to following expressions.

$$\begin{aligned} \omega t_A - \varphi &= 2k'\pi - \frac{\pi}{2} \\ \omega t_B - \varphi &= 2k'\pi + \beta \\ \omega t_C - \varphi &= 2k'\pi + \frac{\pi}{2} \\ \omega t_D - \varphi &= (2k'+1)\pi + \beta \end{aligned} \tag{4}$$

Where, $k' = 0, \pm 1, \pm 2, \pm 3, \cdots$. It is clear that, Jenkins element is in stick condition on the paths AB and CD and is in slip condition on the paths BC and DA. Figure 4 shows the stick-slip switching time (angle) on unit triangle circle. Now, β can be defined using Figure 4. Increasing β indicates the stick duration increases in a cycle of motion.

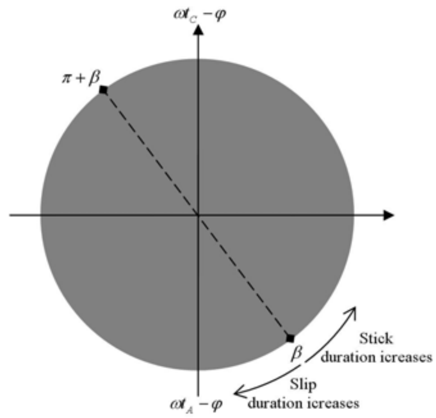

Figure 4: Stick-slip switching time (angle).

By substituting Eq. (3) into Eq. (2) and putting the resultant into equation of motion, the contribution of Jenkins element in the dynamic stiffness of overall system can be extracted as Eq. (5).

$$Z = \frac{4\left(2S_s - k_s X\right)\sqrt{S_s\left(k_s X - S_s\right)} + k_s^2 X^2\left(\pi + 2\beta\right)}{2\pi k_s X^2} + i\,\frac{4S_s\left(k_s X - S_s\right)}{\pi k_s X^2} \tag{5}$$

Real and imaginary parts of Eq. (5) are stiffness and damping terms, respectively. As seen in this equation first order estimation of stiffness and damping are functions of response amplitude so, they are nonlinear. For $X < S_s/k_s$ Jenkins element is completely in stick condition so, it exposes only stiffness effect and Eq. (5) is not valid for this situation. Figure 5 shows the real and imaginary parts of first HB order estimation of dynamic stiffness of Jenkins element. Maximum stiffness is occurs at $X = S_s/k_s$, at this amplitude Jenkins element has no damping efficacy. As response amplitude increases, stiffness decreases but damping coefficient increases until $X = 2S_s/k_s$. At this response amplitude damping coefficient reach to maximum value $\eta_{\max} = k_s/\pi$. After this peak damping coefficient decreases.

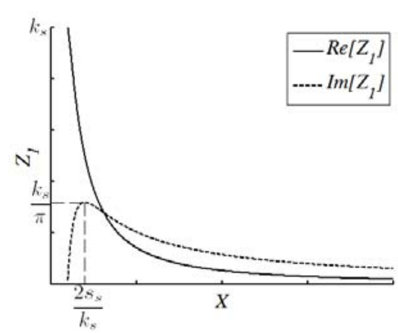

Figure 5: First order HB estimation of dynamic stiffness of Jenkins element.

Figure 6 shows the effect of variation of k_s on dynamic stiffness when S_s is constant and vice versa. In segment (a) of this figure it is seen that sensitivity of real part of dynamic stiffness is very low to the variation of k_s. Despite the stiffness of Jenkins' spring increase, real part decreases more rapidly, with increasing k_s. In fact, real part exhibits a weak softening behaviour as k_s increases. This is due to that the slip occurs at smaller response amplitude. Increasing S_s causes the slider remains at stick condition for greater fraction of a cycle of motion therefore, real part of dynamic stiffness has hardening behaviour in this situation as seen in segment (b). Segment (c) expresses the variation of imaginary part of dynamic stiffness when k_s increases. Imaginary part, or damping coefficient, grows as k_s increases. As seen in Figure 5, the peak of damping coefficient increases, linearly vs. k_s. The amplitude of peak occurrence decreases with inverse of k_s. Variation of S_s can not affect the magnitude of damping coefficient or imaginary part of dynamic stiffness but, the amplitude of peak occurrence incerases linearly with increasing S_s as shown in segment (d).

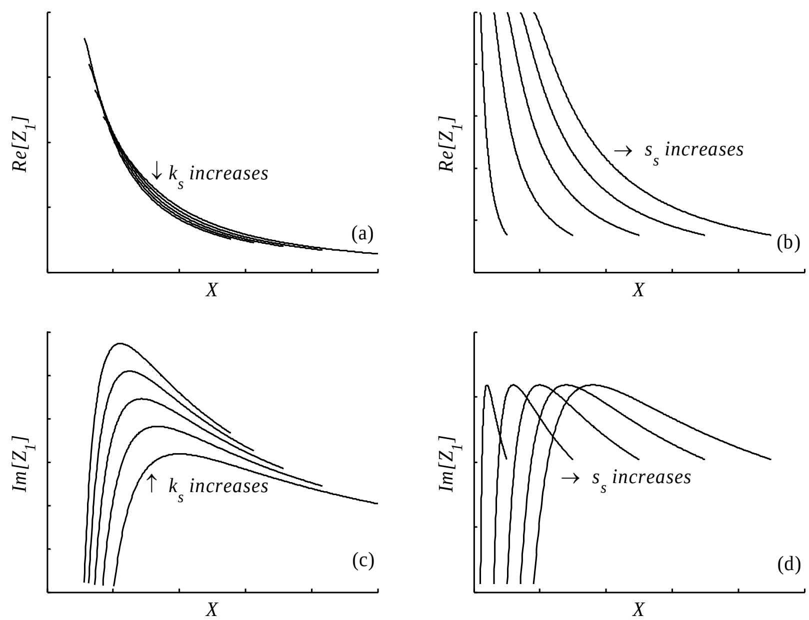

Figure 6: Sensitivity of 1st order HB estimation of dynamic stiffness to Jenkins parameters, a) stiffness sensitivity relative to k_s, b) stiffness sensitivity relative to s_s, c) damping sensitivity relative to k_s, d) damping sensitivity relative to s_s.

2.2 Third order harmonic balance

Third and fifth order HB lead to complicated and huge expression so, here they are not written but their behavior presented graphically. Real and imaginary parts of 3rd order dynamic stiffness are shown in Figure 7. After here horizontal dashed line in graphs indicates the zero level. Real and imaginary parts of dynamic stiffness become negative in some regions of response amplitude.

Figure 8 gives a comparison between the contributions of different orders in overall dynamic stiffness. In segment (a), absolute of real part of the 3rd order estimation is at least ten times smaller than 1st order one. Segment (b) shows that absolute of 3rd order estimation smaller than 1st order one by three times near peak value.

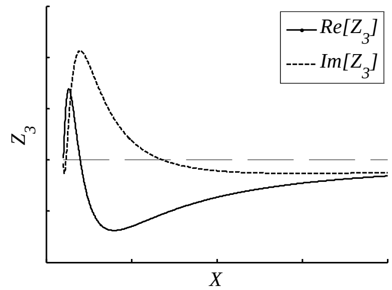

Figure 7: Third order HB estimation of dynamic stiffness of Jenkins element.

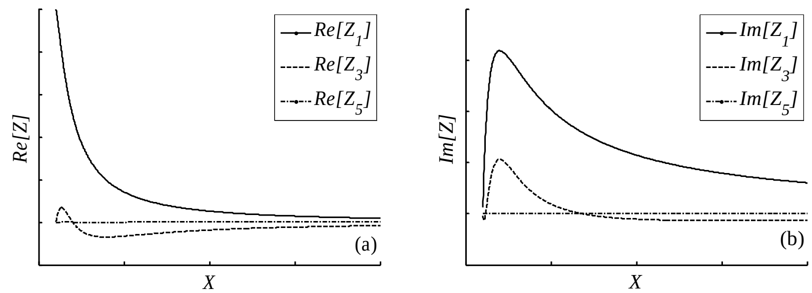

Figure 8: Comparison of dynamic stiffness of different orders, a) real part, b) imaginary part.

Against the real part of 1st order estimation, real part of 3rd estimation is sensitive to k_s variations. As k_s increases real part of dynamic stiffness increases in both positive and negative sides and crossing the zero line takes place at smaller response amplitude as presented in Figure 9(a). If S_s increases, maximum and minimum of 3rd order estimation of real part of dynamic stiffness will not change but, crossing the zero line happens at greater response amplitude. This is shown in segment (b) of Figure 9. Segment (c) expresses that damping coefficient get to a higher peak and greater positive and negative slope when k_s increases. Segment (d) presents the behaviour of imaginary part of 3rd order of dynamic stiffness as S_s increases. Minimum and maximum do not change but, crossing the zero line happens at greater response amplitude.

2.3 Fifth order harmonic balance

Figure 10 shows the real and imaginary parts of 5th order estimation of dynamic stiffness as a function of response amplitude. As seen in this figure number of oscillations around zero line increases relative to two other previous orders. Having a glance on the Figure 8 shows that the magnitude of 5th order dynamic stiffness is much smaller than 3rd and especially, than 1st order estimation. So, fifth and higher orders may have less importance in comparison with two first ones.

Sensitivity of dynamic stiffness to the Jenkins parameters variations is presented in Figure 11. Increasing k_s causes the response amplitudes, in which real part of dynamic stiffness changes its sign, decreases as seen in segment (a). Segment (b) shows, in addition to previous effect, increasing S_s increases the maximums and minimums of real part of dynamic stiffness. Similar effects can be seen in segments (c) and (d) for imaginary parts of 5th order dynamic stiffness estimation.

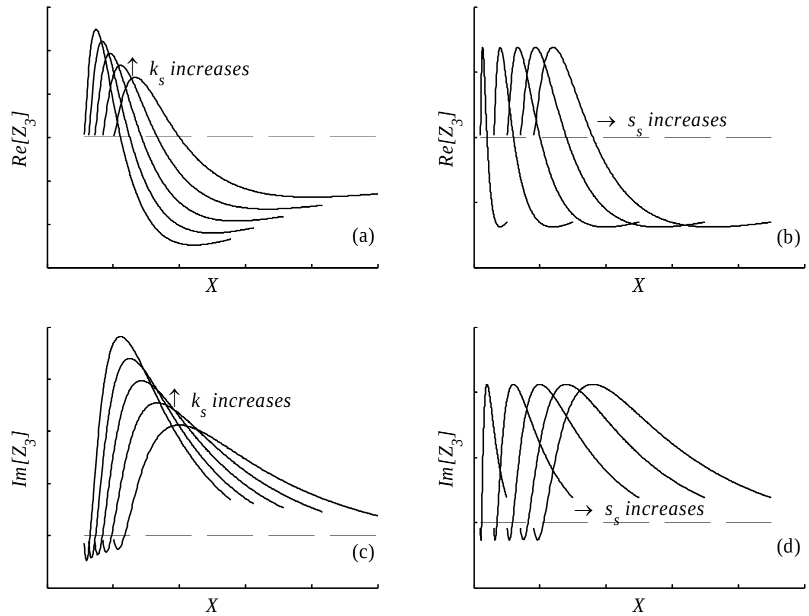

Figure 9: Sensitivity of 3rd order HB estimation of dynamic stiffness to Jenkins parameters, a) stiffness sensitivity relative to k_s, b) stiffness sensitivity relative to s_s, c) damping sensitivity relative to k_s, d) damping sensitivity relative to s_s.

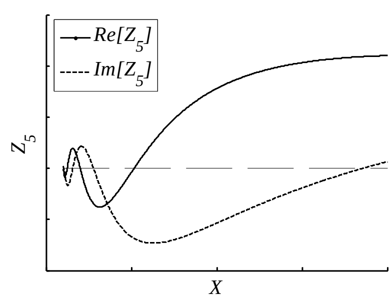

Figure 10: Fifth order HB estimation of dynamic stiffness of Jenkins element.

3 COMPARING THE CONTRIBUTION OF EACH ORDER IN OVERALL DYNAMIC STIFFNESS

It was shown in Figure 8 fifth order dynamic stiffness has negligible contribution in overall dynamic stiffness. It was seen that third and higher order dynamic stiffness are negative in some regions of response amplitude but overall dynamic stiffness composed of three first orders is positive for both real and imaginary parts, as seen in Figure 12. Since the magnitude of higher order decreases rapidly with increasing order, their presence in overall dynamic stiffness cannot make it negative.

4 CONCLUSIONS

- Real part of first order dynamic stiffness, which exhibits stiffness-like behaviour, decreases with the inverse of response amplitude. Imaginary part of dynamic stiffness, which exhibits damping-like behaviour, reaches to a maximum at certain value of response amplitude.

- Third and fifth order dynamic stiffness oscillate around the zero line before settling down. Negativity of stiffness and damping terms in higher order dynamic stiffness can make the system unstable but, since their magnitude is smaller than first order they are not dominant and overall dynamic stiffness is positive in real and imaginary parts.

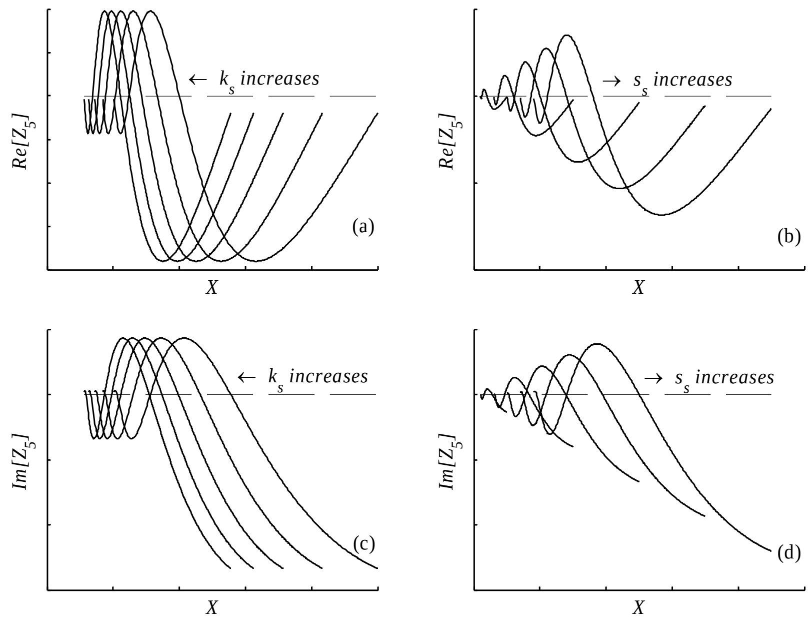

Figure 11: Sensitivity of 5th order HB estimation of dynamic stiffness to Jenkins parameters, a) stiffness sensitivity relative to k_s , b) stiffness sensitivity relative to s_s , c) damping sensitivity relative to k_s , d) damping sensitivity relative to s_s .

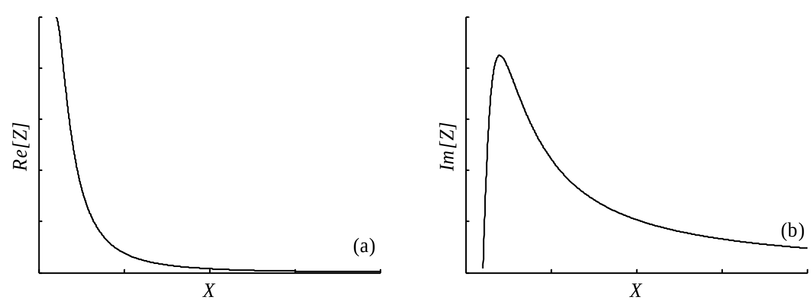

Figure 12: Overall dynamic stiffness composed of three first orders, a) real part, b) imaginary part.

REFERENCES

[1] L. Gaul, R. Nitsche, The role of friction in mechanical joints. *Applied Mechanics Review*, 54, 93-106, 2001.

[2] Y. Song, C. J. Hartwigsen, D. M. McFarland, A. F. Vakaki, L.A. Bergman, Simulation of dynamics of beam structures with bolted joints using adjusted Iwan beam elements. *Journal of Sound and Vibration*, 273, 249-276, 2004.

[3] H. Kashani, A. S. Nobari, Identification of dynamic characteristics of nonlinear joint based on the optimum equivalent linear frequency response function. *Journal of Sound and Vibration*, 329, 1460-1479, 2010.