11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

CRACK IDENTIFICATION IN A BEAM-LIKE STRUCTURE USING THE PROBABILISTIC APPROACH AND TIMOSHENKO BEAM MODEL

Sergey N. Shevtsov [*1], Igor V. Zhilyaev [1], Vladimir A. Akopyan [2], Michail B. Flek [3]

[1] Aircraft Engineering Dept., South Center of Russian Academy
sergnshevtsov@gmail.com

[2] Fracture Mechanics Dept., Institute of Mechanics & Applied Mathematics
akop@math.rsu.ru

[3] Rostvertol Helicopters
michflek@gmail.com

Keywords: Crack identification, Modal analysis, Timoshenko beam, Finite element model, Data uncertainty.

Abstract. *Among the experimental methods of cracks identification there is acoustic modal analysis, which uses different changes of the natural vibration modes of structure caused by presence of crack. The problem of possible crack's parameters estimation is an inverse problem, where the observational data regarding the performance of a system are known, and the characteristics of the system are sought. Because all inverse problems are underdetermined, two different models of structure may predict the same natural mode shape, and data of measurements will never allow to resolve small features of the structure's model. Moreover, there are always experimental uncertainties that allow different models to be acceptable.*

The emphasize of the presented paper is considering of measurement errors, different accelerometers and strain gauges positioning, and also numerical differentiation of the natural vibrations mode shapes as the sources of uncertainty, which affects on the crack identification and positioning results. We assume that a physical theory to solve the forward problem is the Timoshenko beam model with opened crack. With 1D cracked beam finite element model at the different position and depth of the crack we reconstruct some first mode shapes on the fixed points on the beam surface. All measured values are noisy by randomly distributed errors. Once these modes are reconstructed, we calculate their second spatial derivatives (curvatures) which are more sensitive to the presence of crack, but significant uncertainty is inherent for them because of measurements errors and numerical differentiation.

To obtain the probabilistic means for make decision about the damage, the following procedure is performed many times with both intact and cracked beams. We assume that biggest discrepancy between these two modes (their 1st and 2nd derivative) is located close to the probable crack. Next we calculate the histograms and empirical probability distributions which allow to estimate a probability of crack location and to distinguish the cases of presence and absence of defect.

1 INTRODUCTION

Structural Health Monitoring is an important area of research in engineering science because it can increase the safety and reliability and extend the life of structures through condition-based performance and maintenance strategies. So, the damage sensitive and reliable structural health monitoring in industrial and civilian applications attracts considerable research and engineering efforts. The detailed surveys [1-3] categorized all nondestructive damage identification methods as either local or global damage identification techniques. Local damage identification techniques, such as ultrasonic methods and X-ray methods, require that the vicinity of damage is known a priori and readily accessible for testing, which cannot be guaranteed for most cases. The vibration-based damage identification method can overcome these difficulties and commonly used in the practice for global damage identification.

Most researchers classify all damage identification methods as 'model-based method' or 'response based method'. The model-based method assumes that some mathematical model of the structure is assumed for damage identification; while the response-based method depends only on experimental response data from structures. A wide range of damage identification problems that state as 'model-based' can be considered as inverse problems, where data from indirect measurements are used to estimate geometric location and/or severity of the damage, and, probably, to predict the remaining service life of the studied structure [1]. There are many reasons that make the inverse problem underdetermined (nonunique), and most important are the existence of experimental uncertainties [4].

Inverse problems can be solved using a deterministic approach. In this approach, the objective is to find a specific model of a system that its theoretical response best fits the observed data. From the mathematical point of view the detection of damage with model-based methods is a constrained nonlinear optimization problem. But solution of such optimization problems is highly sensitive to experimental noise or numerical errors [5, 6].

In this paper we use the probabilistic approach which characterizes the uncertainties by the *probability* associated with *events* [7]. An event corresponds to any of the possible states a physical system can assume, or any of the possible predictions of a model describing the system. The *probability* of an event will be interpreted in terms of the frequency of occurrence of that event. When a large number of *samples* or *experiments* are considered, the probability of an event is defined as the ratio of the number of times the event occurs to the total number of samples or experiment. So, using a probabilistic approach to the damage detection, we consider each variable in terms of its probability distribution.

Despite the damage identification is actual for many different structures, most often subjects of investigation are the beam-like structures which is very important in aircraft, civilian, and many others application [8-15]. In the works cited above most frequently used for damage identification the Euler–Bernoulli beam theory. Because this theory over-predicts natural frequencies in short beams it is only applicable to a slender beam-type structure [3].

Modeling of the crack in the beam as the massless rotational spring is used very often [16-18]. The authors modeled the effect of location and depth of the crack on the vibratory response of the beam through formulating the loss of energy due to the presence of the crack (i.e. rotational spring). Such crack representation also used in [14], where the method for detection of crack location in short beams is based on the frequency shift measurements, taking into account the effects of shear deformation and rotational inertia through the Timoshenko beam theory. It is known that modeling of crack as a rotational spring based on fracture mechanics will lose its credibility in high frequency modes or deep crack cases [3]. But the biggest drawback of this approach is that the choice of the spring stiffness is quite arbitrary. To eliminate this ambiguity the authors of this paper first proposed the Timoshenko beam model with opened crack, which modeled as the sharp variation of the beam cross-section [13]. 1D FEM implementation of this model show a good capabilities and small computation cost at multiple simulation of the forward problem. Hereafter such model of the damaged structure also used.

The vibration-based methods can be classified by the monitored value into four major categories: natural frequency-based methods, mode shape-based methods, curvature mode shape-based methods, and methods using both mode shapes and frequencies [3], but sometimes the time domain response of vibrated structure is used to extract damage sensitive characteristics. At the natural frequency-based methods the changes in the shape of FRFs is calculated by using the acceleration and dynamic strain responses acquired from intact and damaged states of the structure [19]. One major limitation of the frequency-shift-based method, which requires only the measured natural frequencies, is that the reliable identification of natural frequencies changes of damaged structure encounters the experimental difficulties due to the fact that the frequency changes caused by damage are usually very small and may be buried in the changes caused by environmental and operational conditions. These difficulties can be overcome by using changes in mode shapes due to damage, although the measurement of mode shapes is more difficult than the measurement of frequencies. The convincing feature is that changes in mode shapes are much more sensitive to local damage when compared to changes in natural frequencies. However, using mode shapes also has some drawbacks [15]. It has been observed that the presence of damage does not affect all modes of vibration equally [20]. Some modes are more affected than others. A basis for selecting the affected modes for analysis is, therefore, required. J. Kiddy and D. Pines [8] experimentally established that it is possible to detect damage in the blade if the damage occurs in a region of high modal energy, but damage in regions of low modal energy is not easily detected.

The use of mode shapes curvatures in damage identification is based on the assumption that the changes in the curvatures of mode shapes are highly localized to the region of damage and that they are more pronounced than changes in the displacements of the mode shapes. The curvature is often calculated from the measured displacement mode shapes using a central difference approximation [15]. The damage index was defined as the difference between the curvatures calculated for the monitored and knowingly healthy beams at each point and the largest index indicate the probable damage location [1]. Many results [1, 15, 21] showed that the difference of curvature mode shapes from intact and damaged structure can be a good indicator of damage location. However, it is pointed out that due to experimental and numerical reasons for the higher modes, the difference in modal curvature shows several peaks not only at the damage location but also at other positions, which may lead to a false indication of damage. Indeed, if we use the mode shape curvature as an indicator of the damage, the inherent uncertainty caused by the measurements errors and environmental action will be increased after double numerical differentiation. Hence, the resulting uncertainty (i.e. noise) will be associated both with the number and spatial positions of sensors. The problem of accuracy of damage localization depending on the set of sensors has been studied in [1, 15, 21-23], where it is proved that the number of sensors and the choice of sensors coordinate may have a crucial effect on the accuracy of the damage detection procedure.

Worsening the accuracy of damage localization due to the uncertainty makes it difficult to distinguishing damaged and undamaged states of the structure. To make the good decision at the presence of uncertainties the detection method which consists of the statistical techniques for optimally classifying a signal, the monitoring of modals parameters evolution, the method based on Dempster–Shafer evidence theory, and the probability distributions analysis [4, 24] were developed. Study of these problems on the base of probabilistic approach is emphasized in this paper on example of the cracked Timoshenko cantilever beam.

2 MODEL OF DAMAGED CANTILEVER BEAM

According to the early developed model of Timoshenko beam with crack [13] we assume the crack as notch with not interacted coasts that well grounded for the structures loaded by the big static forces and harmonically excited by small amplitude. Differential equations of Timoshenko beam correctly takes into account the effects of shear and the section rotation even at quite high frequencies of flexural vibrations

$$\begin{aligned}&(EJ\psi_x)_x + kGF(w_x - \psi) = \rho J \psi_{tt}\\&[kGF(w_x - \psi)]_x + q = \rho F w_{tt}\end{aligned}, \tag{1}$$

where E, G, ρ, J, F, L are the Young module, shear module, section moment inertia, section area, and beam length respectively; w, ψ are beam axis deflection and angle of the section rotation; q is distributed load; and k is form-factor equal to 6/5 for rectangular cross section. The boundary conditions for the considered cantilever beam with free load tip are

$$w|_{x=0} = \psi|_{x=0} = \partial\psi/\partial x|_{x=L} = (\partial w/\partial x - \psi)|_{x=L} = 0\,. \tag{2}$$

Assuming for the simplicity the homogenous cross section of the beam (but this assumption does not limit our next consideration), we express the spatial distribution of the section moment inertia and section area in the forms

$$J(x) = J_0 \cdot \zeta(x); \quad F(x) = F_0 \cdot \eta(x), \tag{3}$$

where J_0 and F_0 are the section moment inertia and section area along the defectless part of the beam. Let the beam cross section has the width b and height h.

Transforming to the dimensionless variables according to the rule

$$\xi = x/L \Rightarrow \partial/\partial x = 1/L \cdot \partial/\partial\xi; \quad \tau = t/T \Rightarrow \partial/\partial t = 1/T \cdot \partial/\partial\tau; \quad u = w/L\,, \tag{4}$$

where dimensionless coordinate ξ, time τ and displacement u expressed trough beam length L and pseudo period T to be defined, system (4) is transformed to the dimensionless representation

$$\begin{aligned}&(\zeta\psi_\xi)_\xi + \frac{6kL^2}{h^2(1+\nu)}\eta(u_\xi - \psi) = \frac{\rho L^2}{ET^2}\zeta\psi_{\tau\tau};\\&\frac{k}{2(1+\nu)}[\eta(u_\xi - \psi)]_\xi + \frac{qL}{bhE} = \frac{\rho L^2}{ET^2}\eta u_{\tau\tau}\end{aligned}. \tag{5}$$

A pseudo period T in Eq. 4 is defined as $T = L\sqrt{\rho/E}$. After introducing the dimensionless coefficients

$$A = 6kL^2/[h^2(1+\nu)]; \quad B = k/(1+\nu) \quad \Phi = qL/(bhE) \tag{6}$$

system (5) with boundary conditions is rearranged to the view useful for numerical solving

$$\begin{aligned}&\eta u_{\tau\tau} - B(\eta u_\xi)_\xi + B(\eta\psi)_\xi = \Phi\\&\zeta\psi_{\tau\tau} - (\zeta\psi_\xi)_\xi - A\eta u_\xi + A\eta\psi = 0;\\&u|_{\xi=0} = \psi|_{\xi=0} = \psi_\xi|_{\xi=1} = (u_\xi - \psi)|_{\xi=1} = 0\end{aligned} \tag{7}$$

Description of the crack by the bell–shaped notch with depth $d \in [0;1)$ placed on the dimensionless distance l from the clamped end

$$\delta(\xi) = d \cdot \begin{cases} \cos[\pi(\xi - l)/2\varepsilon]; \xi \in [l - \varepsilon, l + \varepsilon] \\ 0; \xi \notin [l - \varepsilon, l + \varepsilon] \end{cases}, \quad (8)$$

where variable ε is half width of the notch, allow to express the spatial distributions of the beam thick η and section moment inertia ζ in the forms

$$\eta(\xi) = 1 - \delta(\xi); \quad \zeta(\xi) \approx 1 - 3\delta(\xi) \cdot (1 - \delta(\xi))^2 = 1 - 3\delta(\xi) \cdot \eta^2(\xi) \quad (9)$$

The structure of Eq. 7 shows that our beam model satisfies the following requirements:

- The computational cost of the model simulation is minimal due to 1D – dimension;
- Due to dimensionless form model of the damaged structure has the versatility;
- The defect is fully characterized by small number of parameters l, d, ε;
- The output of model is presented by the output variables $u_n(\xi), u_{n,\xi\xi}(\xi)$ that can be interpreted as n-th mode shape and their curvatures;
- These output variables can be properly monitored by direct measurements (optical displacement measurements and surface tenzometry on the set of the beam points) with minimal noise and the influence of environmental factors;
- At decreasing of degree of damage the model of the cracked beam is continuously modified to the model of perfect structure.

The numerical study of the formulated problem showed that Eqs.7 with dependencies (8, 9) is stable at small variation of input data l, d, ε. So, Eqs. (7), (8), (9) can be easily numerically solved and therefore can be used for the multiple forward problem simulation. This feature is very important in order to obtain the probabilistic solution using Monte Carlo approach which requires multiple numerical evaluations at the simulated measurements with noise governable by the Gaussian probability distribution.

3 NUMERICAL MODAL ANALYSIS OF CRACKED TIMOSHENKO BEAM

All numerical calculations were performed using Comsol Multiphysics FE soft package with the Partial Differential Equation (PDE) mode. One dimensional FE mesh consisted of 1000 knots. For the model testing and validation we used early experimentally studied steel beam with dimensions 4cm*5cm*1m and artificially introduced “cracks” – notches with depth: 0.25; 0.4; 0.55, and 0.7 of thick and randomly distributed along the beam. Obtained results (mode shapes) compared with those on the 3D FE model and experiments showed the difference does not exceed 2% [13]. First 5 vibration modes have been calculated and next normalized by dividing the point displacement on the maximum displacement of the free end. To obtain the second spatial derivatives of the modes shape (curvatures) the 5-points symmetric numerical schemes was used

$$u_i'' = [-2u_{i-2} + 32u_{i-1} - 60u_i + 32u_{i+1} - 2u_{i+2}]/(24h^2) + h^4 \cdot u^{(VI)}(\zeta)/90, \quad (10)$$

where h is distance between adjacent measurements points.

The artificial noise has been introduced by adding to the displacement of each point the “measurements error” as randomly distributed value in the range [-0.02; 0.02]. It is obvious from Eq. 10 the calculation of the second spatial derivatives will gain the measurement noise, and this gain depends on the distance h between adjacent displacement sensors. The problem of optimal number of sensors has been studied by the authors early [25], and results of this investigation show the optimal number of measuring points is near 40. This number is used in calculation described below.

In order to obtain the probabilistic estimations for the defected as well as for the ideal beams we simulated a pair of beams (both beams with the artificial noise), and then compared the maximum discrepancies of these beam axis deflections and second derivatives along the beams on the five natural vibration modes. In the cases of intact beam analysis both simulated beams are intact. When a beam with defect is studied, the second simulated beam for compare is intact. Each realization has been modeled 500 times and stored in text file for further statistical analysis.

Two different types of analysis have been performed. First type is a creating of empiric distribution of discrepancies in the form of histograms for all five modes shape and their curvature, then calculation of parameters for matching beta or normal distribution (see Fig. 1). These results allow to estimate the resolution of the vibration modes and their curvatures to determine severity of possible damage and make decision about damage presence or absence.

Second type of analysis has been performed to determine the sensitivity of studied vibration modes and their curvatures to the presence of damages with different levels. To do this the length of studied damaged beams has been divided on the ten identical intervals. Then a number of correct predictions percentage and distribution of deviations from the true crack positions have been calculated. As a result the confidence intervals (p=0.95) for deviations of estimated crack position have been obtained for each interval of beam length and spatial distribution of correct crack localization by each mode and their curvature. Two examples of these results are presented on a Fig. 2 which shows the distributions of the localization errors and percentage of correct crack localization for the cracked beam (d=0.25) according to the analysis of second vibration mode and curvature of the first vibration mode.

Figure 3 demonstrate how to relate the error in localization of crack and accuracy of its spatial position estimation. The similar diagrams for all mode shapes have been used to compare their sensitivity to the damage presence and severity on the different region of the beam.

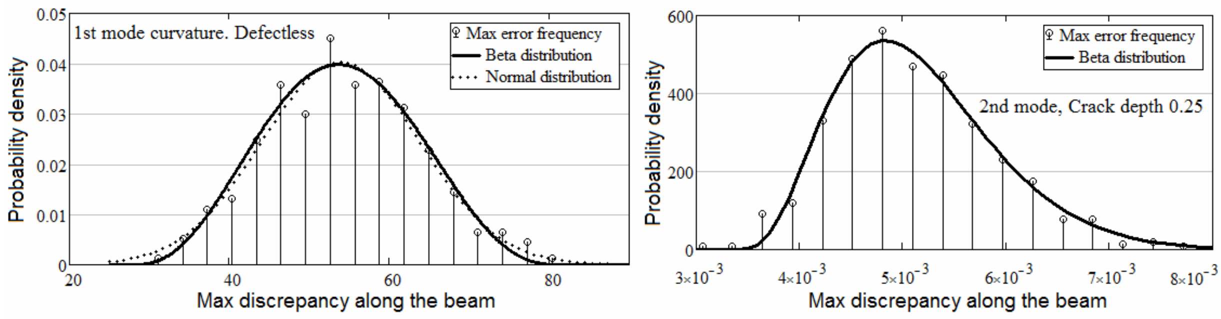

Figure 1: Empiric distributions fitted by the beta and normal probability distributions of the maximum discrepancy along the beam
First mode curvature for the intact beam (left) and second mode for the beam with d=0.25 (right)

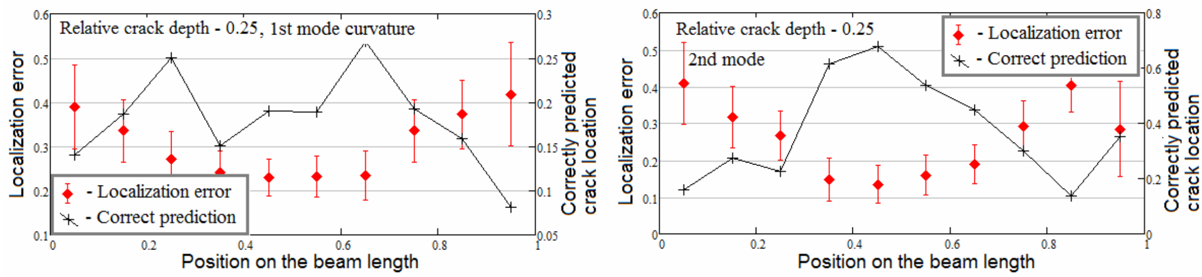

Figure 2: Spatial distributions of confidence intervals for deviations of estimated crack position and percentages of correct crack localization

4 THE PROBABILISTIC ANALYSIS OF FLEXURAL MODES SHAPE AND CURVATURES SENSITIVITY TO THE DAMAGE IN THE BEAM

Common results of measurements variability for intact beams and beams with different severity of damage are presented on a Figure 3 in the form of probability distributions. These plots allow separating the situations with presence or absence of damage and making a statistically confirmed decision.

These diagrams characterize the ability of each mode shape and its curvature to separate the undamaged and damaged state of the studied beam. Indeed, look at the top-left chart and remember that area under each curve on a Figure 3 is unit. Let we performed three (or more) measurements of the first (or other) mode shape (or curvature), and also three maximum discrepancies from the undamaged state are calculated from these measurements. For this set of discrepancies we can determine lower L and upper U bounds of confidence interval at some accepted confidence level (e.g., 0.95). Then the probability of damage described by the probability density p_i where index $i = 0$ matches to undamaged state, and its other values match to the relative crack depth (see Fig. 4) will be obtained as

$$P(i) = \int_L^U p_i(\Delta) d\Delta \,, \tag{11}$$

where Δ is discrepancy. In particular, for the situation shown on a Figure 3 approximate ratio of probabilities will be $P(0)/P(0.25)/P(0.4)/P(0.55)/P(0.7) \approx 0.7/0.3/0.11/0.06/0.015$. As can be seen from Figure 3, the most reliable to identify the healthy state there are mode shapes, but not curvatures. Such preliminary estimation is very useful to identify the undamaged state and for subsequent verification of data predicted by the modes or curvatures as it is presented on a Figure 4.

As it is shown from a Figure 4 a sensitivity of studied mode shapes and curvatures is very different and vary along the beam length. Each plot on a Figure 4 demonstrates spatial distribution of right crack location inside the ranges with length 0.1. Information inside incuts inform about averaged right predicted crack location for each damage severity. The lower graphs for each mode show its corresponding shape and curvature. Comparison of these figures shows that the zones of better sensitivity correspond to the beam areas with greatest curvature, especially for the 2nd, 4th, and 5th modes. For all considered cases the sensitivity near beam ends is minimal. Unfortunately, the sensitivity to locate the small crack does not exceed of 30%. So, to correctly locate the incipient cracks need to use the multi-step enough sophistic algorithms, which take into account the probabilistic properties of monitored data and should be developed in the future.

5 CONCLUSIONS

On the base of cracked Timoshenko beam model the probabilistic approach to the crack identification is proposed. Under conditions of inaccuracy of measuring the vibrations amplitudes the set of distributed sensors this approach assumes the inherent uncertainty of monitored natural mode shapes and their curvatures. More 500 computer simulations of 1D Timoshenko cracked beam finite element model have been performed at randomly varied crack depth, width and location to calculate the histograms and then fit the match empiric probability distribution of the vibration amplitudes and beam curvatures for healthy beam and beams with different damage severity. These probabilistic regularities allow us to estimate relative probability of damaged or undamaged state of the beam. The spatial distribution of correctly predicted crack location has been calculated and studied for the first five natural vibration modes and their curvatures in order to identify the sensitivity of these modes (curvatures) for the different damage position on the beam and varied damage severity. The spatially distributed sensitivity of vibration modes and curvatures to the presence of damage has been established, and the best sensitivity is achieved with the 4th and 5th vibration modes in the beam areas of greatest curvature. However, the low frequency vibration method for damage prediction does not ensure the best sensitivity and resolution for the small crack identification at inherent significant experimental uncertainty. To overcome this drawback is necessary to develop the improved algorithms that take into account all data obtained at the structure vibration monitoring and their probabilistic features.

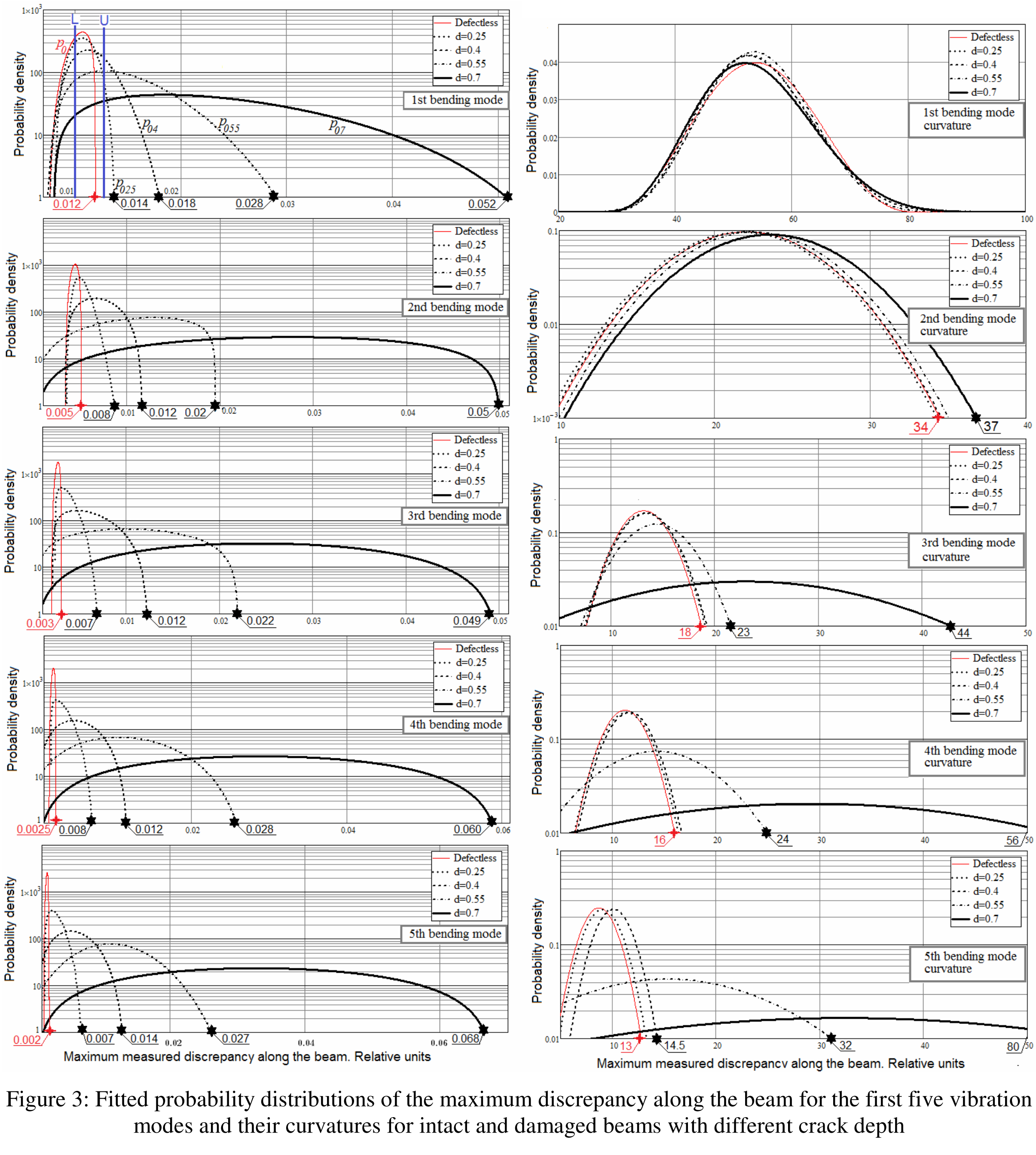

Figure 3: Fitted probability distributions of the maximum discrepancy along the beam for the first five vibration modes and their curvatures for intact and damaged beams with different crack depth

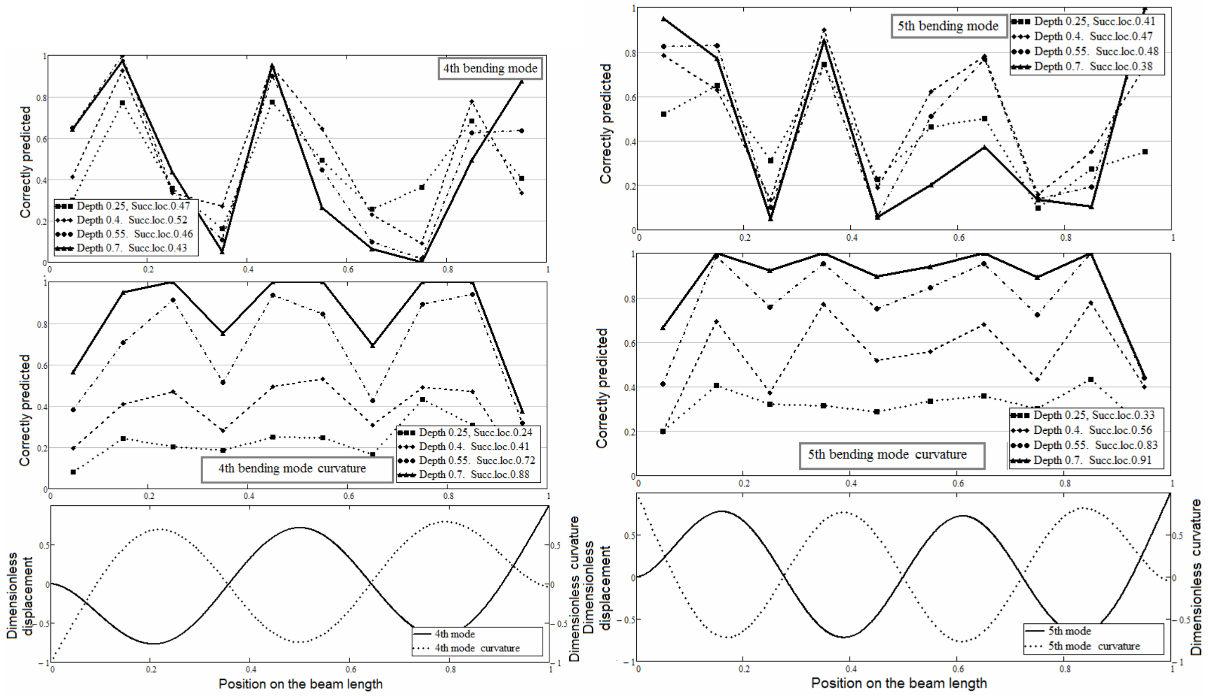

Figure 4: Spatial distributions of correct crack prediction by the mode shapes and their curvatures analysis together with these dimensionless mode shapes and curvatures

REFERENCES

[1] E.P. Carden, P. Fanning, Vibration based condition monitoring: A review. *Structural Health Monitoring,* **3(4)**, 355-357, 2004.

[2] J. Humar, A. Bagchi and H. Xu, Performance of vibration-based techniques for the identification of structural damage. *Structural Health Monitoring*, **5**, 215-242, 2006.

[3] W. Fan and P. Qiao, Vibration-based damage identification methods: A review and comparative study. *Structural Health Monitoring,* **10 (1)**, 83-111, 2011.

[4] A. Tarantola, *Inverse problem theory and methods for model parameters estimation*, SIAM Press, Philadelphia, 2005.

[5] V. Meruane, W. Heylen, Damage detection with parallel genetic algorithms and operational modes. *Structural Health Monitoring*, **9**, 481-497, 2010.

[6] R. Hadidi, N. Gucunski, Probabilistic approach to the solution of inverse problems in civil engineering. *J. of Computing in Civil Engineering*, **22(4)**, 338-347, 2008.

[7] C.W. Gardiner, *Handbook of stochastic methods for physics, chemistry and the natural sciences*, Springer-Verlag, New York, 1983.

[8] J. Kiddy and D. Pines, Experimental validation of a damage detection technique for helicopter main rotor blades. *J. of Systems and Control Engineering*, **215**, 209-221, 2001.

[9] P.M. Pawar, R. Ganguli, Helicopter rotor health monitoring- a review. *Proc. of the Institution of Mechanical Engineers, Part G: J. of Aerospace Engineering*, **221**, 631-650, 2007.

[10] A. Ghoshal, J. Harrison, M.J. Sundaresan, D. Hudghes, and M.J. Schulz, Damage detection testing on a helicopter flexbeam. *Journal of Intelligent Material Systems and Structures*, **12**, 315-330, 2001.

[11] P. Kolakowsky, L.E. Mujica, and J. Vehi, Two approaches to structural damage identification: Model updating versus soft computing. *J. of Intelligent Material Systems and Structures*, **17**, 63-80, 2006.

[12] P.M. Pawar, K.V. Reddy, and R. Ganguli, Damage detection in beams using spatial Fourier analysis and neural networks. *J. of Intelligent Material Systems and Structures*, **18**, 347-359, 2007.

[13] S.N. Shevtsov, V.A. Akopyan, and E.V. Rozhkov, An approach to the problem of damage identification in an elastic rod based on the Timoshenko beam model. *Rus. J. of Nondestructive Testing*, **47(7)**, 480–490, 2011.

[14] S.P. Lele and S.K. Maiti, Modelling of transverse vibration of short beams for crack detection and measurement of crack extension. *J. of Sound and Vibration*, **257,** 559–583, 2002.

[15] M. Chandrashekhar, R. Ganguli, Structural damage detection using modal curvature and fuzzy logic. *Structural Health Monitoring*, **8**, 267-283, 2009.

[16] M. Afshari, D.J. Inman, Continuous crack modeling in piezoelectrically driven vibrations of an Euler-Bernoulli beam. *J. of Vibration and Control*, **19**, 341-356, 2013.

[17] G. Gounaris, A.D. Dimarogonas, A finite element of cracked prismatic beam for structural analyses, *Computer and Structures*, **28**, 309-313,1988.

[18] H. Mohammad, F. Dado, O.A. Shpli, Crack parameter estimation in structures using finite element modeling. *Int. J. Solid and Structures*, **40**, 5389-5406, 2003.

[19] J. Lee, S.Kim, Structural damage detection in the frequency domain using neural networks. *J. of Intelligent Material Systems and Structures*, **18**, 785-793, 2007.

[20] L.M. Khoo, P.R. Mantena, and P. Jadhav, Structural damage assessment using vibration modal analysis. *Structural Health Monitoring*, **3(2)**, 177–194, 2004.

[21] C.C. Chang, T.Y.P. Chang, Y.G. Xu, and M.L. Wang, Structural damage detection using an iterative neural network. *J. of Intelligent Material Systems and Structures*, **11**, 32-44, 2000.

[22] S. Das, A. Chattopadhyay, and X. Zhou, Acoustic based structural health monitoring for composites using optimal sensor placement: Analysis and experiments. *J. of Reinforced Plastics and Composites*, **28**, 83-97, 2009.

[23] K. Mendrok and T. Uhl, Experimental verification of the damage localization procedure based on modal filtering. *Structural Health Monitoring*, **10**, 157-173, 2011.

[24] W. Zhang, X. Lin, J. Pei, Y. Zhang, Managing uncertain data: Probabilistic approaches. *Proc. on the 9th Int. Conf. on Web-Age Information Management*, WAIM '08, 405–412, 2008.

[25] S. Shevtsov, I. Zhilyaev, V. Akopyan, and A. Soloviev, Identification of the damage in the beam-like structure on the basis of Timoshenko beam model. *Proc. on the Int. Conf. on Innovative Technologies*, In-Tech 2012, Rijeka, Croatia, 517-521, 2012.