11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

A STUDY ON THE NEAR-RESONANCE RESPONSE OF MACHINERY STRUCTURAL SYSTEMS TO FREQUENCY-SWEEP INPUT

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Keywords: Rotary machinery systems, Frequency-sweep vibration, Near-resonance response.

Abstract. *Many of rotary machinery systems during their start-up and shut-down phases generate a vibration with frequency-sweep nature. This type of vibration causes a near-resonance response in the supporting structure, which usually gives an undesired vibration to the machine as well as an unpleasant sensation to the operators. The duration and extent of this so-called pseudo-resonance response depends basically on the trend of variation of the input frequency. In the present paper, a parametric study is performed to find the parameters that characterize the maximum response of a structural system subjected to a frequency-sweep excitation. In this regard, a simple single-degree-of-freedom structural model is employed and the structural responses are then calculated based on the various input excitation patterns and also the model parameters. The results indicate that the maximum frequency of the input excitation, the duration of frequency change in the excitation and the damping of the structure are the three parameters that have a strong influence on the peak response of the structural system. Finally, an approximate closed form model is presented for expressing the relationships between the input and system parameters, and the peak responses of the system*.

1 INTRODUCTION

Many of rotary machinery systems during their start-up and shut-down phases generate a vibration with frequency-sweep nature. This type of vibration causes a near-resonance response in the supporting structure, which usually gives an undesired vibration to the machine as well as an unpleasant sensation to the operators. The response of a system to frequency-sweep excitation has been subject of study in literature (e.g., in [1–3]). The mathematical solutions for the response of a system under frequency-sweep excitation are of a level of complexity that makes them unsuitable to be used in hand calculations.

This paper aims to perform a study on the response of a structure subjected to a frequency-sweep excitation in order to find a simple model for maximum response estimation of the structure. In this regard, the fundamental parameters from the input excitation that characterize the response of the structure are presented in section 2. The structure response corresponding to specified ranges of the parameters are analysed in section 3.1. It is intended in section 3.2 to provide a relationship between the parameters from the input excitation and the maximum response as an approximate closed form formula. The results obtained from the analyses aid in better understanding of the near-resonance of a system under frequency-sweep excitation.

2 NUMERICAL MODELING

In this section, simple models are considered for both structure and engine. The parameters that determine these models are then used in section 3 for the parametric study on the response of the structural system.

2.1 Structural model

To analyse the dynamic response of industrial structures under operational excitations, a linear time-invariant (LTI) model can be considered. Based on dynamic properties of LTI systems, the response of an *n*-degree of freedom (*n*-DOF) structure can be equivalently expressed as the sum of responses from a set of single degree of freedom (SDOF) structures. As a result, the calculation of the response of an *n*-DOF system can be reduced to the ones for SDOF systems [4]; hence, the response of a SDOF structure to a sweep-frequency excitation is focused in this study. An SDOF structure with mass, damping and stiffness of *m*, *c* and *k* is shown in Figure 1 in which $F(t)$ represent the ambient force excitation (e.g., from a rotary engine). The natural period of the system, T_n, is $2\pi\sqrt{m/k}$ which is the reciprocal of the natural frequency of the system, ω_n; also, the damping can be expressed as $2\xi m\omega_n$ in which ξ stands for the damping ratio of the system.

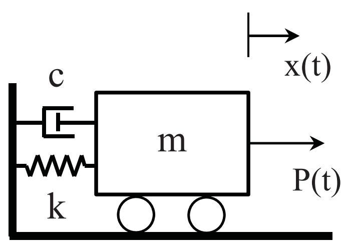

Figure 1: A typical model for a SDOF structure.

In the next section, T_n and ω_n are efficiently used to normalize the key parameters of a frequency-sweep excitation.

2.2 Engine model

Figure 2 shows a typical model of a rotary engine. In the model, two elements with mass of m_s rotate contrariwise in circles of radii of r and generate one-directional excitation across the y axis.

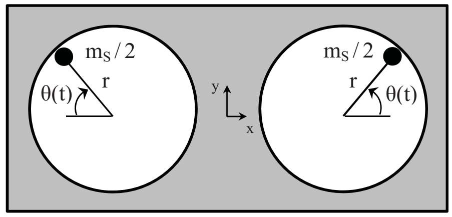

Figure 2: A typical model for a rotary engine.

The type of the excitation generated by the model is directly dependent to the definition of the phase function, $\theta(t)$. In the start-up stage, the angular speed (i.e., $d\theta(t)/dt$) varies from zero to a final value which is corresponding to the operational speed of the engine, $\omega_{\max}$. The variation function of the angular speed determines the type of the swept frequency. In linear swept frequency (which is in the scope of the present work), the angular speed follows a linear pattern between zero and the operational speed. The schematic pattern of the excitation in start-up and operational stages respectively with durations of t_1 and t_2 are depicted in Figure 3. In the present study, the phase function is defined as λt^2 in which λ is a constant value; λ can be expressed in terms of the operational speed and duration of the start-up stage, $\lambda = \omega_{\max}/2t_1$.

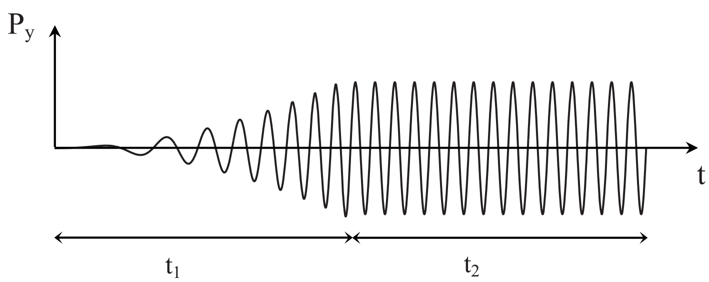

Figure 3: The schematic pattern of excitation in start-up and operational stages.

The excitation equation in start-up stage is presented as Eq. (1) in which p_0 is a constant coefficient equal to $2m_s r\lambda$ that has the same dimension as force.

$$P_{y,1}(t) = p_0\sqrt{1+4\lambda^2 t^4}\,\sin(\lambda t^2) \tag{1}$$

In the operational stage, the engine reaches the steady speed of $2\lambda t_1$ and the nature of the excitation changes to a constant frequency excitation which its equation is as follows:

$$P_{y,2}(t) = 2p_0 t_1\lambda\,\sin(2t_1\lambda t) \tag{2}$$

It is seen from the above equations that λ and t_1 are the parameters by which the frequency-sweep excitation can be fully defined. The dynamic equations of motion of a SDOF system under the frequency-sweep and operational excitations are presented in Eqs. (3-4).

$$\ddot{x}_1 + 2\xi\omega_n\dot{x}_1 + \omega_n^2 x_1 = \frac{p_0}{m}\sqrt{1+4\lambda^2 t^4}\,\sin(\lambda t^2) \tag{3}$$

$$\ddot{x}_2 + 2\xi\omega_n\dot{x}_2 + \omega_n^2 x_2 = 2\frac{p_0}{m} t_1\lambda\,\sin(2t_1\lambda\,t) \tag{4}$$

From the above equations, the response of the structure can be determined.

3 PARAMETRIC STUDY

To perform the parametric analysis on the response of the SDOF structure, independent parameters of interest should be addressed. Formation of these parameters are subject of the section 3.1. The parametric study on the response of the structure is presented in section 3.2.

3.1 Formation of independent parameters

Beside the parameters related to the input excitation (i.e., t_1 and $\omega_{\max}$), the natural frequency and damping ratio of the structure are also the significant parameters which determine the response of the structure (i.e., $x(t)$). At first glance, four parameters of ξ, T_n, t_1 and λ may be considered as independent parameters. But, on the other hand, the number of parameters can be reduced to three by a proper normalization procedure. Using T_n and ω_n as the normalizing parameters, t_1 and $\omega_{\max}$ can be re-expressed as $\alpha_1 T_n$ and $\beta_{\max}\omega_n$ respectively, in which α_1 and $\beta_{\max}$ are dimensionless parameters. In addition, the maximum displacement response of the structure, $x_{\max}$, can also be normalized as $x_{\max}/(p_0/k)$. Mathematically it can be simply shown that the variation of T_n has no effect on the normalized response of the structure. Applying the above normalizations, the three dimensionless parameters of ξ, α_1 and $\beta_{\max}$ are obtained to be used in parametric analysis of $x_{\max}/(p_0/k)$.

3.2 Parametric analysis

To examine the variation of $x_{\max}/(p_0/k)$, other parameters are varied within specified ranges. To include a wide range for the input parameters, α_1 and $\beta_{\max}$ are varied from 2 to 20. ξ is varied through 0.05 to 0.50 which covers a wide range of damping for usual structures. Having the specified ranges, the structural analysis is performed to determine the maximum response of the structure. In numerical calculation of the response, the time sampling of the input is taken equal to $T_n/100\beta_{\max}$ to get a very close solution to the exact solution. Figure 4 show the variation of $x_{\max}/(p_0/k)$ with respect to the variation of the input parameters corresponding to $\xi = 0.05$ and $\xi = 0.30$.

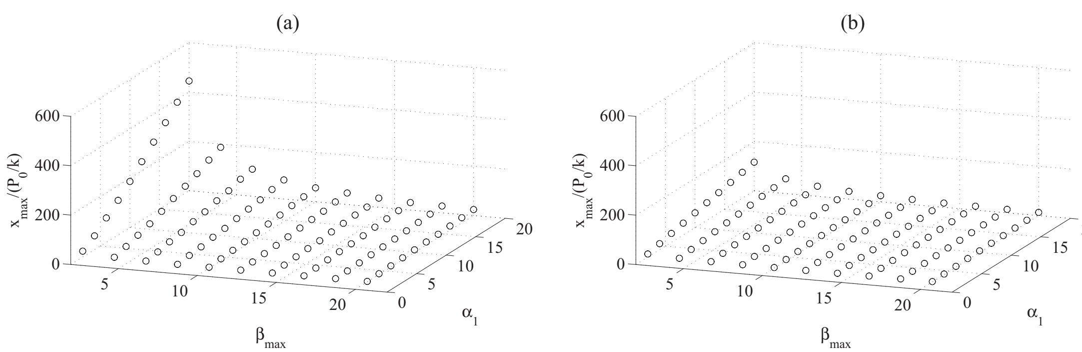

Figure 4: The variation of the normalized maximum displacement with respect to variation of the input parameters obtained by: (a) $\xi = 0.05$ and (b) $\xi = 0.30$.

It can be found from Figure 4 that for a specified damping ratio, the response points form a surface that can be fitted by an approximate model in terms of α_1 and $\beta_{\max}$. Through the investigation of different classes of algebraic functions, it is found that the maximum response can be property approximated by a closed form model as follows:

$$\frac{x_{\max}}{p_0 / k} \approx d_1 \alpha_1^{d_2} \exp(d_3 / \beta_{\max}^{d_4}) \qquad (5)$$

in which d_1 to d_4 are positive constants which are determined by means of a regression analysis. Eq. (5) indicates that the normalized maximum displacement varies as a power function along the α_1 axis, whereas its variation along the $\beta_{\max}$ axis is of an exponential nature. For the domain $10 \le \alpha_1 \le 20$, $2 \le \beta_{\max} \le 10$ in which the maximum values build up significantly, the regression analysis is done and the constants d_1 to d_4 are determined for the various values of the damping ratio. Table 1 shows the constant values obtained from the fitting procedure.

ξ	d_1	d_2	d_3	d_4
0.05	1.147e-007	1.269	19.25	0.07381
0.10	1.606e-007	1.163	18.74	0.06851
0.15	1.518e-007	1.091	18.64	0.06381
0.20	9.564e-008	1.035	18.97	0.05920
0.25	2.959e-007	0.9934	17.73	0.06099
0.30	9.384e-008	0.9635	18.76	0.05559
0.35	5.128e-007	0.9409	16.97	0.06066
0.40	5.217e-007	0.9253	16.86	0.06021
0.45	1.096e-006	0.9152	16.04	0.06319
0.50	1.130e-006	0.9096	15.94	0.06372

Table 1: The constant values of the approximate model obtained from the regression analysis for the domain $10 \le \alpha_1 \le 20$, $2 \le \beta_{\max} \le 10$.

In the regression analysis, the response points are weighted in a way to limit the error within a range of $\pm 6\%$. With the proposed model, the maximum response of a structure under the frequency sweep excitation can be approximated over arbitrary ranges of the input parameters. To compare the maximum response in start-up stage with the one in operational stage, the ratio between the peak responses in start-up and operational stages, *R*, obtained by $\xi = 0.05$ and $\xi = 0.30$ is presented in Figure 5.

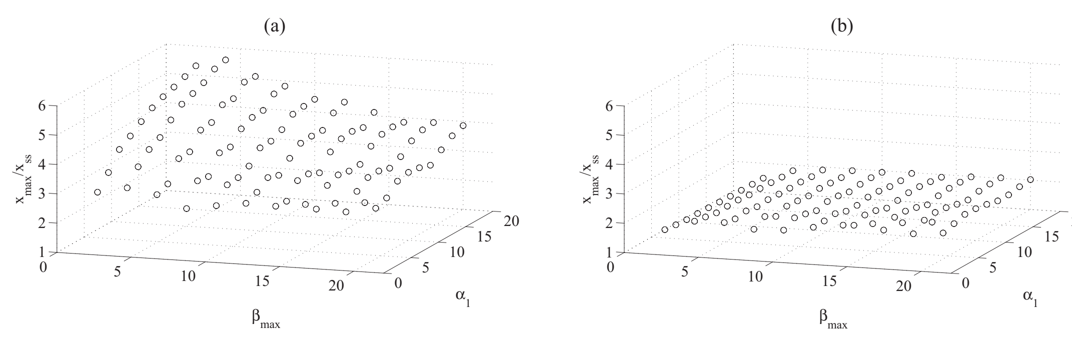

Figure 5: The ratio between the peak responses in start-up and operational stages, obtained by: (a) $\xi = 0.05$ and (b) $\xi = 0.30$.

It is observable from the above figure that *R* increases as the damping ratio of the structure decreases; also, comparing Figure 5(a) and Figure 5(b), it can be found that firstly, increase in the damping ratio globally reduces the *R* values over the entire domain and secondly, the variation of the damping ratio has a strong effect on the distribution of *R* with respect to the variation of α_1 and $\beta_{\max}$.

4 CONCLUSIONS

- The problem of structures supporting a rotary engine is addressed.
- Simple models are considered for both the structure and the engine.
- A parametric study on the maximum response of the structure with respect to variation of the input and system parameters is performed.
- An approximate closed-form model for estimation of the maximum displacement response of the structure is proposed.
- The proposed model can be easily used for hand calculation of the maximum response of a structure under frequency-sweep excitation exerted by a rotary engine.

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