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NONLINEAR DYNAMICS OF A HYDRAULIC PRESSURE CONTROL VALVE

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Abstract. *Hydraulic valves are known to show interesting dynamic behavior. Nevertheless they have not yet been investigated extensively from the viewpoint of nonlinear dynamics and are not sufficiently understood. The elementary hydraulic valve can be described as a system of third order with a non-smooth nonlinearity. The transition from an ideally impermeable valve to a valve allowing for leakage flow is investigated in this paper for different valve geometries. Leakage changes the character of the equilibrium position from a set-valued equilibrium position to a unique one. Stability of the equilibrium position is investigated and the birth of a limit cycle due to leakage is shown. Subsequently, a bifurcation analysis reveals the different solution types for the system under external forcing, yielding evidence of period-doubling phenomena up to quasi-periodic solutions.*

1 INTRODUCTION

Hydraulic valves are used in a broad range of technical applications. While researched extensively from an experimental viewpoint, there is only little theoretical treatment of the dynamic behavior of systems involving hydraulic parts. Among the few theoretical investigations, a focus lies on control theory and impact oscillations. Eyres et al. [2], for example analyze an hydraulic damper system and give evidence of grazing bifurcation phenomena in such systems. Licskó et al. [1] take a similar perspective and investigate bifurcation phenomena in a pressure relief valve. Leakage phenomena remain a rarely researched topic in the field, although they are known to cause issues in hydraulic circuits.

Applications of hydraulics frequently involve control of the pressure in a system. The valves used for such purposes are vulnerable to leakage. In this paper, a model of a simple hydraulic pressure control valve is presented and its dynamic behavior is discussed. Firstly, an ideal pressure control system is modeled. It is shown how the system behavior changes as leakage flow is taken into account.

2 MODELLING PRESSURE CONTROL VALVES

2.1 The ideal pressure control valve

As the system under investigation (see Figure 1(a)) is a hybrid system, the equations describing the system behavior are twofold: by Newton's second law one obtains the equation of motion for the valve and formulating a balance equation for the fluid flow yields an equation for the hydraulics of the system coupled with the mechanical behavior by the dependence of the fluid flow on the position of the valve. Under the assumption that the excitation is harmonic, one gets:

$$m\ddot{x} + d\dot{x} + kx = -F_0 - k(L_0 - L_1) - F_1 \sin(\omega t) + \bar{A}(p_0 + p(t)), \tag{1}$$

with $p(t)$ being the pressure deviation from the desired working pressure p_0. The lengths L_0 and L_1 are the lengths generating the desired pre-stress of the spring. Considering the fluid flow balance, one obtains

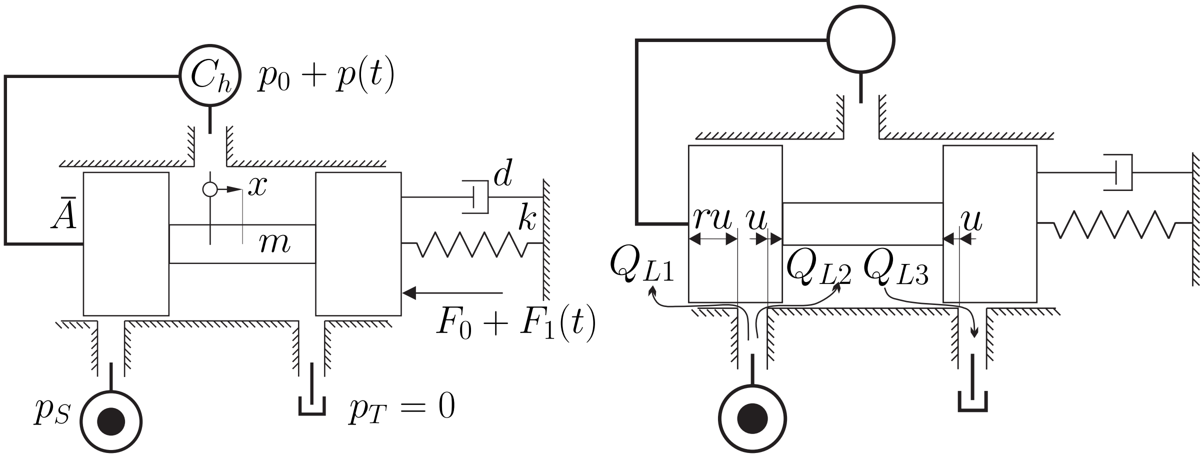
$$C_h \dot{p}(t) + \bar{A}\dot{x} = \bar{Q}, \tag{2}$$

where $\bar{Q}$ is the net fluid flow into capacity C_h. The net fluid flow $\bar{Q}$ depdends on the groove geometry of the valve (see Figure 2) and consists of the fluid flows Q_S from the pressure supply into the capacity and of the flow Q_T from the capacity into the tank. Note that for an ideal valve the flows are mutually exclusive outside the dead region $-u \leq x^* \leq u$, that is, if $Q_S \neq 0$, then $Q_T = 0$ and vice versa. Practically, this is the only source of nonlinearity considered here. The fluid flows Q_S and Q_T are determined from modelling the valve control edges as throttles.

After non-dimensionalization with $x = uX$, $t = T\tau$ and $p(t) = p_0 P(\tau)$, the equation of motion for the system derives to:

$$X''' + 2D\omega X'' + (\omega^2 + a_A A)X' - A\bar{Q}(X'', X', X', \tau) = -F\eta cos(\eta\tau). \tag{3}$$

Here,



(a) Pressure control valve system. (b) Leakage flows and valve geometry.

Figure 1: System.

$$\omega = \frac{kT^2}{m}, \quad D = \frac{dT}{2}\sqrt{\frac{m}{k}}, \quad a_A = \frac{\bar{A}u}{C_h p_0}, \quad A = \frac{\bar{A}T^2}{mu}p_0, \quad F = \frac{F_1 T^2}{mu}, \quad \eta = \Omega T,$$

and T is a characteristic time allowing for scaling the problem. For a rectangular groove, the non-dimensional volume flow $\bar{Q}$ reads

$$\bar{Q} = \bar{Q}^{\square}(X'', X', X', \tau) = \begin{cases} -\gamma^{\square}(1+X)\sqrt{\kappa-1}\left(1 - \frac{1}{2(\kappa-1)}P(\tau)\right) & \text{if } X < -1 \\ 0 & \text{if } -1 \leq X \leq 1 \\ -\gamma^{\square}(X-1)\left(1 + \frac{1}{2}P(\tau)\right) & \text{if } X > 1, \end{cases} \tag{4}$$

for which the constants

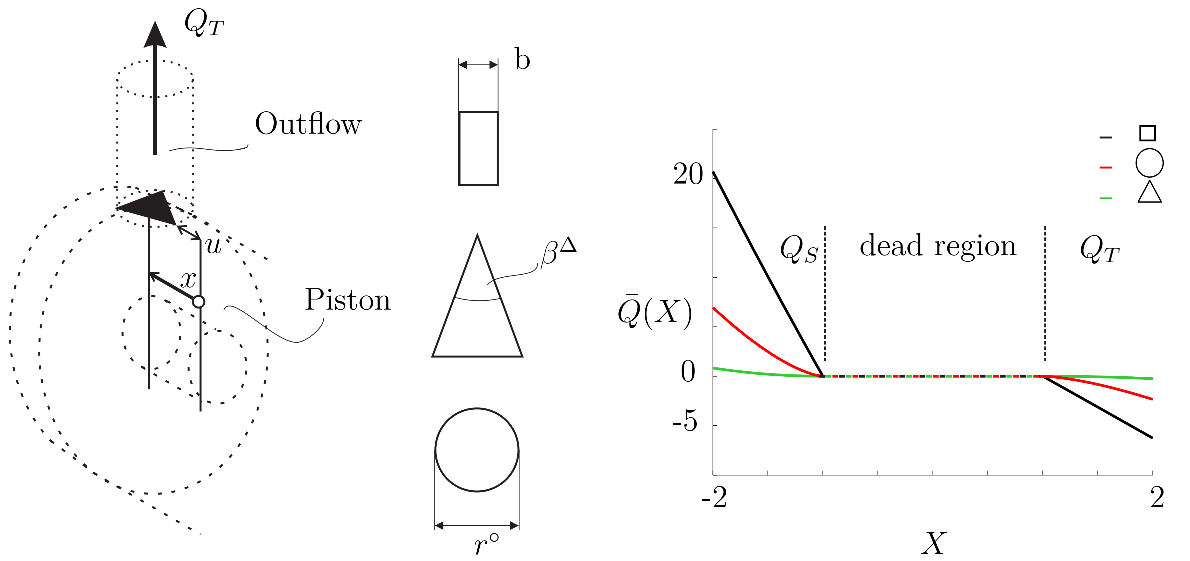
$$\gamma^{\square} = \alpha_D \sqrt{\frac{2}{\rho}} \frac{T}{C_h \sqrt{p_0}} bu, \quad \kappa = \frac{p_S}{p_0}$$

were introduced and

$$P(\tau) = \frac{1}{A}\left(X'' + 2D\omega X' + \omega^2 X + F\sin(\eta\tau)\right). \tag{5}$$

In deriving the above equations, it was assumed that that the valve is symmetric, i.e. that the valve overlap u is identical for both control edges. In addition, it was assumed that the volume flows at the respective control edges are linear functions of x and of the pressure difference which implies the linearization of the pressure change in consumer C_h.

Here and in the remainder of this article, superscripts $i = \{1, 2, 3\}$ denote the different regions of the flow function and the system behavior with $i = 1$ for the interval $(-\infty, -1)$, $i = 2$ for $[-1, 1]$ and $i = 3$ for $(1, \infty)$. Flow geometries are indicated by superscripts $\square$,$\triangle$ and $\circ$.



(a) Groove geometries (for the tank side control edge).

(b) Volume flows for different groove geometries.

Figure 2: Non-dimensional volume flow characteristics.

2.2 The pressure control valve with leakage

Ideal control valves cannot be realized by real-word manufacturing technology. In practice, tolerance errors will always lead to leakage flow within the valve. While the overall structure of the equation of motion for the pressure control valve system remains unchanged, the description of the volume flow changes from $\bar{Q}$ to $\bar{Q}^L$, where the superscript L indicates the extension of the flow model to leakage flow superposed to the ideal valve characteristic.

To model leakage flow, it is assumed that leakage is relevant only in the region $X \in [-1, 1]$, i.e. in the dead region of an ideal valve. This assumption is justified by the circumstance that leakage flows are "small" volume flows relative to the power volume flows $\bar{Q}^\square$, $\bar{Q}^\triangle$ and $\bar{Q}^\circ$.

The flow situation at a control edge as shown in Figure 1(b) is modelled as a sequential coupling of an orifice and a circular throttle of variable length, see table 1 for a description of the parameters used. Via a hydraulic diameter calculation, one finds the equivalent throttle cross section to be $A_T = \pi \Delta r^2$.

According to Figure 1(b), leakage flows at three points in the system. After non-dimensionalizing, the leakage flow reads:

$$\begin{aligned} Q_L(X) \approx \lambda q_{L1}\Bigg(&-(r+X) + \sqrt{(r-X)^2 + \lambda^2 q_{L2}(\kappa-1)} - \frac{\lambda^2 q_{L2}}{2\sqrt{(r-X)^2+\lambda^2 q_{L2}(\kappa-1)}} P(\tau) \\ &+ \sqrt{(X+1)^2 + \lambda^2 q_{L2}(\kappa-1)} - \frac{\lambda^2 q_{L2}}{2\sqrt{(X+1)^2+\lambda^2 q_{L2}(\kappa-1)}} P(\tau) \\ &- \sqrt{(1-X)^2 + \lambda^2 q_{L2}} - \frac{\lambda^2 q_{L2}}{2\sqrt{(1-X)^2+\lambda^2 q_{L2}}} P(\tau)\Bigg). \end{aligned} \tag{6}$$

Table 1: Parameters of the pressure control valve.

Parameter	Symbol	Unit
Valve displacement	x	m
Valve mass	m	kg
Damping coefficient	d	Ns/m
Spring stiffness	k	N/m
Force components	F_0, F_1	N
Excitation frequency	Ω	rad/s
Capacity of hydraulic consumer	C_h	m^3/ bar
Piston area	A	m^2
Working pressure/supplied pressure	p_0, p_S	bar
Valve overlap	u	m
Flow coefficient	α_d	-
Fluid density	ρ	kg/m^3
Throttle cross section	A_T	m^2
Diameter of piston bore	D	m
Gap	Δr	m
Fluid viscosity	η_{HP}	m^2/s

Here, the coefficients are

$$\lambda = \frac{\Delta r}{D}, \quad q_{L1} = 12\frac{\pi\alpha_D^2 u\eta_{HP}T}{\rho C_h p_0}, \quad q_{L2} = \frac{D^4\rho p_0}{72\eta_{HP}^2 u^2}.$$

With these expressions, the model of a pressure control valve with ideal valve characteristic can be extended by simply adding the leakage flow Q_L in the dead region and shifting the flows in the regions $\mathbb{X}^1 = (-\infty, -1)$ and $\mathbb{X}^3 = (1, \infty)$ accordingly to match the flow at the end of the dead region's interval. Hence,

$$\bar{Q}^{L\square}(X) = \begin{cases} -\gamma^\square(1+X)\left(1 - \frac{1}{2(\kappa-1)}P(\tau)\right) + Q_L(-1) & \text{if } X < -1 \\ Q_L(X) & \text{if } -1 \leq X \leq 1 \\ -\gamma^\square(X-1)\left(1 + \frac{1}{2}P(\tau)\right) + Q_L(1) & \text{if } X > 1. \end{cases} \tag{7}$$

3 EQUILIBRIUM ANALYSIS

3.1 The transition from a non-smooth to a smooth system by increasing leakage

The zero(s) of the flow function are the equilibrium positions of the system. The transition from non-smooth to smooth takes places in a well-ordered manner: for a pressure ratio $\kappa > 1$, the leakage flow is a strictly monotonous function in X.

This leads to a unique equilibrium position independent of gap height. By increasing the leakage parameter λ, the equilibrium position moves to the tank-side control-edge. The same effect can be achieved by increasing κ.

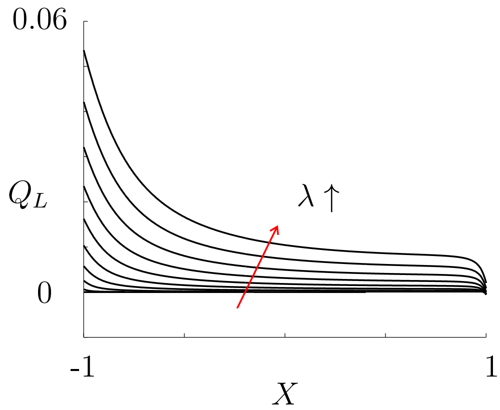


Figure 3: Non-dimensional leakage flow in $\mathbb{X} = [-1, 1]$ for increasing non-dimensional gap heights λ. Parameters are: $D = 1.29$, $\omega = 38.73$, $a_A = 0.04$, $A = 2.06 \times 10^4$, $q_{L1} = 55.83$, $q_{L2} = 1.76 \times 10^2$, $\gamma^{\square} = 14.81$, $\kappa = 57.14$. Parameter range: $\lambda = 0 - 3 \times 10^{-3}$.

3.2 Stability effects of leakage flow

Stability of equilibria of the ideal control valve For ideal valves, the flow function has infinetely many zeros: any point $X^* \in [-1, 1]$ is a zero of the flow function and thus an equilibrium position. This is a feature of the dead region resulting from perfect impermeability when the valve is closed. Equilibria in the dead region of the ideal valve are indifferent.

Stability of the control valve with leakage and equilibria in the former dead region The situation changes when allowing for leakage. By introducing leakage flow, the flow function has a unique zero. Depending on the choice of parameters, this zero lies in the interval $X^* \in [-1, 1]$ or in the interval $X^* \in (1, \infty)$. Note that for an equilibrium position in $X^* \in (1, \infty)$ the valve is open.

The Jacobian of the system with $X^* \in [-1, 1]$ has the characteristic equation

$$-\lambda^3 - 2D\omega\lambda^2 - (\omega^2 + a_A A)\lambda + A\tfrac{\partial \bar{Q}_L(X^*)}{\partial X} = 0, \tag{8}$$

which leads to the following Hurwitz criterion for dimensional parameters:

$$dTu\left(kC_h + A^2\right) + AC_h m p_0 \tfrac{\partial \bar{Q}_L(X^*)}{\partial X} \geq 0. \tag{9}$$

In practice, this criterion is easily fulfilled for equilibrium positions $X^* \in [-1, 1]$, so that these equilibrium positions are stable.

Stability of the control valve with rectangular groove and equilibria outside of the former dead region For certain levels of q_{L2}, increasing the leakage parameter λ leads to equilibrium positions in $\mathbb{X} \in (1, \infty)$.

Because for a rectangular groove the flow function is constant when ignoring first order changes in $P(\tau)$, the stability of the equilibrium position is independent of the location of $X^* \in (1, \infty)$. Substituting back dimensional parameters and rearranging, one finds the maximum allowable working point pressure p_0 for which equilibrium positions $X^* \in (1, \infty)$ are stable:

$$p_0 < p_{0MAX} = \frac{\rho}{2} \left(\frac{\left(C_h k + A^2\right) d}{\alpha_D m A b} \right)^2 . \tag{10}$$

In practice, the working point of pressure control valves usually exceeds the maximum allowable pressure p_{0MAX}. Stability of the equilibrium changes from stable in the dead region to unstable as soon as the equilibrium position lies within $X \in (1, \infty)$.

Stability of the control valve with triangular groove and equilibria outside of the former dead region A means to achieve stable behavior is to modify the valve opening characteristic in such a way that the stability criterion is fulfilled for any equilibrium position X^*.

The ideal valve geometry results from a triangular drilling through which the fluid flows. Here, the flow is a quadratric function of the valve displacement. Taylor-expanding the flow function's squareroot expressions and non-dimensionalizing them, one obtains

$$\bar{Q} = \bar{Q}^{L\Delta} = \begin{cases} \gamma^{\Delta}(X+1)^2 \sqrt{\kappa - 1} \left(1 - \frac{1}{2(\kappa-1)} P(\tau)\right) + Q_L(-1) & \text{if } X < -1 \\ Q_L(X) & \text{if } -1 \le X \le 1 \\ -\gamma^{\Delta}(X-1)^2 \left(1 + \frac{1}{2} P(\tau)\right) + Q_L(1) & \text{if } X > 1 \end{cases} \tag{11}$$

and

$$\gamma^{\Delta} = \alpha_D \sqrt{\frac{2}{\rho}} \tan\left(\frac{\beta^{\Delta}}{2}\right) \frac{T u^2}{C_h \sqrt{p_0}}.$$

Here, β^{Δ} is the opening angle of the triangular flow cross section. Again neglecting first order pressure changes $P(\tau)$, the Hurwitz criterion for equilibrium positions $X^* \in (\infty)$ can be rearranged into the region within which X^* is stable:

$$X^* < 1 + \frac{\left(C_h k + A^2\right) d}{2 \alpha_D m A \sqrt{p_0} \tan\left(\frac{\beta^{\Delta}}{2}\right) u} \sqrt{\frac{\rho}{2}}. \tag{12}$$

Stability of the control valve with circular groove and equilibria outside of the former dead region The desired ideally triangular control edge cannot be realized by current means of manufacturing technology. In practice, the control edge will resemble a circle in some region of the triangle's tip. Therefore, the flow through a circular drilling of radius r° is considered, for which the flow function becomes:

$$\bar{Q} = \bar{Q}^{L\circ} = \begin{cases} Q^{1L\circ} + Q_L(-1) & \text{if } X < -1 \\ Q_L(X) & \text{if } -1 \le X \le 1 \\ Q^{3L\circ} + Q_L(1) & \text{if } X > 1 \end{cases}$$

For the volume flows, after linearizing in $P(\tau)$, one finds

$$Q^{1L\circ} = \gamma^\circ \left(-(\chi^\circ(X+1)+1)\sqrt{1-(\chi^\circ(X+1)+1)^2} + \arcsin(\chi^\circ(X+1)+1) + \frac{\pi}{2} \right) \sqrt{\kappa^2-1}(1 - \frac{1}{2(\kappa-1)}P(\tau)) \tag{13}$$

and

$$Q^{3L\circ} = \gamma^\circ \left((\chi^\circ(X-1)-1)\sqrt{1-(\chi^\circ(X-1)-1)^2} + \arcsin(\chi^\circ(X-1)-1) + \frac{\pi}{2} \right) (1 + \frac{1}{2}P(\tau)). \tag{14}$$

Here, the following constants were introduced:

$$\gamma^\circ = \alpha_D \sqrt{\tfrac{2}{\rho}} \tfrac{r^2 T}{C\sqrt{p_0}}, \quad \chi^\circ = \tfrac{u}{r}.$$

From the stability condition, one finds the region of stability:

$$X^* < 1 + \frac{r^\circ}{u} \left(1 - \sqrt{1 - \frac{\rho}{2} \left(\frac{(C_h k + A^2)d}{2mA\, r^\circ\ \sqrt{p_0}} \right)^2} \right), \tag{15}$$

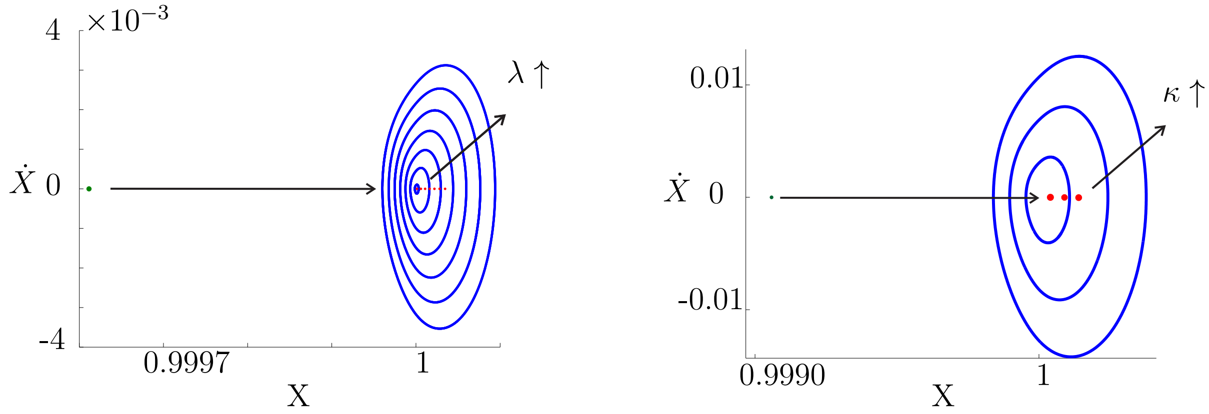
so that just like in the case of a triangular groove there always exists a region for $X^* \in (1, \infty)$ in which equilibria are stable.

4 STABLE LIMIT CYCLES WITH RELAXATION OSCILLATIONS

4.1 Birth of a limit cycle from the transition of the nonsmooth system to the system smoothened by leakage

According to section 3.2, the transition from the nonsmooth to the smooth system results in a unique equilibrium position moving towards the tank-side control edge while increasing leakage flow through augmenting λ or the pressure ratio κ. For a rectangular groove, once the equilibrium position lies in $\mathbb{X}^3 = (1, \infty)$, the equilibrium becomes unstable for a certain minimum level of $\gamma^\square$ and a stable limit cycle occurs to which the trajectory is drawn.

Figure 4(a) shows the transition from nonsmooth to smooth for $\gamma^\square$ exceeding the level required by condition (10). As predicted, the equilibrium position is stable while $X^* \in [-1, 1]$ and is moving along the X-axis of the phase plane while increasing the gap height λ. As λ is big enough for $X^* \in (1, \infty)$, the trajectory evolving from the unstable equilibrium position is drawn towards a limit cycle corresponding to gap height λ. Clearly, the limit cycle's amplitude depends on the gap height. The physical interpretation of the limit cycle is as follows: Let the initial conditions of the valve arbitrarily be such that it starts moving from a point $X \in [-1, 1]$. As leakage inflow exceeds leakage outflow, pressure $P(\tau)$ slowly increases and the valve opens. The relaxation from the excessive pressure takes place in a short time, causing the valve to overswing back into the interval $[-1, 1]$ where the process continues, resulting in the observed limit cycle.



(a) Birth of a limit cycle from increasing the gap height parameter λ. Parameters are: $D = 1.29$, $\omega = 38.73$, $a_A = 0.04$, $A = 2.06 \times 10^4$, $q_{L1} = 55.83$, $q_{L2} = 1.76 \times 10^2$, $\gamma^{\square} = 14.81$, $\kappa = 57.14$. Parameter range: $\lambda = 2.20 \times 10^{-3} - 2.50 \times 10^{-3}$.

(b) Birth of a limit cycle from increasing the pressure ratio κ. Parameters are: $D = 1.29$, $\omega = 38.73$, $a_A = 0.04$, $A = 2.06 \times 10^4$, $q_{L1} = 55.83$, $q_{L2} = 1.76 \times 10^2$, $\gamma^{\square} = 14.81$, $\lambda = 3 \times 10^{-3}$. Parameter range: $\kappa = 20 - 60$.

Figure 4: Stable limit cycles with relaxation oscillations for a valve with rectangular groove. Equilibrium positions $X^* \in [-1, 1]$ are stable (green points) and move to to tank side control edge as a_R or a_5^1 are increased. There, they become unstable (red points) and give rise to a limit cycle.

4.2 Birth of a limit cycle from increasing supplied pressure

For the respective stable limit cycle to appear, not only a certain gap height λ has to be given, but also a certain levels for q_{L2} and $\gamma^{\square}$. If q_{L2} is too low, the equilibrium position will always be $X^* \in [-1, 1]$. Hence, κ is a suitable parameter for a bifurcation scenario. Figure 4(b) shows the evolution of a limit cycle from increasing supplied pressure for a given λ and $\gamma^{\square}$. Qualitatively, the limit cycle is similar to the one depicted in Figure 4(a). Increasing supplied pressure also moves equilibrium positions. The stability map in figure 5 summarizes the regions of stability for varied parameters.

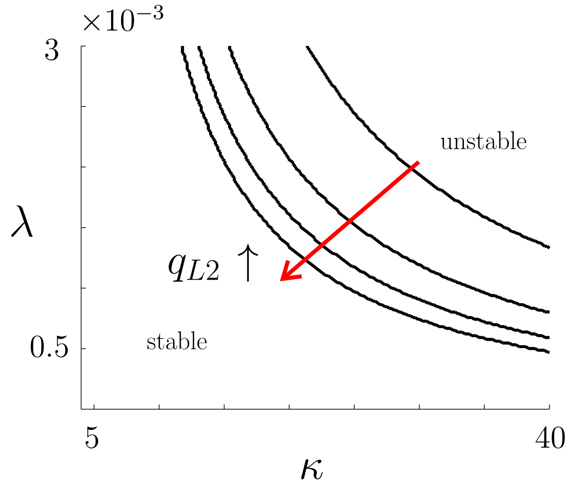


Figure 5: Stability map for varied working point of the system. $D = 1.29$, $\omega = 38.73$, $a_A = 0.04$, $A = 2.06 \times 10^4$, $q_{L1} = 55.83$, $\gamma^{\square} = 14.81$. Parameter range: $q_{L2} = 1 \times 10^3 - 5 \times 10^3$.

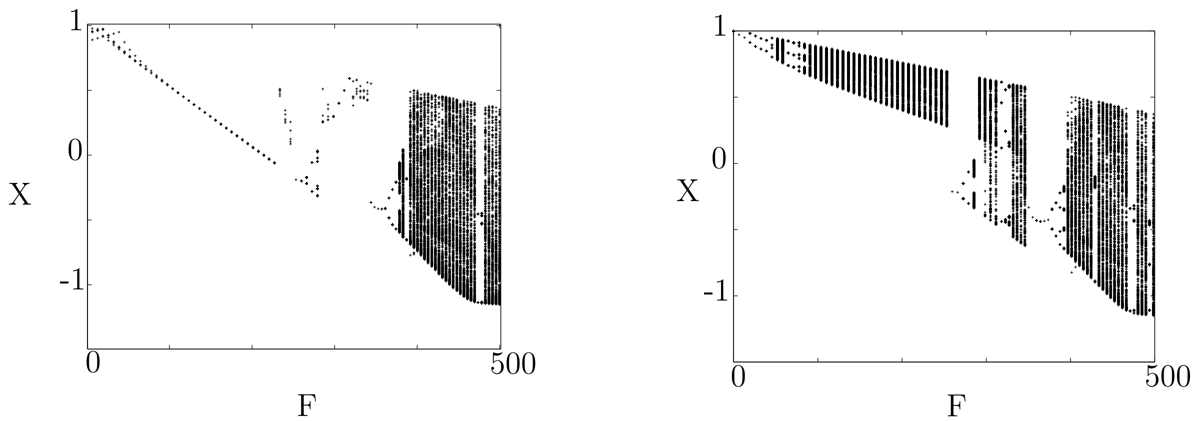
5 FORCED RESPONSE OF THE PRESSURE CONTROL VALVE

5.1 External Forcing

In applications, spool valves of the type presented are often subjected to high frequency excitation. This means an excitation with high frequencies relative to the eigenfrequencies of a system and low amplitudes. This is done to smoothen the frictional behavior of the system and to improve performance. Here, the system's behavior under excitation at a non-dimensional frequency $\eta = 47.12$ with forcing amplitudes ranging between $F = 0 - 500$ is investigated.

To draw conclusions on the dynamic response of the pressure control system to such forcing, a numerical bifurcation analysis is performed by means of Monte-Carlo simulations. Even though this method allows only for showing bifurcations to stable solutions, it is robust and and gives an often sufficiently detailed picture of the system behavior, see Parker/Chua [3] for further reference.

Bifurcation scenarios of the ideal pressure control valve with rectangular groove From the bifurcation diagram in Figure 6(a) it can be seen that the system behaves relatively well-mannered up to a forcing amplitude of around $F = 340$. Except for a minor interval of period-doubling between $F = 250$ and $F = 280$, the solutions are period-one overall. From distinct period one solutions for distinct initial conditions, the solutions converge to a unique period one solution between $F = 100$ and $F = 225$. Up from $F = 340$, period doubling leads to quasi-periodic and possibly chaotic behavior.



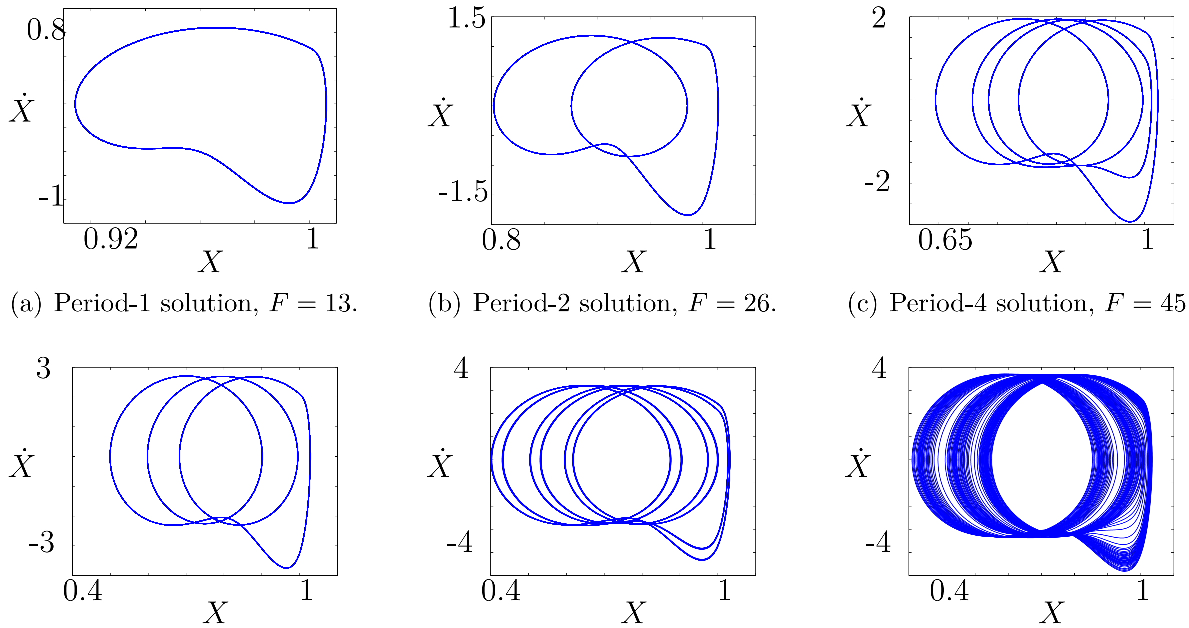
(a) Bifurcation diagram for the ideal valve model.

(b) Bifurcation diagram for the valve model featuring leakage.

Figure 6: Monte-Carlo bifurcation diagram for the model of an ideal valve with a rectangular groove and the model featuring leakage. Parameters are: $D = 1.29$, $\omega = 38.73$, $a_A = 0.04$, $A = 2.36 \times 10^5$, $q_{L1} = 4.88$, $q_{L2} = 2.01 \times 10^3$, $\gamma^{\square} = 4.30$, $\kappa = 20$, $\lambda = 3 \times 10^{-3}$, $\eta = 47.12$.

Bifurcation scenarios of the pressure control valve with rectangular groove and leakage In Figure 6(b) the computational results for the bifurcation scenario of the hydraulic valve with rectangular groove and leakage are shown.

Period-One solutions (Figure 7(a)) exist for forcing amplitudes up to $F = 20$. At $F \approx 20$, period doubling takes place for the first time, see Figure 7(b) for an example of two-period behavior. This leads to a short region of quasiperiodic behavior of the system for forcing amplitudes ranging between $F = 50 - 60$. Increasing the forcing amplitude further, the system finds back into periodic behavior with period three, see Figure 7(d). Via period-doubling, Figure 7(e), the system falls back into quasi-periodic behavior, Figure 7(f). At an excitation amplitude of around 260, the system becomes periodic again with period one, doubling the period again as the forcing amplitude is further increased.



(a) Period-1 solution, $F = 13$. (b) Period-2 solution, $F = 26$. (c) Period-4 solution, $F = 45$.

(d) Period-3 solution, $F = 65$. (e) Period-6 solution, $F = 78$. (f) Quasi-periodic solution, $F = 91$.

Figure 7: Stationary solution behaviors from the bifurcation diagram 6(b).

Comparing the bifurcation scenario of the model of an ideal valve with that of a valve model featuring leakage, it is evident the behavior of the model with leakage is more complex especially for low forcing amplitudes up to $F \approx 21$. The reason for this is that in this range the forcing amplitude is not big enough for the valve to leave the dead region. The oscillations simply take place in the dead region. In the case of an ideal valve, the system behaves like a one degree of freedom oscillator. When leakage is present, there is an interaction between hydraulics and mechanics leading to the period-doubling and quasi-periodic behavior that can be observed in the respective forcing interval.

As for the control valve with triangular or circular groove, no bifurcations occur under high frequency excitation in the parameter range considered. All solutions observed were periodic with the excitation's period.

6 Conclusion

In this paper, the model of a simple hydraulic pressure control device is presented. The transition from an ideally impermeable valve to a valve allowing for leakage flow is investigated for different valve geometries. Leakage changes the character of the equilibrium position from a set-valued equilibrium position to a unique one. Stability of the equilibrium position is investigated and the birth of a limit cycle from introducing leakage and increasing the pressure of the pressure supply is demonstrated. Subsequently, a bifurcation analysis shows the different solution types for the system under external forcing, yielding evidence of period-doubling phenomena up to quasi-periodic solutions. Future effort should be directed towards determining the nature of the solutions in more detail.

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