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DYNAMIC ANALYSIS OF CRACKED TIMOSHENKO BEAM HAVING GEOMETRIC NONLINEARITY

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Abstract. *In the present study linear and nonlinear dynamic responses of free bending vibration of a Timoshenko beam having a transverse crack have been obtained for clamped-clamped boundary condition. It is assumed that crack always remains open. Properties of material are restricted to have constant throughout the transverse and longitudinal direction. Beam is modelled as two sub beams connected by a mass less rotational spring. A jump in rotation is incorporated at the crack section. Transverse and longitudinal displacements are assumed to be continuous throughout. Governing Eigen value equations are derived from the potential and kinetic energy expressions using Ritz method and solved numerically by an iterative process to obtain linear and nonlinear vibration characteristics. For different order of modes different iterations are performed for mode shapes and Eigen frequencies. Effect of crack on the system characteristics are obtained by varying crack size and position. Frequencies corresponding to first three mode of vibration are compared for different crack parameters and different maximum amplitudes. Comparison of higher order mode shapes and frequencies are done for linear and nonlinear vibration of cracked beams. Effects of slenderness ratio on first three natural frequencies are obtained to show the effect of shear deformation and rotary inertia on the vibration characteristics of a Timoshenko beam.*

1 INTRODUCTION

Structural elements have a very wide area of application. Its characteristics have a huge impact on the capacity and efficiency of a structural system. It becomes much more important to analyze, detect and identify if any damage is present in the element.

In the ‘inverse problem’ crack parameters are estimated by knowing its dynamic characteristics. In ‘direct problems’ dynamic characteristics are estimated for known crack parameters. Effect of crack on linear and nonlinear vibration characteristics are observed by so many researchers in last few decades and introduced various techniques to identify and reduce the effects of damages in a structural system.

Dimarogonas [1] published a review of the state of the art of vibration based methods for testing cracked structures. Chondros et al [4] has analyzed the lateral vibration of cracked Euler-Bernoulli beams with single or double edge cracks. Resaee and Hassannejad [5] have proposed a new approach to free vibration analysis of a beam with a breathing crack based on mechanical energy balance method. They modeled the effect of opening and closing of a crack as fatigue crack. Variation in flexibility is determined from experimental data, using this flexibility relation and balancing mechanical energy dynamic response is carried out. Chasalevris and Papadopoulos [3] have studied the dynamic behaviour of a cracked beam with two transverse surface cracks. Each crack is characterized by its depth, position and relative angle. A local compliance matrix of two degrees of freedom, bending in the horizontal and the vertical planes is used to model the rotating transverse crack in the shaft. Dynamic response is calculated based on the available expressions of the stress intensity factors and the associated expressions for the strain energy release rates. Loya et al. [10] developed a model for cracked beam with a rotational and an extensional spring and solved for natural frequencies by a perturbation technique. Ramezani et al. [8] has shown effect of rotary inertia and shear deformation on vibration characteristics of a micro beam without any crack. Extended Hamilton’s principle is used to develop equation of motion and then solved it by multiple scale method. Bikrie [6] has developed theoretical investigation of the geometrically non-linear free vibrations of a clamped–clamped beam containing an open crack. The approach uses a semi-analytical model based on an extension of the Rayleigh–Ritz method to non-linear vibrations. The general formulation is established using new admissible functions, called ‘‘cracked beam functions’’ (CBF), which satisfy all the boundary condition. Iterative solution of a set of non-linear algebraic equations is obtained numerically. Then, an explicit solution is derived and proposed as an alternative procedure, simple and ready to use for engineering applications. Kitipornchai et al. [7] modeled the functionally graded Timoshenko beam as two sub beam connected by a rotational mass less spring. Using Ritz method followed by a direct iterative process, fundamental nonlinear frequencies are obtained.

Most of the literature on nonlinear dynamics of cracked beam focused on the fundamental (lowest) mode of vibration. Literatures explaining nonlinear effects on higher mode of vibration for Timoshenko cracked beam are rare. In the present work an attempt is made to capture the effect of crack on the fundamental as well as higher order modes of free vibration of an isotropic Timoshenko beam under large amplitude motion with clamped-clamped boundary condition. A parametric study is performed on natural frequencies of different higher order modes and when the beam vibrates with different maximum amplitude. Effects on first three order natural frequencies are obtained for different crack location. Effect of shear deformation and rotary inertia also influence vibration characteristics and these effects are estimated by observing vibration characteristics by varying slenderness ratio.

2 PROBLEM FORMULATION

Figure 1 (a) shows a beam having a transverse open crack of depth ‘a’ at a distance L_1 from fixed end. Beam length and depth are denoted by L and h, respectively.

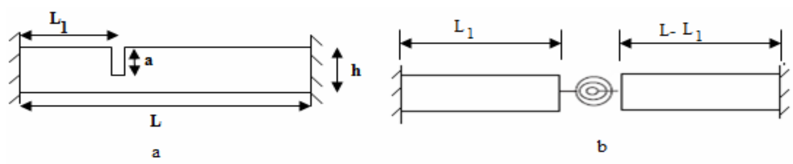

Figure 1: (a) Clamped-clamped beam with a crack and (b) Cracked beam modeled as rotational spring.

Beam with a transverse open crack is modeled as two sub beams connected by a mass less rotational spring as shown in Figure 1 (b). Continuity is assumed in the longitudinal and transverse displacements. Bending stiffness of the rotational spring can be given as in Eq. (1)

$$K_t = \frac{1}{G} \tag{1}$$

where flexibility, G due to crack can be expressed as

$$G = \int_0^{\varsigma} \frac{72\pi\left(1-\upsilon^2\right)\varsigma F^2(\varsigma)}{Eh^2} d\varsigma \,. \tag{2}$$

υ and E being the poisons ratio and young’s modulus of elasticity of the beam, respectively. $F(\varsigma)$ is a function of crack depth ratio given by Erdogan and Wu [11]. For isotropic material it can be expressed as

$$F(\varsigma) = 1.150 - 1.662\varsigma + 21.667\varsigma^2 - 192.451\varsigma^3 + 909.375\varsigma^4 - 3124.31\varsigma^5 + 2395.83\varsigma^6 - 1031.75\varsigma^7 \tag{3}$$

where crack depth ratio, ς is given by

$$\varsigma = \frac{\text{crack depth}}{\text{beam thickness}} = \frac{a}{h}$$

2.1 Displacement field, strain displacement relation

Displacement fields in space and time coordinates of Timoshenko beam can be given by

$$U(x,z,t) = U(x,t) + z\{\Psi(x,t)\} \text{ and } W(x,z,t) = W(x,t)) \tag{4}$$

where expression U(x, t), W(x, t) and $\Psi(x,t)$ represent longitudinal stretching, transverse and rotational displacement of midplane in space and time coordinates, respectively. Von Kerman type nonlinearity is incorporated in Strain-displacement relation. Strain and stress field may be expressed in terms of following relationships

$$\varepsilon_x = \frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2, \gamma_{xz} = \frac{\partial W}{\partial x} + \Psi,\ \sigma_{xx} = \frac{E}{1-\upsilon^2}\left[\frac{\partial U}{\partial x} + z\frac{\partial \Psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2\right], \sigma_{xz} = \frac{E}{2(1+\upsilon)}\left(\frac{\partial W}{\partial x} + \Psi\right) \tag{5}$$

where ε_x, γ_{xz}, σ_{xx} and σ_{xz} are normal strain, shear strain, normal and shear stress, respectively.

2.2 Energy equations

While deriving the Energy equations it is assumed that a part of the strain energy is associated due to crack and is proportional to the square of rotational discontinuity at the crack section. Thus kinetic and potential energy functional of the cracked beams are given by

$$KE=\frac{1}{2}\int_{0}^{L_1}\int_{-h/2}^{h/2}\rho\left\{\left[\frac{\partial U_1}{\partial t}+z\frac{\partial \Psi_1}{\partial t}\right]^2+\left(\frac{\partial W_1}{\partial t}\right)^2\right\}dzdx+\frac{1}{2}\int_{L_1}^{L}\int_{-h/2}^{h/2}\rho\left\{\left[\frac{\partial U_2}{\partial t}+z\frac{\partial \Psi_2}{\partial t}\right]^2+\left(\frac{\partial W_2}{\partial t}\right)^2\right\}dzdx$$

$$PE=\frac{1}{2}\int_{0}^{L_1}\int_{-h/2}^{h/2}\frac{E}{1-\upsilon^2}\left\{\left[\frac{\partial U_1}{\partial x}+z\frac{\partial \Psi_1}{\partial x}+\frac{1}{2}\left(\frac{\partial W_1}{\partial x}\right)^2\right]^2+\frac{E}{2(1+\upsilon)}\left(\frac{\partial W_1}{\partial x}+\Psi_1\right)^2\right\}dzdx+ \tag{6}$$

$$\frac{1}{2}\int_{L_1}^{L}\int_{-h/2}^{h/2}\frac{E}{1-\upsilon^2}\left\{\left[\frac{\partial U_2}{\partial x}+z\frac{\partial \Psi_2}{\partial x}+\frac{1}{2}\left(\frac{\partial W_2}{\partial x}\right)^2\right]^2+\frac{E}{2(1+\upsilon)}\left(\frac{\partial W_2}{\partial x}+\Psi_2\right)^2\right\}dzdx+\frac{1}{2}K_t(\Delta\Psi)^2.$$

In Eq. (6) subscripts 1and 2 stand for left and right sub beams, respectively. $\Delta\psi=\psi_2-\psi_1$. In order to simplify Eq. (6) harmonic motion is assumed $(\text{temporal function}=e^{i\Omega t})$ and stiffness and inertial parameters are taken as given below

$$\{K_1,K_2,K_3\}=\int_{-h/2}^{h/2}\frac{E}{1-\upsilon^2}\{1,z,z^2\}dz,\ K_4=\int_{-h/2}^{h/2}\kappa\frac{E}{2(1+\upsilon)}dz,\{M_1,M_2,M_3\}=\int_{-h/2}^{h/2}\rho\{1,z,z^2\}dz \tag{7}$$

For isotropic material K_2 and M_2 in Eq. (7) being zero. Since cracked beam is divided into two different sub beams, two different coordinates $(\xi and\eta)$ are assigned to represent longitudinal direction of left and right sub beam, respectively. Both the sub beams are solved individually using the following non-dimensional scheme

$$\xi=\frac{x}{L_1}\text{ and }\eta=\frac{x-L_1}{L-L_1} \tag{8}$$

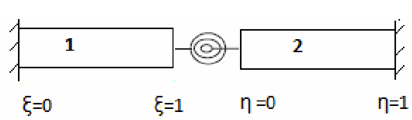

Figure 2: Sub beams with its normalized coordinates.

$$u=\frac{U}{h},w=\frac{W}{h},\psi=\Psi,\alpha=\frac{L_1}{h},\alpha_1=\frac{L-L_1}{h},\omega=\Omega\sqrt{L_1(L-L_1)\frac{M_1}{K_1}},\beta=\frac{L-L_1}{L_1}$$
$$\{m_3,k_3,k_4\}=\left\{\frac{M_3}{M_1h^2},\frac{K_3}{K_1h^2},\frac{K_4}{K_1}\right\}. \tag{9}$$

Incorporating Eqs. (8) and (9) into Eq. (6) and writing this equation for maximum kinetic and potential energy then multiplying each expression by $\frac{L-L_1}{K_1h^2}$, equation reduces to

$$KE^*_{\max} = \frac{\omega^2}{2}\left[\int_0^1 \left(u_1^2 + m_3\psi_1^2 + w_1^2\right)d\xi + \beta\int_0^1 \left(u_2^2 + m_3\psi_2^2 + w_2^2\right)d\eta\right]$$

$$PE^*_{linear} = \frac{\beta}{2}\int_0^1 \left\{\left(\frac{\partial u_1}{\partial \xi}\right)^2 + k_3\left(\frac{\partial \psi_1}{\partial \xi}\right)^2 + k_4\left(\frac{\partial w_1}{\partial \xi}\right)^2 + k_4\alpha^2\psi_1^2 + k_4\alpha\frac{\partial w_1}{\partial \xi}\psi_1\right\}d\xi$$
$$+\frac{1}{2}\int_0^1 \left\{\left(\frac{\partial u_2}{\partial \eta}\right)^2 + k_3\left(\frac{\partial \psi_2}{\partial \eta}\right)^2 + k_4\left(\frac{\partial w_2}{\partial \eta}\right)^2 + k_4\alpha_1^2\psi_2^2 + k_4\alpha_1\frac{\partial w_2}{\partial \eta}\psi_2\right\}d\eta + \frac{1}{2}k_t^*(\Delta\psi)^2 \tag{10}$$

$$PE^*_{nonlinear} = \frac{\beta}{2}\int_0^1 \left\{\frac{1}{\alpha}\left(\frac{\partial u_1}{\partial \xi}\right)\left(\frac{\partial w_1}{\partial \xi}\right)^2 + \frac{1}{4\alpha^2}\left(\frac{\partial w_1}{\partial \xi}\right)^4\right\}d\xi + \frac{1}{2}\int_0^1 \left\{\frac{1}{\alpha_1}\left(\frac{\partial u_2}{\partial \eta}\right)\left(\frac{\partial w_2}{\partial \eta}\right)^2 + \frac{1}{4\alpha_1^2}\left(\frac{\partial w_2}{\partial \eta}\right)^4\right\}d\eta$$

where $\left\{KE^*_{\max}, PE^*_{linear}, PE^*_{nonlinear}, k_t^*\right\} = \dfrac{L-L_1}{K_1 h^2}\left\{KE_{\max}, PE_{linear}, PE_{nonlinear}, K_t\right\}$.

Energy functional of the beam can be obtained as

$$\Pi = PE^*_{linear} + PE^*_{nonlinear} - KE^*_{\max} \tag{11}$$

In case of a clamped-clamped beam transverse, longitudinal and rotational displacement may be assumed to be zero at both ends, namely, at $\xi = 0$ and $\eta = 1$. At the crack location, continuity is assumed in transverse and longitudinal displacement. Discontinuity is allowed only in rotation leading to a moment of $k_t^*(\psi_2 - \psi_1) = M_{\text{at crcaked section}} = k_3\dfrac{\partial \psi_2}{\partial \eta}$ at cracked location. In order to satisfy the condition mentioned above, following trial functions are chosen as

$$u_1 = \sum_{j=1}^{j=N} A_j\xi^j(1-\xi) + \xi\sum_{j=1}^{j=N} a_j,\ u_2 = \sum_{j=1}^{j=N} a_j(1-\eta)^{j+1}$$
$$w_1 = \sum_{j=1}^{j=N} B_j\xi^j(1-\xi) + \xi\sum_{j=1}^{j=N} b_j,\ w_2 = \sum_{j=1}^{j=N} b_j(1-\eta)^{j+1} \tag{12}$$
$$\psi_1 = \sum_{j=1}^{j=N} C_j\xi^j(1-\xi) + \xi\sum_{j=1}^{j=N}\left\{1 + (j+1)\frac{k_3}{k_t^*}\right\}c_j,\ \psi_2 = \sum_{j=1}^{j=N} c_j(1-\eta)^{j+1}$$

Substituting Eq. (12) into Eq. (11) and minimizing the energy functional by taking derivative with respect to six unknown parameters one may obtain a set of nonlinear governing equations of cracked beam, which is given below

$$[K_{linear}]\{UC\} + [K_{nonlinear}]\{UC\} = \lambda[M]\{UC\}. \tag{13}$$

In Eq. (13) $[K_{linear}]$, $[K_{nonlinear}]$, $\{UC\}$, $[M]$ and λ are linear stiffness matrix, nonlinear stiffness matrix, unknown coefficients, mass matrix and frequency parameter (ω^2), respectively.

Elements of nonlinear stiffness matrix contain terms of unknown coefficients as well. In order to solve nonlinear problem, an iterative technique is employed with the following steps:

1. First all the nonlinear terms are neglected and a linear set of unknown coefficients are determined.
2. Linear set of unknown coefficients are scaled up in such a manner that the maximum nonlinear displacement occurs at the middle for first mode of vibration and is equal to the

assumed maximum amplitude of vibration (w_{max}). For the higher order nonlinear mode scaling is carried out according to the nature of that mode shape and assumed maximum amplitude.

3. Using the scaled unknown vectors nonlinear set of vectors are found out.
4. Nonlinear set of unknown coefficient vectors are again scaled up like step 2. Repeating the above process until relative error between two consecutive values of the non-linear frequency is less than 0.0001.

3 RESULTS AND VALIDATION

Above beam model is solved numerically by MATLAB code using gauss quadrature integration technique. Twenty four numbers of Gauss points are generated for each part of the beam in normalized co-ordinates (ξ and η). Geometric and material properties of beam are taken as follows E =70 Gpa, ν=0.33, □ =2780 Kg/m3, L/h=16, h=0.1 m, a/h = 0.2, κ =5/6. For all further calculation this properties will remain unchanged until and unless mentioned otherwise. Figure 3 (a) shows the results of the convergence study undertaken for first three linear natural frequencies with increasing number of polynomial terms. It is found that converged results are obtained when number polynomial terms are more than eight. Therefore numbers of polynomial terms are taken as eight for all subsequent analysis.

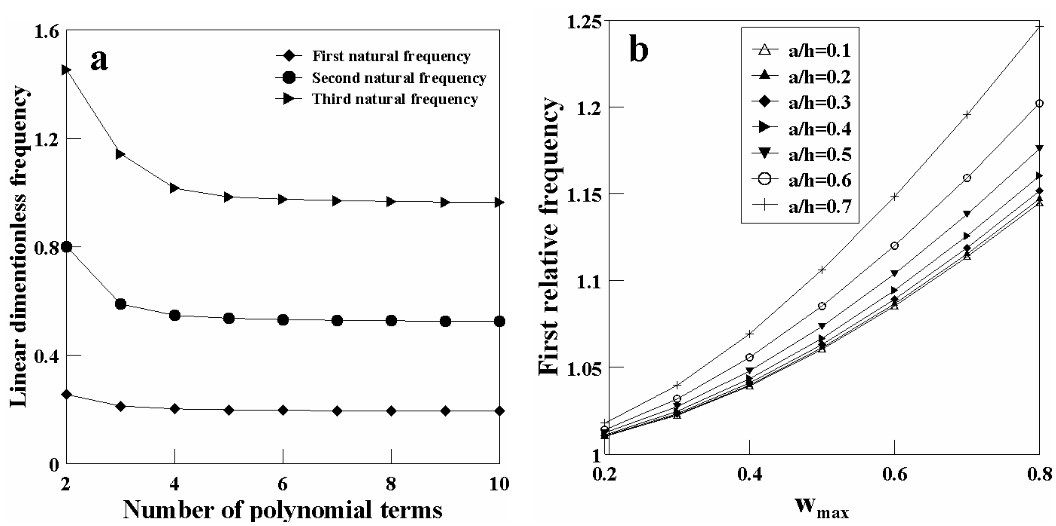

Figure 3: (a) Convergence of first three linear natural frequencies with increasing number of polynomials (b) First relative natural frequency vs.max amplitudes of vibration for diff. crack depth ratio.

Although linear frequencies are found to be independent of the amplitude of vibration but nonlinear frequencies are strictly dependent on the amplitude of vibration. Using the numerical technique mentioned in the previous section, nonlinear natural frequencies of the beam having mid crack with parameters $\frac{a}{h} = 0.1, \frac{L}{h} = 6, h = 0.1$ are obtained for different amplitude of vibration of the beam. Table.1 shows the results of the present analysis along with the results obtained by Kitipornchai et al [7]. It is observed that the present results matched fairly well with that of ref [7].

Figure 3 (b) shows the variation in the ratio of first nonlinear to linear frequency with respect to maximum amplitude of vibration for different crack depth ratio. L/h =16, h=0.1 and crack is located at the middle of the beam. It is observed that for larger amplitude of vibration nonlinear frequency increased significantly. Relative natural frequencies (ratio of nonlinear to linear) are increased with increasing crack depth ratio. Similar phenomenon was observed by Bikri et al. [6].

Non Dimention-alisation scheme	Dimension less linear freq. (ω_l)	Absolute linear freq. (Ω_l)	$\frac{\omega_{nonlinear}}{\omega_{linear}}$			
			$w_{\max} = 0.2$	$w_{\max} = 0.4$	$w_{\max} = 0.6$	$w_{\max} = 0.8$
Present $\omega = \Omega\sqrt{L_1(L-L_1)\frac{M_1}{K_1}}$	0.434455	7698.165	1.01621	1.07055	1.15213	1.24910
Ref [7] $\omega = \Omega L\sqrt{\frac{I_{10}}{A_{110}}}$	0.86882	7697.363	1.01842	1.07133	1.15294	1.25704

Table 1: Comparison of present results with reference [7].

Variation of second and third natural frequency for different maximum amplitude and depth for mid crack are given in Table 2 and 3, respectively. Second natural frequency is not influenced by crack when it is located at middle of the beam length. This is due to the fact that node of the second mode lies at the mid position of the beam having no vibration at all. Similar phenomenon is observed for all even numbers of natural frequencies.

Crack depth	Linear (ω_l)	nonlinear		
		$w_{\max}=0.15$	$w_{\max}=0.2$	$w_{\max}=0.25$
0.1	0.5265600000	0.5332574466	0.5383158727	0.5446258763
0.2	0.5265420000	0.5332574568	0.5383158821	0.5446258758
0.3	0.5265440000	0.5332574436	0.5383158623	0.5446258683
0.5	0.5265426000	0.5332574123	0.5383158682	0.5446258668
0.7	0.5265420000	0.5332572354	0.5383158536	0.5446258650

Table 2: Dimensionless second nonlinear frequency for different maximum amplitude.

Crack depth	Linear (ω_l)	nonlinear			
		$w_{\max}=0.12$	$w_{\max}=0.15$	$w_{\max}=0.20$	$w_{\max}=0.25$
0.1	0.9817800000	0.9931292114	0.9994196900	1.0127749855	1.0294613578
0.3	0.9443930000	0.9545468417	0.9602078105	0.9723156242	0.9876135084
0.5	0.8779400000	0.8874995783	0.8928748333	0.9044449661	0.9192273488
0.7	0.7987220000	0.8094631339	0.8154101867	0.8283114918	0.8448613929

Table 3: Dimensionless third nonlinear frequency for different maximum amplitude.

Figure 4 (a) shows effect of crack depth on the first normalized nonlinear mode shape for midcracked Timoshenko beam vibrating with significantly large dimensionless amplitude ($w_{\max}$=0.4). It can be observed that the curvatures increased at the end boundary and decreased at the mid position (cracked position) as crack depth increased. Similar phenomenon as observed for first nonlinear mode can be observed for third normalized nonlinear mode-shape at cracked section in Figure 4 (b). Location of crack affects nonlinear dynamic response of a cracked Timoshenko beam significantly. In present non-dimentionalisation scheme non-dimensional frequency contains a term of crack location distance from left fixed end (L_1). Therefore, variation of absolute frequency (in Hz) is shown instead of dimensionless frequency to show the exact variation with crack location. Figure 5 (a) shows variation in linear first natural frequency with crack depth for different crack location for beam with length L=1.6m and Figure 5 (b) shows the same for nonlinear first frequency.

First natural frequency of vibration for a clamped-clamped beam is less affected by crack when crack is situated near the fixed ends. Reduction in natural frequency increases when crack location shifts towards middle of the beam. Maximum reduction in the first natural frequency is obtained when the crack is in the middle of the beam. Approximately 0.4 percent reduction is found in linear first natural frequency for increasing crack depth ratio from 0.1 to 0.7 when crack is located at 0.3m from left fixed end. Reduction rate increased to approximately 12 percent when crack location is 0.56m from left fixed end. When crack is located at middle reduction in linear first frequency is found to be maximum approximately 22 percent. Figure 6 (a) shows variation in second absolute frequency (linear and nonlinear) with crack depth for crack at different location. When crack is located at middle, frequency changes insignificantly with the crack depth and when crack is located at 0.56m from the left fixed end, considerable decrease in second natural frequency is observed with increasing crack depth. From figure 6 (b) it can be observed that crack has negligible effect on third absolute frequency, when it is located at 0.56m (approximately a node of third mode) from the left fixed end. Linear and nonlinear third natural frequencies are found to be decreased significantly with increasing crack depth, when crack is located at 0.3m from the left fixed end.

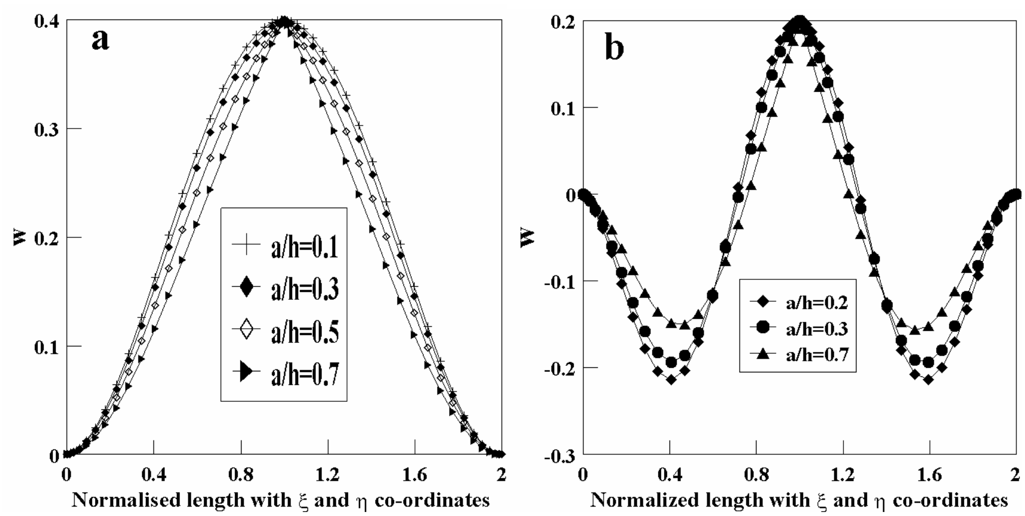

Figure 4: (a) First nonlinear modeshape of midcracked beam for diff. crack depth and w_{max}=0.4 (b) First nonlinear modeshape of midcracked beam for diff. crack depth and. w_{max}=0.2.

Effect of slenderness ratio is observed on dynamic characteristics of cracked Timoshenko beam. First natural frequency in dimensionless form is shown in Figure 7 (a) with varying L/h ratio for different crack depth ratio keeping beam depth h=0.1m as constant. Reduction in first natural frequencies is observed with increase in slenderness ratio. It can be observed that less slender beam or thicker beam is more affected by nonlinearity. As slenderness ratio is increased difference between nonlinear and corresponding linear frequency is decreased and this difference became very less when L/h ratio is increased to twenty.

Figure 7 (b) and 7 (c) shows the effect of slenderness ratio on second and third natural frequency, respectively. It has been observed again as in first mode of vibration that natural frequency decreased when slenderness ratio is increased and the nonlinear effect also decreased when slenderness ratio is increased.

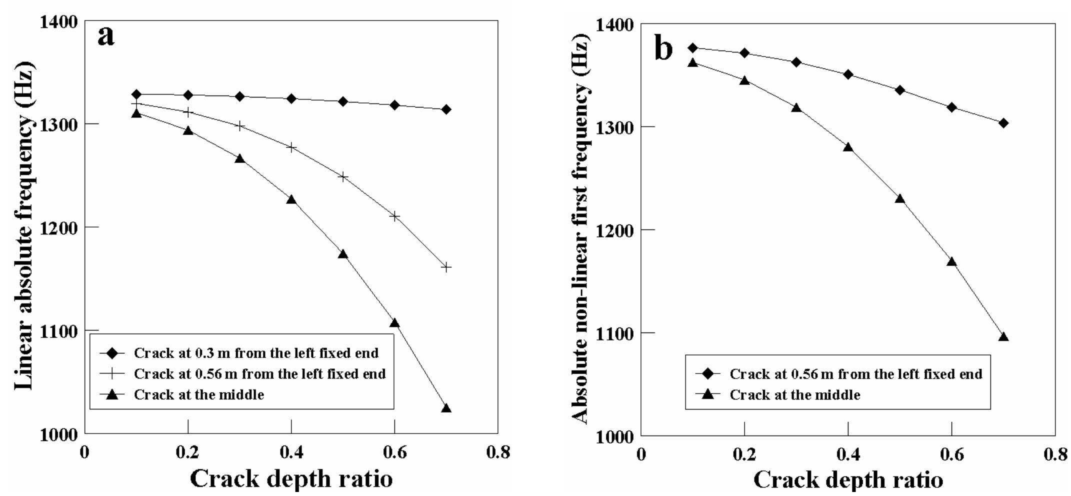

Figure 5: (a) First absolute linear freq. vs. crack depth ratio for different crack location (b) First absolute non-linear freq. vs. crack depth ratio for different crack location.

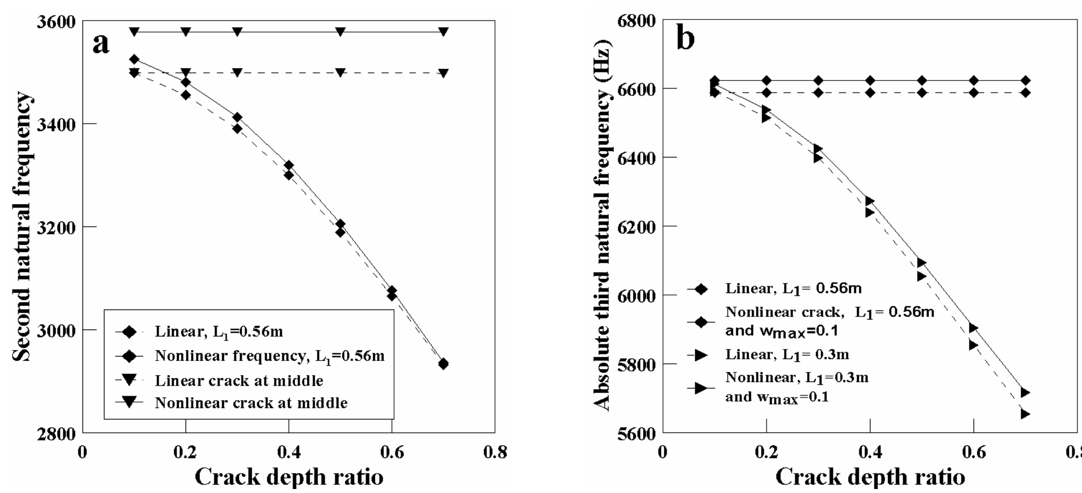

Figure 6: (a) Second absolute linear and non-linear freq. vs. crack depth ratio for different crack location (b) Third absolute linear and non-linear freq. vs. crack depth ratio for different crack location.

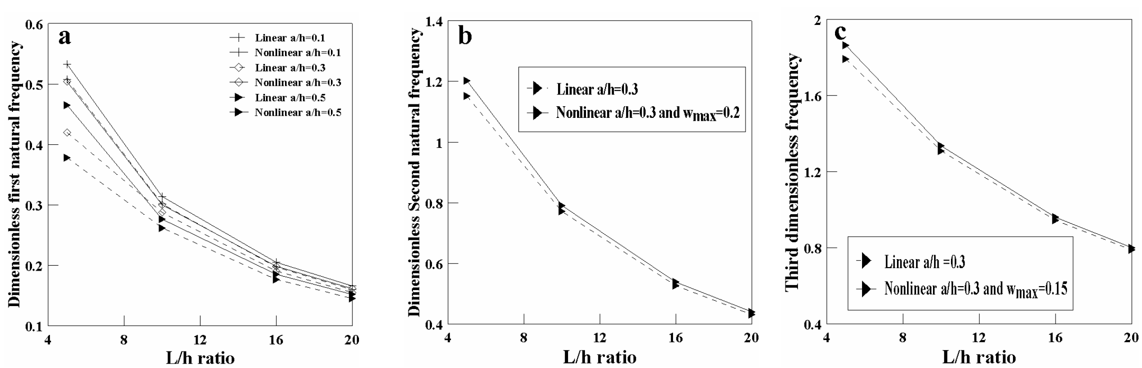

Figure 7: (a) First frequency vs. L/h ratio for different crack depth (b) Second frequency vs. L/h ratio for different crack depth (c) Third frequency vs. L/h ratio for different crack depth.

4 CONCLUSION

Timoshenko beam having a transverse crack is modeled as a rotational spring model. Linear and nonlinear behaviour of first three mode of free vibration of cracked Timoshenko beam is obtained. For Isotropic material nonlinear frequency of free vibration is not affected by sign of the maximum amplitude, both positive and negative cycle shows similar behaviour. Linear frequency does not depend on the amplitude but nonlinear frequency increased with increasing max amplitude for each mode of nonlinear vibration. For first mode of vibration, effect of crack is found severe for crack located at mid position and this effect reduced when crack location is shifted away from the mid position. For higher mode of vibration effect of crack depends on the position of node of vibration mode. If crack is located at any of the node of vibration mode its effect is negligible on both linear and nonlinear vibration characteristics. Natural frequencies are significantly reduced by increasing slenderness ratio. This is because of effect of shear deformation and rotary inertia reduces with increasing slenderness ratio and beam behaves like an Euler Bernoulli beam.

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