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MODELING OF MAIN LANDING GEARS SHIMMY AND SHIMMY-LIKE VIBRATIONS ON THE BASIS OF THE MULTI-COMPONENT ANISOTROPIC DRY FRICTION THEORY

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Abstract. *A new model for the shimmy and the high-frequency shimmy-like vibrations of main landing gears of modern aircrafts is proposed. The oscillations of wheels of main landing gears observed during operation of many aircrafts appear shortly after touchdown and differ significantly from the well-known shimmy of nose gears. The translational slip and spin of wheels result the inapplicability of the traditional models of the shimmy that are based on the non-holonomic constraint. On the other hand, the tire's deformations at the first stage of landing are small so that the effect of the friction forces to the rolling stability can be equal or even rank over the effect of the elastic forces in tires. This effect must be investigated on the basis of new models of rolling wheels considering the friction forces and torque and using no non-holonomic constraints.*

A qualitatively new model of the shimmy of rigid wheels was formulated by V. Ph. Zhuravlev and D. M. Klimov on the basis of the theory of combined dry friction. They have proven analytically that the shimmy of rigid gears can be caused by dry friction forces. Later this model was used to simulate the dynamics of the landing gears of the real aircrafts. It was shown both numerically and experimentally that the friction forces can induce the high-amplitude transient oscillations of main landing gears at the earliest stages of landing.

Here the development of these models is proposed. The friction anisotropy, non-circular form of contact spot and non-Hertzian contact pressure profile are allowed. Some numerical results are shown.

1 INTRODUCTION

A shimmy of nose landing gears of aircrafts is well investigated theoretically and experimentally. Contrary to nose gears, the oscillations of uncontrollable main landing gears observed during operations of some modern aircrafts have higher frequencies and higher velocities of wheels' running. These oscillations occur often shortly after touchdown, when the vertical force and the tire's deformations are relatively small. The combined frictional sliding and spinning at the low rotary speeds of the wheel are characteristic to first stages of non-steady rolling. This specificity makes inapplicable the well-known classical shimmy model developed by M. V. Keldysh [1], V. S. Gozdek [2], H. B. Pacejka [3] and many other authors that use the hypothesis of wheel's rolling with no sliding and spinning. The nonholonomic constraint being the groundwork of all traditional models becomes inconsistent with the observed character of wheel's motion. The qualitatively different shimmy theory proposed by V. Ph. Zhuravlev and D. M. Klimov and based on the model of combined rolling with both frictional sliding and spinning [4] was derived from the theory of combined dry friction of P. Contensou [5] and Th. Erissmann [6]. In [4, 7] it was proved that the unstable rolling of rigid wheels can be induced by the dry friction forces. The applicability of the model of rigid wheel's shimmy based on the combined dry friction theory [4, 7, 8, 9, 10] to real objects' computing has been investigated both theoretically and experimentally in [11] and [12]. It was shown that the shimmy-like high-frequency oscillations observed directly after touchdown of aircrafts can be induced by the dry friction forces. Here the improvement of the shimmy models [7, 11] is proposed.

2 STATEMENT OF THE PROBLEM

Let us consider a typical main landing gear with lateral wheels gab and longitudinal inclination (see Figure 1).

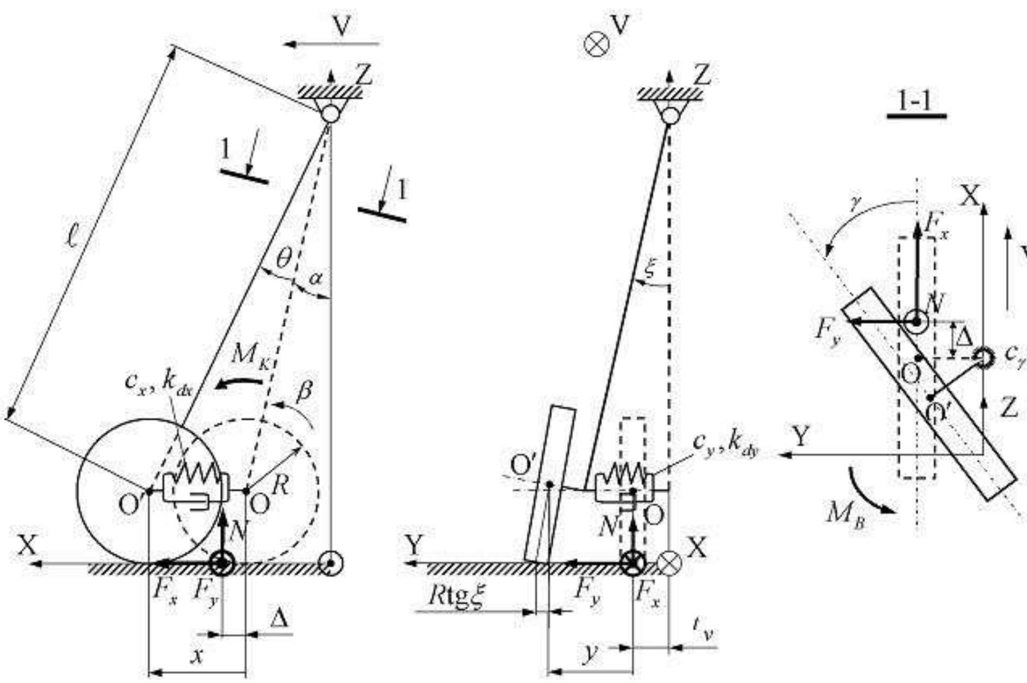

Figure 1: Structural layout of typical main landing gear

The aircraft have constant translational velocity $\mathbf{V}$ with no sideslip. Let the runaway's surface be plane and absolutely rigid and the wheel be slightly deformable so that the Hertzian approximation of the wheel deformed state can be used. The absolutely rigid gear is connected to the aircraft's frame by the elastic joint with small viscous damping. Let us also limit the mathematical model to the small linear oscillations of the wheel. Let us use the Cartesian frame attached to the aircraft as the main coordinate system with the longitudinal axe x co-directed to the plane velocity vector $\mathbf{V}$ (see Figure 1).

3 DYNAMIC EQUATIONS

The considered mechanical system is defined within the configurational space Ω by four generalized coordinates: two translational degrees of freedom of the wheel (the longitudinal displacement x and the lateral displacement y of the wheel centre of mass), the gear torsion angle γ and the wheel rotation angle denoted by β.

The potential energy of joint's deformation is defined by the formula

$$U\left(x,y,\gamma\right)=\frac{1}{2}\left(c_x x^2+c_y y^2+c_\gamma \gamma^2\right) \tag{1}$$

where c_x, c_y, c_γ are the reduced stiffness of the elastic joint. For the kinetic energy we have

$$T\left(\dot{x},\dot{y},\dot{\gamma},\dot{\beta}\right)=\frac{m}{2}\left[\left(\dot{x}-t_c\dot{\gamma}\right)^2+\dot{y}^2\right]+\frac{1}{2}J_{xy}\left(\frac{\dot{y}}{l\cos\alpha}-\dot{\gamma}\sin\alpha\right)^2+\frac{J_{z0}}{2}\dot{\gamma}^2\cos^2\alpha+$$
$$+J_{xz_0}\left(\frac{\dot{y}}{l\cos\alpha}-\dot{\gamma}\sin\alpha\right)+\dot{\gamma}\cos\alpha+\frac{1}{2}J_{y0}\left(\dot{\beta}-\gamma\frac{\dot{y}}{l}+\frac{1}{2}\gamma\dot{\gamma}\sin 2\alpha\right)^2; \tag{2}$$

here $t_c = t_v\cos\alpha$, m is the mass of the wheel, J_{ij} are the mass moments of inertia. The coefficients of the linear damping are denoted by k_{dx}, k_{dy} and $k_{d\gamma}$.

The contact interaction of the rolling wheel and the runaway surface is described by the normal reaction N, the forces of dry friction F_x, F_y, the dry friction torque M_z and the rolling friction torque M_y defined in the cartesian frame attached to the wheel's mass centre. Here the generalized forces of dry friction are considered as active forces [13] so that the d'Alembert-Lagrange principle can be used.

The dynamic equations for the system defined by Eqs. (1) and (2) can be written as Lagrange equations of the second kind in the next general representation:

$$m\left(\ddot{x}-t_c\gamma\right)+k_{dx}\dot{x}+c_x x=-F_x; \tag{3}$$

$$\left(m+\frac{J_{x0}}{l^2\cos^2\alpha}\right)\ddot{y}+\frac{J_{x0z0}-J_{x0}\tan\alpha}{l}\ddot{\gamma}-\frac{J_{y0}}{l}\left(\gamma\ddot{\beta}+\dot{\gamma}\dot{\beta}\right)+k_{dy}\dot{y}+c_y y=-F_y; \tag{4}$$

$$J_{y0}\ddot{\beta}-\frac{J_{y0}}{l}\left(\gamma\ddot{y}+\dot{y}\dot{\gamma}\right)+\frac{J_{y0}\sin 2\alpha}{2}\left(\ddot{\gamma}+\dot{\gamma}^2\right)=-M_k+F_x R; \tag{5}$$

$$J_\gamma\ddot{\gamma}-mt_c\ddot{x}+\frac{J_{x0z0}-J_{x0}\tan\alpha}{l}\ddot{y}+\frac{J_{y0}\sin 2\alpha}{2}\gamma\ddot{\beta}+\frac{J_{y0}}{l}\dot{\beta}\dot{y}+k_{d\gamma}\dot{\gamma}+c_\gamma\gamma=$$
$$=-M\cos\alpha+F_x\Delta-F_y\left(\Delta\gamma\cos\alpha-R\frac{y}{l\cos\alpha}-t_c\right). \tag{6}$$

Here the hyroscopic forces are taken into account. Let us note that the sleep velocity [14] $w=V-\dot{\beta}R+\dot{x}-t_c\dot{\gamma}$ need to define dry friction forces can be used as a new kinematic variable [7].

4 DRY FRICTION FORCES

The dry friction forces in the area of contact of the slightly deformed wheel and the runaway can be described on the basis of the combined dry friction theory proposed by P. Contensou [5] and Th. Erissmann [6] and improved later by V. Ph. Zhuravlev [8, 9, 10] and A. A. Kireenkov [15, 16]. At the first stage of landing shortly after touchdown the contact area can be approximately considered as a circle. This hypothesis together with the Hertzian solution for the contact presure was used by V. Ph. Zhuravlev and D. M. Klimov in their earliest publications [4, 7, 14] to define the stability boundary for the simplest landing gear model. At the later stages the contact spot becomes elliptic so that the simplest model used in [4, 7] cannot be used. The exact solution and the Pad'e approximations for the dry friction forces in the arbitrary contact spot were proposed by A. A. Kireenkov [15, 16] for the sliding without rolling. On the other hand the effect of the tread must be considered for the real tyres of aircraft landing gears together with the elliptic form of the contact spot. An anisotropic dry friction model proposed by V. V. Kozlov [13] can be introduced here as a first approximation.

Let us define the forces in the contact spot D by the differential formula proposed in [9, 10] and corresponding to the anisotropic friction law [13]:

$$d\mathbf{F} = -\mathbf{f} \cdot \mathbf{v} \frac{1}{|\mathbf{v}|} \sigma dS \quad (\mathbf{v} \neq 0) \tag{7}$$

where $\mathbf{v}$ denotes the total sliding velocity, σ is the contact pressure and $\mathbf{f} = f^{ij}\mathbf{e}_i\mathbf{e}_j$ is the dry friction tensor coefficient satisfying the non-negative definiteness condition [13]:

$$f^{ij} v_i v_j \geq 0. \tag{8}$$

Let us use the five-dimensional model of combined sliding, spinning and rolling friction [10] together with the anisotropic dry friction model (Eq. 7) [13] to construct the formulae for the friction forces and torque. Let us introduce two cartesian frames attached to the centre of the contact spot D, the frame $0x_0y_0z_0$ that is codirectional with the main cartesian frame $Oxyz$ and the rotating frame $O\xi^1\xi^2\xi^3$ hardly connected to the contact spot D [15]. The first frame is useful for the problem of dynamics (eqs. (3–6)) formulation and at the second one the contact spot's boundary ∂D depends not from the rotation angle $\gamma \cos\alpha$.

On the basis of [10] and taking into account that for the rolling wheel $\omega_x \equiv 0$ we have the total sliding velocity

$$\mathbf{v} = \left(V + \dot{x} - t_c\dot{\gamma} - \dot{\beta}R - \dot{\gamma}\cos\alpha\varepsilon R\bar{y}_0^2, \quad \dot{y} + \dot{\gamma}\cos\alpha\varepsilon R\bar{x}_0^2 \right), \tag{9}$$

where the dimensionless coordinates $\varepsilon\bar{x}_0 = x_0$, $\varepsilon\bar{y}_0 = y_0$ are introduced and $\varepsilon = \mathrm{diam}D$. For the rotating coordinates $\varepsilon\eta^i = \xi^i$, $i = 1, 2$ we have the orthogonal transformation [15]

$$\begin{aligned} \bar{x}_0 &= \eta^1 \cos\alpha \cos\gamma - \eta^2 \cos\alpha \sin\gamma; \\ \bar{y}_0 &= \eta^1 \cos\alpha \sin\gamma + \eta^2 \cos\alpha \cos\gamma. \end{aligned} \tag{10}$$

For the linear problem statement for small oscillations $\cos\gamma \approx 1$, $\sin\gamma \approx \gamma$, so that the Eq. 10 can be linearized as follows:

$$\begin{aligned} \bar{x}_0 &\approx \cos\alpha \left(\eta^1 - \eta^2\gamma\right); \\ \bar{y}_0 &\approx \cos\alpha \left(\eta^1\gamma + \eta^2\right). \end{aligned} \tag{11}$$

Let us suppose that the axis $Ox_0y_0z_0$ are the the principal axis of the dry friction tensor **f**:

$$(f_{ij})_{2\times 2} = \begin{pmatrix} f_1 & 0 \\ 0 & f_2 \end{pmatrix} \tag{12}$$

and $f_1 > 0$, $f_2 > 0$ to secure the non-negative definiteness provided by the Eq. 8.

For the rolling wheel we can define the deformed distribution of the contact pressure σ as follows [10]:

$$\sigma(\eta^1, \eta^2) = \sigma_0(\eta^1, \eta^2)\left(1 + h\varepsilon\eta^1\right) \tag{13}$$

where σ_0 is the static (or quasi-static) contact pressure and h is the dimensionless rolling friction coefficient.

Now we can define the resultant force and moment vector in the contact of the wheel and runaway. For the normal reaction and the rolling friction moment we have

$$N = \varepsilon^2 \int\int_D \sigma(\eta^1, \eta^2)\, d\eta^1 d\eta^2; \tag{14}$$

$$M_y = -\varepsilon^3 \int\int_D \eta^1 \sigma(\eta^1, \eta^2)\, d\eta^1 d\eta^2. \tag{15}$$

The longitudinal and lateral components of the dry friction force can be defined as follows:

$$\begin{aligned} F_x = -\varepsilon^2 \int\int_D \Big\{ &\left(f_1 + \gamma^2 f_2\right)\left[V + \dot{x} - t_c\dot{\gamma} - \dot{\beta}R - \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^2 + \gamma\eta^1\right)\right] + \\ &+ \gamma\left(f_1 - f_2\right)\left[\dot{y} + \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^1 - \gamma\eta^2\right)\right]\Big\} \frac{\sigma(\eta^1, \eta^2)}{v}\, d\eta^1 d\eta^2; \end{aligned} \tag{16}$$

$$\begin{aligned} F_y = -\varepsilon^2 \int\int_D \Big\{ &\gamma\left(f_1 - f_2\right)\left[V + \dot{x} - t_c\dot{\gamma} - \dot{\beta}R - \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^2 + \gamma\eta^1\right)\right] + \\ &+ \left(f_1 + \gamma^2 f_2\right)\left[\dot{y} + \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^1 - \gamma\eta^2\right)\right]\Big\} \frac{\sigma(\eta^1, \eta^2)}{v}\, d\eta^1 d\eta^2, \end{aligned} \tag{17}$$

where $v = v(\eta^1, \eta^2)$ denotes the modulus of the total sliding velocity in the arbitrary point $M(\eta^1, \eta^2)$ of the contact area D:

$$v^2 = \left[V + \dot{x} - t_c\dot{\gamma} - \dot{\beta}R - \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^2 + \gamma\eta^1\right)\right]^2 + \left[\dot{y} + \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^1 - \gamma\eta^2\right)\right]^2. \tag{18}$$

For the friction torque M_z we have the following expression:

$$\begin{aligned} M_z = -\varepsilon^3 \int\int_D \Big\{ &\left[V + \dot{x} - t_c\dot{\gamma} - \dot{\beta}R - \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^2 + \gamma\eta^1\right)\right] \times \\ \times &\left[\eta^1\gamma\left(f_1 - f_2\right) - \eta^2\left(f_1 + \gamma^2 f_2\right)\right] + \left[\dot{y} + \dot{\gamma}R\varepsilon\cos\alpha\left(\eta^1 - \gamma\eta^2\right)\right] \times \\ \times &\left[\eta^1\left(f_1 + \gamma^2 f_2\right) - \eta^2\gamma\left(f_1 - f_2\right)\right]\Big\} \frac{\sigma(\eta^1, \eta^2)}{v}\, d\eta^1 d\eta^2. \end{aligned} \tag{19}$$

The represented integral expressions, Eqs. (15–19), can be used directly for the computation of the dry friction force and moment's components during the numerical simulation of the landing gear's transient dynamics. Another effective way that was proposed in [9, 15] and used in [4, 7, 11] consists in use of the Padé approximations for the integral expressions (15–17), (19).

Below we suppose that the contact spot D is elliptic with the longest half-axe codirected to the main axe $O\eta^1$.

5 NUMERICAL EXAMPLES

Using the model of main landing gear defined by Eqs. (3–6) the transient dynamics of the gear at the first stage of landing was simulated (see Figure 2–3). The homogeneous Cauchy's problem was solved numerically for the landing gear with the properties that secures the stable rolling on the basis of most traditional shimmy models [1, 2, 3]. The main dynamic properties of the model including the viscous damping coefficients were obtained in the bench test using the standard drum bench. The friction coefficients $f_1,\ f_2$ and the dimensions of the contact spot were obtained from the statical test for some different reaction forces N for the first stage of landing.

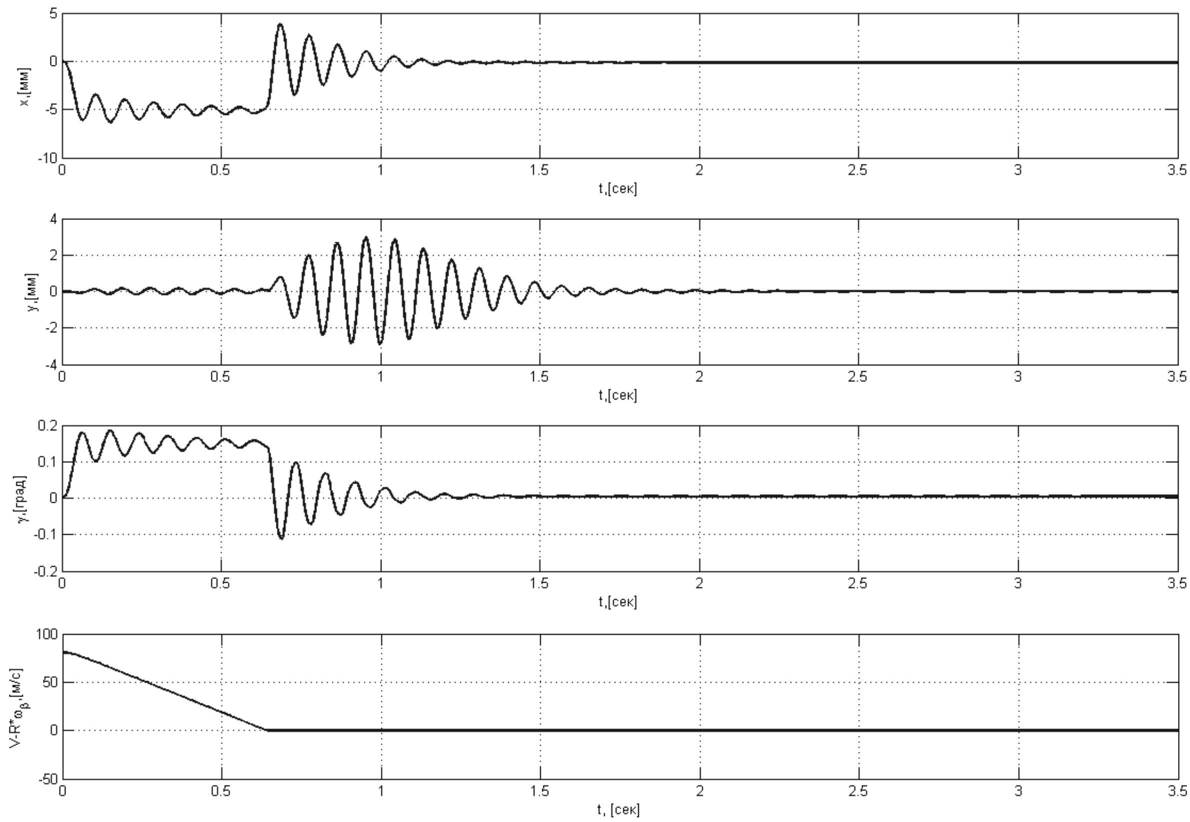

Figure 2: Dynamics of the wheel at the touchdown. Translations x, y, gear torsion γ and slip velocity $V - \dot{\beta}R$.

It was shown that even into the stability domain of the considered system the transient process due to non-steady rolling of the wheel with both translational slip and spin can be observed. The maximum amplitudes were observed directly after the end of non-steady wheel rolling when $\dot{\beta} = V/R$. The beating-like oscillation type can be observed on the Figure 2 mainly for the lateral vibration y with the 'slow' vibration period ≈ 0.7 s.

The effect of the dimensionless coefficient of rolling friction to the transient dynamics of the landing gear was investigated using the numerical simulation. It was shown that the increase of this coefficient reduces the lateral amplitudes rise so that confirms the result obtained in [4]. It can be also shown that the increase of the sliding friction coefficient f_1 reduces the spin-up time if the rolling friction coefficient is fixed and increases significantly lateral amplitudes y of the wheel vibrations.

The friction-induced oscillations can result dangerous accelerations n_x, n_y of the wheel in both longitudinal and lateral directions (see Figure 3) so that is often treated in engineering practice as a ”quasi-shimmy” vibrations.

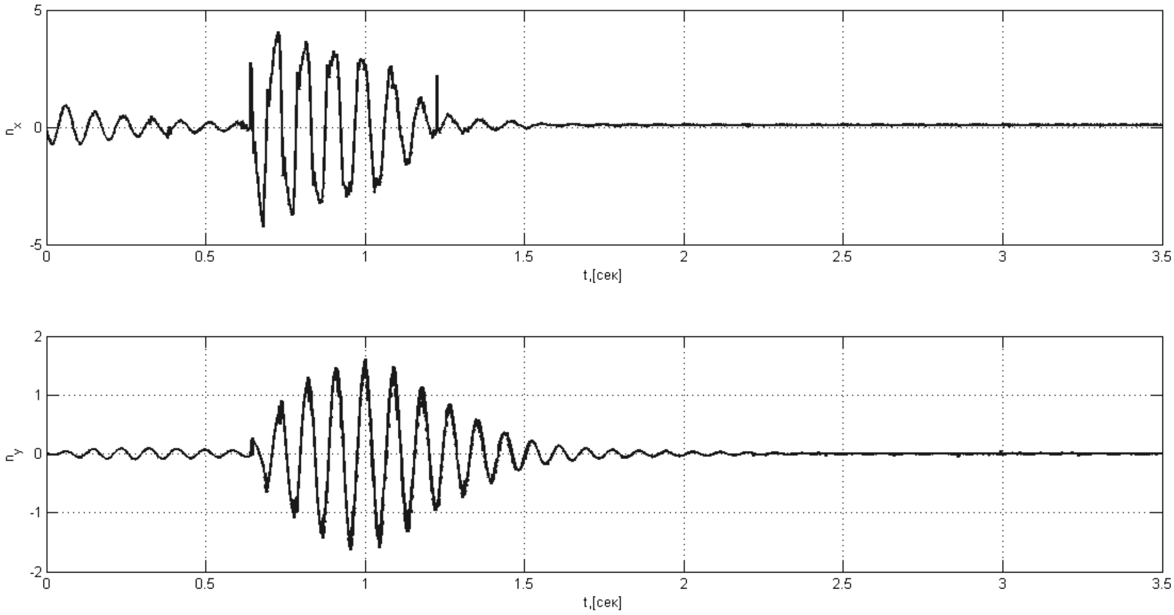

Figure 3: Dynamics of the wheel at the touchdown. Longitudinal n_x and lateral n_y accelerations of the wheel.

The maximum amplitude $n_x \approx 4.2g$ corresponds to the longitudinal oscillations and was observed at the point $t \approx 0.71$ s directly after the end of the wheel spin-up. Increase of the dimensionless rolling friction coefficient up to $10 \ldots 12$ reduces the lateral acceleration that exceeds $10g$; therefore the ”quasi-shimmy” appears if the rolling friction rises even if the well-damped system demonstrates the asymptotic stability.

6 CONCLUSIONS

- A new model of the main landing gear with lateral wheel’s gab and longitudinal inclination is constructed with no non-holonomic constraints so that the combined rolling and sliding can be simulated.
- The dry friction forces are defined on the basis of the combined dry friction and anisotropic dry friction theories, the sliding and rolling interaction is taken into account. The elliptic contact spot is considered.
- Some numerical solutions for the problems of transient dynamics of the landing gear are obtained. The initial stage of the landing ia simulated and it is shown that the dry friction forces at the stage of wheel’s spin-up and shortly after them can induce high-amplitude vibrations if even the system is stable from the point of view of the traditional shimmy theory. The effect of both sliding and rolling friction coefficients to the transient dynamics of the system is investigated.
- It is shown that the friction-based models can be useful to simulate the quasi-shimmy parametric vibrations of main landing gears of modern aircrafts.

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