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FURTHER STUDIES ON NONLINEAR VIBRATION ANALYSIS OF SHALLOW SHELLS

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Abstract. *When plates or shells are subjected to vibrate with large amplitudes, the analysis becomes at most difficult due to the complicated nature of the governing differential equations. Following Mazumdar [1], Banerjee, et.al [2] extended this Method to non-linear vibration analysis of plates. This simple but efficient method is known as the CDC (Constant Deflection Contour) Method. The beauty of this approach is to transform a set of 4th order PDE to a set of ordinary differential equations. A brief report on the Shell vibrations has already been published by the present authors [3]. The present paper includes briefly the non-linear vibration analysis of elastic-plastic shallow shells of arbitrary shapes. The Basic governing equations for nonlinear vibration analysis for such shells of arbitrary shapes have been deduced. Further numerical studies on elastic shell problem have also been made as an extended investigation of the previous attempt by the present authors [3] as by-product of the present investigation.*

1 INTRODUCTION

Due to the very complicated nature of the basic equations and their solution Mazumdar in 1970 proposed a new approach, which appeared to be quite suitable for bending analysis of elastic plates of arbitrary shapes based on the concept of iso-deflection contour lines on the bent surface of the plate [1]. This simple but efficient method is best known as Constant Deflection Contour Method or CDC-Method. Subsequently, the same method has been extended to the vibration analysis of plates and shallow shells [4,5].

The CDC method has so far been restricted to linear analysis until an attempt has been made recently to extend it to nonlinear analysis of plates [2]. Recently a similar approach as in [3] is undertaken for extension of the study to shallow shell analysis . The present paper may, therefore, be regarded as a sequel to earlier papers and deals with the non-linear vibration of shallow shells based on the CDC Method. Some specific examples on nonlinear vibrations of shallow shells have been included to show the efficacy of the method, and that the results are in excellent agreement with known results in the literature.

2 DERIVATION OF BASIC EQUATIONS

Consider an elastic, isotropic shallow shell of uniform thickness h subject to a continuously distributed normal load q . Let the equation of the middle surface of the shell referred to a system of orthogonal coordinates *xyz,* be given by [5]

$$z = \frac{x^2}{2R_x} + \frac{xy}{R_{xy}} + \frac{y^2}{2R_y} \tag{2.1}$$

where $r = \sqrt{x^2 + y^2}$ is small compared to the least of the radii of curvature, R_x, R_y and R_{xy} (supposed to be constants). If the shell is assumed to be comparatively thin and the displacements (u, v, w) are predominantly flexural.

Following Ilyushin's approach to Elastic-Plastic deformation [6], the bending moments $M_{x,}$, M_y and M_{xy} are given by the following relationships

$$M_x = -D_0(1-\Omega)\lfloor w_{xx} + \nu\, w_{yy} \rfloor,\; M_y = -D_0(1-\Omega)\lfloor w,_{yy} + \nu\, w,_{xx} \rfloor$$

$$\text{and } M_{xy} = -D_0(1-\Omega)(1-\nu)\, w,_{xy} \tag{2.2a}$$

Furthermore, since D and E are related as $D = Eh^3/12(1-\nu^2)$, when h and ν are constants, *D/E is a constant* . So when D assumes the form $D = D_0(1-\Omega)$, E changes to $E = E_0(1-\Omega)$, where

$$\Omega = \lambda\left(1 - \frac{3}{2e} + \frac{1}{2e^3}\right) \tag{2.2b}$$

Here $\Omega = 0$, when $e \le 1$ and the region is elastic and the region is plastic when $e \ge 1$, and

$$e^2 = \frac{h^2}{3e_s}\left(w_{xx}^2 + w_{yy}^2 + w_{xx}w_{yy} + w_{xy}^2\right) \tag{2.2c}$$

in which e_s is the yield strain and λ is a material constant. With usual notations using the expressions for the total strain energy, the kinetic energy and the work done and then formulating the Lagrangian and applying Hamilton's principle to it, a straightforward application of the variational calculus yields the following equations of motion [3]

$$(1-\Omega)\nabla^4 w - \frac{\partial\Omega}{\partial x}\left(2w_{xxx} + 2w_{xyy}\right) - \frac{\partial\Omega}{\partial y}\left(2w_{yyy} + 2w_{xxy}\right) - \frac{\partial^2\Omega}{\partial x^2}\left(w_{xx} + \nu w_{yy}\right) - \frac{\partial^2\Omega}{\partial y^2}\left(w_{yy} + \nu w_{xx}\right)$$

$$-2(1-\nu)\frac{\partial^2\Omega}{\partial x\partial y} w_{xy} = \frac{h}{D_0} S(w,F) - \frac{h}{D_0}\left(\frac{F_{yy}}{R_x} + \frac{F_{xx}}{R_y} - 2\frac{F_{xy}}{R_{xy}}\right) - \rho\frac{h}{D_0}\ddot{w} + \frac{q}{D_0} \qquad (2.3)$$

and

$$\nabla^4 F = E_0(1-\Omega)[\left(w^2{}_{xy} - w_{xx}w_{yy}\right) + \left(\frac{w_{yy}}{R_x} + \frac{w_{xx}}{R_y} - 2\frac{w_{xy}}{R_{xy}}\right)] - E_0\frac{\partial\Omega}{\partial x}\left(w_y w_{xy} - w_x w_{yy} + 2\frac{w_x}{R_y} - 2\frac{w_y}{R_{xy}}\right) -$$

$$-E_0\frac{\partial\Omega}{\partial y}\left(w_x w_{xy} - w_{xx}w_y + 2\frac{w_y}{R_x}\right) - E_0\frac{\partial^2\Omega}{\partial x^2}\left(\frac{1}{2}w^2{}_y + \frac{w}{R_y}\right) - E_0\frac{\partial^2\Omega}{\partial y^2}\left(\frac{1}{2}w^2{}_x + \frac{w}{R_x}\right)$$

$$+E_0\frac{\partial^2\Omega}{\partial x\partial y}(w_x w_y + 2\frac{w}{R_{xy}}) \qquad (2.4)$$

where suffixes in 'w' and 'F' denote partial differentiation w.r.t. the variables.

$$S(w,F) \equiv \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2} \cong w_{xx}F_{yy} - 2w_{xy}F_{xy} + w_{yy}F_{xx}$$

Here 'F' denotes well-known Airy-Stress function.

3 BASIC THEORY OF CDC-METHD

When the plate or the shallow shell vibrates in a normal mode, then at any instant t_0, the intersections between the deflected surface and the parallels *z = constant* yield contours which after projection onto the base plane z = 0 are a set of level curves, *U(x,y) = constant*, called the "Lines of Equal Deflections" [5], which are, in fact , iso-amplitude contour lines . The boundary of the plate or the shell irrespective of any combination of support is also a simple curve belonging to the family of lines of equal deflections.

As defined by Mazumdar [6] this family of nonintersecting curves may be denoted by C_u, where $0 \le U \le U^*$, so that C_0 (*U = 0*) is the boundary and C_U^* coincides with the point(s) at which the maximum *U = U** is attained (Figure 1.).

Let *U = U(x,y) = constant* be a member of the family of iso-deflection or iso-amplitude contour lines. Using the following transformations

$$\frac{\partial w}{\partial x} = w_x = \frac{dw}{dU}U_x \ , \ \frac{\partial w}{\partial y} = w_y = \frac{dw}{dU}U_y \ ,$$

$$w_{xx} = \frac{d^2 w}{dU^2}U_x^2 + \frac{dw}{dU}U_{xx} \ w_{xy} = \frac{d^2 w}{dU^2}U_x U_y + \frac{dw}{dU}U_{xy} \text{, etc…}$$

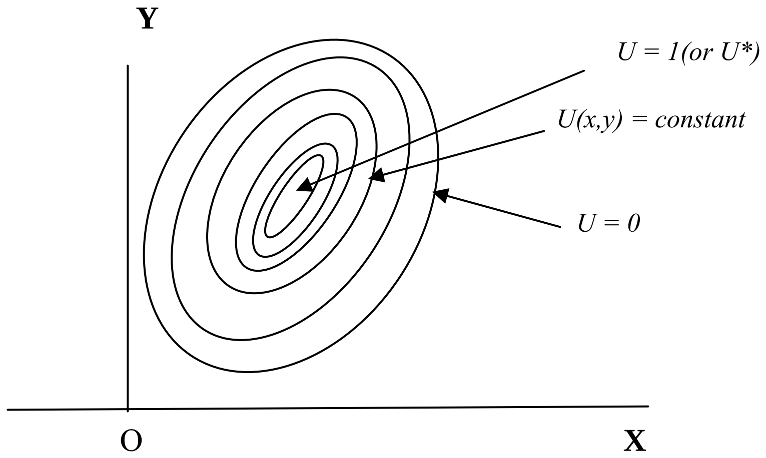

Figure 1. Iso-deflection Curves

Equations (2.6, 2.7) can be transformed into the following

$$\sum_{i=1}^{4} \lambda_i \frac{d^{5-i} w}{du^{5-i}} - \frac{d\Omega}{dU}\left[2\lambda_5 \frac{d^3 w}{dU^3} + \lambda_6 \frac{d^2 w}{dU^2} + \lambda_7 \frac{dw}{dU}\right] - \frac{d^2\Omega}{dU^2}\left[\lambda_8 \frac{d^2 w}{dU^2} + \lambda_9 \frac{dw}{dU}\right]$$
$$= \frac{h}{D_0}[\ \lambda_{10} \frac{d}{dU}\left(\frac{dw}{dU}\frac{dF}{dU}\right) + \lambda_{11} \frac{dw}{dU}\frac{dF}{dU}]$$
$$- \frac{h}{D_0}\left[\lambda_{12} \frac{d^2 F}{dU^2} + \lambda_{13} \frac{dF}{dU}\right] - \frac{q}{D_0} + \frac{\rho\ h}{D_0} w,_{tt} \tag{3.1}$$

$$\sum_{i=1}^{4} \lambda_i \frac{d^{5-i} F}{dU^{5-i}} = E_0 (1-\Omega)\left[\lambda_{14} \frac{d^2 w}{dU^2}\frac{dw}{dU} + \lambda_{15}\left(\frac{dw}{dU}\right)^2 + \lambda_{16} \frac{d^2 w}{dU^2} + \lambda_{17} \frac{dw}{dU}\right]$$
$$- E_0 \frac{d\Omega}{dU}[\frac{1}{2}\lambda_{14}\left(\frac{dw}{dU}\right)^2 + 2\lambda_{16}\frac{dw}{dU} + \lambda_{17} w] - E_0 \frac{d^2\Omega}{dU^2}[\lambda_{16} w] \tag{3.2}$$

where

$$\lambda_1 = \left(U_x^2 + U_y^2\right)^2 \qquad \lambda_2 = U_x^2\left(6U_{xx} + 2U_{yy}\right) + U_y^2\left(6U_{yy} + 2U_{xx}\right) + 8U_x U_y U_{xy},$$
$$\lambda_3, \ldots\ldots, \lambda_{11}, \ldots\ldots \lambda_{14}, \lambda_{15}, \ldots\ldots \lambda_{17} \quad with$$

$$\lambda_{12} = \left(\frac{U_y^2}{R_x} + \frac{U_x^2}{R_y} - 2\frac{U_x U_y}{R_{xy}}\right) = \lambda_{16} \qquad \lambda_{13} = \left(\frac{U_{yy}}{R_x} + \frac{U_{xx}}{R_y} - 2\frac{U_{xy}}{R_{xy}}\right) = \lambda_{17} \tag{3.3}$$

are all functions of partial derivatives of *U*

Since Eqns. (3.1) and (,3.2) are valid for all points on the surface of the shell, we can have

$$\iint_A \Big(\sum_{i=1}^{4} \lambda_i \frac{d^{5-i} w}{dU^{5-i}} - \frac{d\Omega}{dU}\left[2\lambda_5 \frac{d^3 w}{dU^3} + \lambda_6 \frac{d^2 w}{dU^2} + \lambda_7 \frac{dw}{dU} \right] - \frac{d^2\Omega}{dU^2}\left[\lambda_8 \frac{d^2 w}{dU^2} + \lambda_9 \frac{dw}{dU} \right] - \frac{h}{D_0}\Big[\lambda_{10} \frac{d}{dU}\left(\frac{dw}{dU}\frac{dF}{dU} \right) + \lambda_{11} \frac{dw}{dU}\frac{dF}{dU} \Big] + \frac{h}{D_0}\left[\lambda_{12} \frac{d^2 F}{dU^2} + \lambda_{13} \frac{dF}{dU} \right] + \frac{q}{D_0} - \frac{\rho h}{D_0} w,_{tt} \Big) dA = 0 \tag{3.4}$$

and

$$\iint_A \sum_{i=1}^{4} \lambda_i \frac{d^{5-i} F}{dU^{5-i}} dA$$

$$= \iint_A \Big\{ E_0 (1-\Omega) \left[\lambda_{14} \frac{d^2 w}{dU^2}\frac{dw}{dU} + \lambda_{15} \left(\frac{dw}{dU} \right)^2 + \lambda_{16} \frac{d^2 w}{dU^2} + \lambda_{17} \frac{dw}{dU} \right] - E_0 \frac{d\Omega}{dU}\Big[\frac{1}{2}\lambda_{14} \left(\frac{dw}{dU} \right)^2 + 2\lambda_{16} \frac{dw}{dU} + \lambda_{17} w \Big] - E_0 \frac{d^2\Omega}{dU^2}\left[\lambda_{16} w \right] \Big\} dA \tag{3.5}$$

where the integration is taken over the region bounded by any contour C_u. While performing the above integrals it would be more convenient to utilize the formula in the modified form care should be taken to evaluate first the contour integral [3] .

4 METHOD OF SOLUTION

The method of solution has been explained in Ref. [3] to obtain the *Time Differential Equation* as

$$\ddot{\Psi}(t) + L\,\Psi(t) + M\,\Psi^2(t) + N\,\Psi^3(t) = q^*, \tag{4.2}$$

from which all sorts of static and dynamic analysis of the structure can be made.

6 SPECIFIC ILLUSTRATION

6.1 Large Vibration of a shallow Dome upon an Elliptic Base

Consider the vibration of a shallow dome of nonzero Gaussian curvature upon an elliptic base. Fig. 2 depicts the geometry of the shell. The edges are clamped and immovable. When the shell vibrates in a normal mode, the lines of equal deflections, as described in Sec.3, may reasonably be taken as

$$U(x,y) = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{6.1.1}$$

Performing the required integrals in equations (3.4) and (3.5) the resulting equation will take the forms

$$(1-\Omega)\left[(1-U)^2\frac{d^3w}{dU^3}-2(1-U)\frac{d^2w}{dU^2}\right]-\frac{d\Omega}{dU}\left[(1-U^2)\frac{d^2w}{dU^2}-2\frac{P^*}{P}\frac{dw}{dU}\right]-$$
$$=-\alpha(1-U)\left(\frac{dw}{dU}\frac{dF}{dU}\right)-\beta(1-U)\frac{dF}{dU}$$
$$-\frac{q}{2D_0P}(1-U)-\frac{\rho h}{2D_0P}\int_1^U w_{tt}\,dU=0$$

(6.1.2)

and

$$(1-U)^2\frac{d^3F}{dU^3}-2(1-U)\frac{d^2F}{dU^2}$$
$$=(1-\Omega)\left[\gamma(1-U)\frac{d^2w}{dU^2}dU+\delta(1-U)\frac{dw}{dU}\right] \quad (6.1.3)$$
$$-\frac{d\Omega}{dU}[\int_1^U \gamma(1-U)\left(\frac{dw}{dU}\right)^2+\delta(1-U)w]$$

where

$$\alpha=\frac{4h}{DPa^2b^2},\qquad \beta=\frac{h}{DP}\left(\frac{\kappa}{a^2b^2}\right),\quad \gamma=\frac{2E_0}{Pa^2b^2},\quad \delta=\frac{E_0\kappa}{Pa^2b^2}$$

To solve for the stress function, earlier in Ref.[3] we differentiated both sides of the governing eqations w.r.t. U after considering the average value of the parameters involving *‘e’* in terms of e_0, the central plastic strain. It is assumed here that once the steady state is reached these quantities do not change with time. The value of e_0 is determined from Eq. (2.2c) for $U = 1$ [7].

Compatible with boundary conditions for ‘*w*’ may be assumed as a power series [2]. A single term approximation with a Galerkin procedure yields quite satisfactory and excellent comparable results minimizing the computational hazards. So, let us assume ‘w’ for the said structure with clamped immovable edges as [2]

$$w(U,t)=h\,W(U)\,\Psi(t)=h\,\Psi(t)\sum_{j=2}^{n}U^j\approx hU^2\Psi(t) \quad (6.1.4)$$

With this value of “w’ Eqn. (6.1.3), the first and the second integrals may be obtained. and finally

$$\left\{(1-U)\frac{dF}{dU}\right\}=(1-\Omega)\left[\frac{1}{4}lU^3+\frac{1}{3}mU^2\right]+\frac{d\Omega}{dU}\left[p\left(-\frac{9}{5}U^5+U^4+2U^3+16U^2\right)+\frac{1}{4}QU^3\right]+C_1U+C_2$$

(6.1.5)

where

$$l=\frac{4\gamma h^2\Psi^2}{3},\quad m=h\delta\,\Psi\,,\quad p=(\gamma h^2\,\Psi^2)/36\,,\ Q=\frac{h\,\delta\,\Psi}{3} \quad (6.1.6)$$

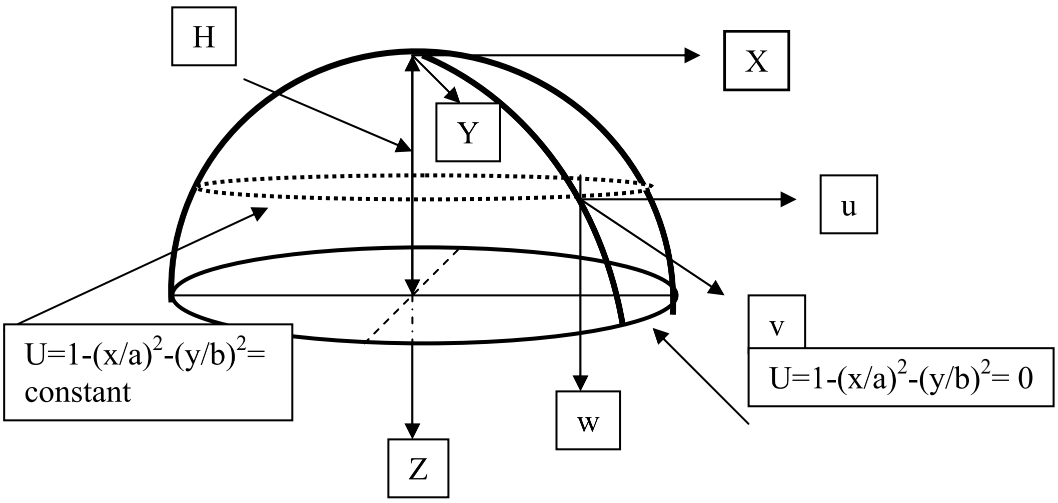

Figure 2: Schematic Diagram of the Shell

Considering the case for a clamped immovable edge condition set the following conditions on *"F"* [3]

$$\left.\frac{dF}{du}\right|_{u=0} = 0 \qquad and \qquad \left.\left\{(1-u)\frac{d^2F}{du^2} - 2(1-\nu)\frac{dF}{du}\right\}\right|_{u=0} = 0$$

which makes both C_1 and C_2 to be zero and Eqn. (6.1.5) reduces to

$$\left\{(1-U)\frac{dF}{dU}\right\} = (1-\Omega)\left[\frac{1}{4}lU^3 + \frac{1}{3}mU^2\right] + \frac{d\Omega}{dU}\left[p\left(-\frac{9}{5}U^5 + U^4 + 2U^3 + 16U^2\right) + \frac{1}{4}QU^4\right] \quad (6.1.7)$$

Our next step is to solve Eqn. (6.1.2) . To avoid the integral in the term involving w_{tt} let us differentiate both sides of Eqn.(6.1.2) w.r.t. *U,* to obtain

$$\begin{aligned}&(1-\Omega)_0\left[(1-U)^2\frac{d^4w}{dU^4} - 4(1-U)\frac{d^3w}{dU^3} + 2\frac{d^2w}{dU^2}\right] \\ &- \left[\frac{d\Omega}{dU}\right]_0 \frac{d}{dU}\left[(1-U^2)\frac{d^2w}{dU^2} - 2\frac{P^*}{P}(1-U)\frac{dw}{dU}\right] - \\ &\quad + \alpha\frac{d}{dU}\left\{(1-U)\left(\frac{dw}{dU}\frac{dF}{dU}\right)\right\} + \frac{d}{dU}\left\{\beta(1-U)\frac{dF}{dU}\right\} \\ &\quad - \frac{q}{2D_0P} + \frac{\rho h}{2D_0P}w_{tt} = 0\end{aligned}$$

(6.1.8)

On substitution of assumed values of 'w' and $\left\{(1-U)\frac{dF}{dU}\right\}$ from Eqns (6.1.4) and (6.1.7) in Eqn. (6.1.8), the error function is obtained, which on application of Galerkin process yields the following time differential equation as

$$\rho h^2\,\ddot{\Psi}+\alpha_1\Psi+\alpha_2\Psi^2+\alpha_3\,\Psi^3=Q^* \tag{6.1.9}$$

where

$$\left.\begin{aligned}
\alpha_1 &= [\,(10D_0P)\left(\frac{4}{3}+\frac{1}{5}\beta\,\delta\right)+\left(\frac{1}{3}+\frac{2}{3}\frac{P^*}{P}+\frac{1}{18}\beta\delta\right)\left(\frac{d\Omega}{dU}\right)_0]h\\
\alpha_2 &= (10D_0P)\left[(1-\Omega)_0\left(\frac{4}{9}\alpha\delta+\frac{2}{9}\beta\lambda\right)+\left(\frac{5}{43}\alpha\delta+\frac{94}{945}\beta\gamma\right)\left(\frac{d\Omega}{dU}\right)_0\right]h^2\\
\alpha_3 &= \frac{10}{21}D_0P(\alpha\gamma)\left[10(1-\Omega)_0+\frac{1889}{90}\left(\frac{d\Omega}{dU}\right)_0\right]h^3 \qquad Q^*=\frac{5}{3}q
\end{aligned}\right\} \tag{6.1.10}$$

Equation (6.1.9) with Eqn. (6.1.10) is set for all kind of Static and Dynamic analysis of Shells for this particular case.

7 VERIFICATION OF THE RESULTING GOVERNING EQUATIONS

A. **Nonlinear Analysis of Elastic Plates**

Set $\Omega=0$, when (6.1.8) and (6.1.9) turn out exactly equal to be those as previously obtained as Eqs. (6.3) and (6.4) in ref. [3].

B. Linear Static Analysis of Elastic-Plastic Shells

Deleting the inertial term and the stress function from Eqn. (6.1.2) one gets

$$(1-\Omega)\left[(1-U)\frac{d^3w}{dU^3}-2\frac{d^2w}{dU^2}\right]-\frac{d\Omega}{dU}\left[(1-U)\frac{d^2w}{dU^2}-2\frac{P^*}{P}\frac{dw}{dU}\right]$$
$$=-\frac{q}{2D_0P} \tag{7.3}$$

This is exactly the same governing equations obtained in Ref. [7]. This confirms that though the approach for deduction of governing differential equations are different the present approach appears to be a simpler one.

8 SOME DEDUCTIONS

8.1 Free Linear Vibration

Set $\alpha_2,\alpha_3 and\ Q^*$ equals to zero, when the linear frequency ω^*_{LP} is given by

$$\omega^{*\,2}_{LP}=\frac{Eh^2P}{8\rho}\left[\frac{(320/3)}{12(1-\nu^2)}+4\left(\frac{2\gamma_1}{h}\right)^2\right]+\frac{E_0h^2P}{72\rho}\left\{\frac{2}{(1-\nu^2)}+\frac{4}{(1-\nu^2)}\frac{P^*}{P}+\left(\frac{2\gamma_1}{h}\right)^2\right\}\left(\frac{d\Omega}{dU}\right)_0 \tag{8.1.1}$$

where $\gamma_1 = \left(\dfrac{\kappa}{Pa^2b^2}\right)$ and $\dfrac{2\gamma_1}{h} = \gamma^*$ represents the measure of shallowness of the shell. For Elastic shells, the corresponding linear frequency ω^*_{LE} is obtained from (8.1) by setting $\Omega = 0$, as

$$\omega_{LE}^2 = \frac{Eh^2P}{8\rho}\left[\frac{(320/3)}{12(1-\nu^2)} + 4\left(\frac{2\gamma_1}{h}\right)^2\right]$$

which exactly equals to those obtained in Refs. [3,5,8] after making some simplification or with a little rearrangement of terms involving the parameters.

8.2 Free Nonlinear Vibrations

Set $Q^* = 0$ in Eqn.(6.1.9) to get

$$\ddot{\Psi} + \frac{\alpha_1}{\rho h^2}\Psi + \frac{\alpha_2}{\rho h^2}\Psi^2 + \frac{\alpha}{\rho h^2}\Psi^3 = 0 \quad (8.2.1)$$

Eqn.(8.2.1) is a familiar form of time differential equation and for which the frequency ratio (Nonlinear to Linear) is given by

$$\frac{\omega^*_{PNL}}{\omega^*_{PL}} = \left[1 + \left\{\frac{3}{4}\frac{\alpha_3}{\alpha_1} - \frac{5}{6}\left(\frac{\alpha_2}{\alpha_1}\right)^2\left(\frac{A}{h}\right)^2\right\}\right]^{1/2} \quad (8.2.2)$$

where *(A/h)* represents the relative amplitude.

8.3 Static Analysis

Set $\ddot{\Psi} = 0$ in Eqn.(6.1.9) **,** when Ψ represents the maximum deflection. On substitution of the values of the coefficients in the transformed equation the following equation depicts the load-deflection relationship with a little bit of simplifications as

$$\begin{aligned} &L_1\left[(1-\Omega)_0\left\{\frac{2}{3(1-\nu^2)} + \frac{3}{10}\gamma^{*2}\right\} + \frac{1}{2(1-\nu^2)}\left\{\frac{1}{3} + \frac{2}{3}\frac{L_3}{L_2} + \frac{1}{6}(1-\nu^2)\gamma^{*2}\right\}\left(\frac{d\Omega}{dU}\right)_0\right]\Psi \\ &+ L_1\gamma^*\left[(1-\Omega)_0\left\{\frac{16}{3L_2} + \frac{4}{3}\right\} + \left\{\frac{60}{43L_2} + \frac{188}{315}\right\}\left(\frac{d\Omega}{dU}\right)_0\right]\Psi^2 \\ &+ \frac{L_1}{L_2}\left[(1-\Omega)_0\frac{160}{7} + \frac{1889}{1890}\left(\frac{d\Omega}{dU}\right)_0\right]\Psi^3 = \frac{qa^4}{E_0h^4} \end{aligned} \quad (8.3.1)$$

where

$$L_1 = \left[3\left(\frac{a}{b}\right)^4 + 2\left(\frac{a}{b}\right)^2 + 3\right],\ L_2 = \left[3\left(\frac{a}{b}\right)^2 + 2 + \left(\frac{b}{a}\right)^2\right],\ L_3 = \left[\left(\frac{a}{b}\right)^2 + 2\nu + \left(\frac{b}{a}\right)^2\right]$$

9 CONCLUSION

It can therefore be concluded that the CDC method appears to be a simple tool to deal with the problems of nonlinear vibration of plates and shallow shells of arbitrary shapes. The application of polynomial expressions for the deflection and the stress functions in conjunction with the Galerkin procedure appears to produce highly accurate results. There remains further scope for future investigations.

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