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OPTIMIZATION OF THE FREQUENCY AVERAGED INPUT POWER INTO PLATES WITH DYNAMIC VIBRATION ABSORBERS USING THE WAVE BASED METHOD

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Abstract. *When it comes to the vibro-acoustic analysis of panels there is a strong interest in reducing the radiated sound power which can be effectively decreased by minimizing the power injected into the plate. In this paper, a thin plate with Dynamic Vibration Absorbers (DVA) attached to it, is considered and optimized in order to reduce the average input power injected. Instead of classic numerical quadrature schemes, the residue theorem is used to evaluate the average input power over a frequency band. This results into a considerable reduction of computational efforts, as it requires only few function evaluations at complex frequencies, regardless of the bandwidth considered. Properties and location of the DVAs are chosen as design variables to minimize the average power injected into the plate by using genetic algorithms. The dynamic system behaviour is simulated by using the Wave Based Method (WBM). Next to an increased convergence rate as compared to classical element based techniques, the WBM is also free in determining the optimal position of the DVAs unlike the element based approaches, which restrict the optimum position to nodal grid locations. Moreover, when point connections are taken into account, only a small part of the WB system matrices needs to be recomputed at each iteration, resulting in a strong reduction of the computational effort. Numerical examples illustrate the benefits and the efficiency of the proposed optimization strategy.*

1 INTRODUCTION

Reducing the vibration of a mechanical structure and its radiated noise is a challenging goal for vibro-acousticians. In these regards, optimization techniques are very helpful to identify the best performing configuration. However, when the target is a band rather than a single frequency, the computation can be prohibitively demanding. In fact, classic quadrature techniques may require a large number of samples to achieve accurate estimates of the band-averaged values and this dramatically slows down the computation.
In order to overcome this limitation, a recently proposed approach is adopted to evaluate frequency averaged input power [1]. Instead of using quadrature schemes [2], the residue theorem [3] is applied and, as a result, a few computations at complex frequencies allow for an accurate estimate of the average value, regardless of the bandwidth.
This powerful tool can be used in combination with optimization strategies [4] to minimize the input power over a band and indirectly benefit from a reduction of the global vibration of the structure and the radiated acoustic power [5].

In the following paper, this approach is used to identify the characteristics of a Dynamic Vibration Absorber (DVA) and minimize the band-averaged input power into a bare plate. Genetic Algorithms (GAs) [6] are used as an optimization tool and the Wave Based Method (WBM) [7] is applied to simulate the dynamic behaviour of the plate. Compared to traditional element-based techniques [8], the WBM shows a higher convergence rate and does not restrict the DVA position to nodal locations. Moreover, when dealing with point connections, only a few rows and columns have to be added to the system of equations of the bare structure. These advantageous features perfectly fit the framework of an optimization scheme.

After having described the problem formulation in Section 2, the main features of the WBM for plate bending problems are illustrated in Section 3. The proposed optimization strategy is explained in Section 4 and validated by numerical examples in Section 5 to find the best characteristics for a DVA to minimize the input power into a plate. Finally, conclusions are drawn in Section 6.

2 PROBLEM FORMULATION

The steady-state vibration of a thin plate excited by a harmonic point force and connected to a DVA can be expressed by using the Kirchhoff theory as follows,

$$\nabla^4 w(\mathbf{x}) - k_b^4 w(\mathbf{x}) = \frac{F}{D}\delta(\mathbf{x}_f) + \frac{f_{pc}w_{pc}}{D}\delta(\mathbf{x}_{pc}), \qquad \mathbf{x} \in \Omega_p \tag{1}$$

where w is the out-of-plane displacement, ∇^4 represents the differential operator $\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$, k_b is the plate bending wavenumber, D the bending stiffness and δ the Dirac delta function. The symbol F represents the complex amplitude of the harmonic point force $F_0 = Fe^{i\omega t}$ applied at $\mathbf{x_f}$. Finally, the term $f_{pc}w_{pc}$ expresses the force exerted on the plate by the point connected DVA and is computed by multiplying the plate displacement at the connection point $\mathbf{x}_{pc} = (x_{pc}, y_{pc})$ by the following force-displacement relation for mass-spring-dampers,

$$f_{pc}(\omega) = \frac{m_{pc}\omega^2 \left(k_{pc} + i\omega c_{pc}\right)}{k_{pc} + i\omega c_{pc} - m_{pc}\omega^2}, \tag{2}$$

where k_{pc}, m_{pc} and c_{pc} are the stiffness, mass and damping coefficient of the DVA, respectively, and ω the angular frequency of analysis.
As Eq. (1) is a fourth order partial differential equation, two boundary conditions have to be imposed on each partition of the boundary Γ_p. These can be either kinematic conditions, i.e. for clamped edges, or mechanical boundary conditions, i.e. for free or loaded edges. Finally, a mixed boundary condition can be defined by imposing the displacements and bending moments. When the plate is simply supported, prescribed displacements and bending moment are equal to zero.

The structure is immersed in a fluid in which the acoustic problem is governed by the homogeneous Helmholtz equation

$$\nabla^2 p(\mathbf{x}) + k_a^2 p(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega_a \tag{3}$$

where p is the acoustic pressure and k_a is the acoustic wavenumber $\frac{\omega}{c}$, with c the speed of sound in the media of density ρ_a. The three-dimensional unbounded fluid is represented by the domain Ω_a with boundary $\Gamma_a \cup \Gamma_\infty$.
A set of boundary conditions can be applied to Γ_a, namely pressure, impedance and normal velocity. Finally, as the considered problem is unbounded, the Sommerfeld condition is imposed at infinity on Γ_∞, which ensures no acoustic energy is radiated from infinity.

Since the plate has high stiffness and the fluid low density, it is reasonable to assume that the mutual interaction is weak and negligible. Under this assumption the problem can be treated as uncoupled. According to the following procedure, the plate problem is solved by means of the WBM. Successively, structural velocities are imposed as boundary condition for Eq. (3), which is solved for the pressure distribution by means of the indirect Boundary Element Method (BEM). The radiated acoustic power can eventually be computed by integrating the acoustic intensity $\mathbf{I}$ over the surface of the plate

$$W_a = \int_{\Omega_p} \mathbf{I} \cdot \mathbf{n} \, d\Omega_p, \tag{4}$$

where

$$\mathbf{I}(\mathbf{x}) = \frac{1}{2} \Re \left\{ p(\mathbf{x}) \cdot v^*(\mathbf{x}) \right\}, \tag{5}$$

with $\mathbf{n}$ the normal to the surface $d\Omega_p$, $\Re\{\bullet\}$ the real part of the complex number $\bullet$, $(\bullet)^*$ the complex conjugate operator and v the plate velocity. Apart from the capability of inherently satisfying the Sommerfeld condition, the BEM properties are not exploited in the following paper. For this reason its formulation is not presented here and the reader is referred to dedicated literature.

3 THE WAVE BASED METHOD FOR PLATE BENDING PROBLEMS

The WBM belongs to the family of Trefftz approaches [7, 9], in which the field variables are expanded in terms of wave functions, which satisfy *a priori* the governing partial differential equation. Consequently, the out-of-plane displacement field w is approximated by the following expansion,

$$w(\mathbf{x}) \approx \hat{w}(\mathbf{x}) = \sum_{b=1}^{n_b} c_b \Psi_b(\mathbf{x}) + \hat{w}_f(\mathbf{x}) = \mathbf{\Psi}(\mathbf{x}) \cdot \mathbf{c} + \hat{w}_f(\mathbf{x}), \tag{6}$$

with $\mathbf{\Psi}$ and $\mathbf{c}$ vectors containing n_b wave functions Ψ_b, and c_b the corresponding contribution factors. The function $\hat{w}_f$ is the particular solution of Eq. (1) subject to the force of amplitude F and is defined as

$$\hat{w}_f(\mathbf{x}) = -\frac{iF}{8k_b^2 D}[H_0^{(2)}(k_b r_f) - H_0^{(2)}(-ik_b r_f)] \tag{7}$$

where r_f is the Euclidean distance between the excitation point $\mathbf{x}_f$ and the generic point $\mathbf{x}$. $H_0^{(2)}$ is the zero order Hankel function of the second kind. The wave functions Ψ_b have mathematical form

$$\begin{aligned} \Psi_{b_1}(\mathbf{x}) &= cos(k_{b_{1,x}} x) e^{-ik_{b_{1,y}} y} \qquad b_1 = 0, 1, \ldots, n_{b_1}, \\ \Psi_{b_2}(\mathbf{x}) &= e^{-ik_{b_{2,x}} x} cos(k_{b_{2,y}} y) \qquad b_2 = 0, 1, \ldots, n_{b_2}. \end{aligned} \tag{8}$$

This set of functions has been proven to be sufficient for convergence of the WBM, provided that the domain is convex [7].

From an infinite number of wave functions, a truncated set must be selected based on the dimension of the smallest rectangular box, $(L_x \times L_y)$, circumscribing the plate geometry. The first wavenumber is chosen such that an integer number of half wavelengths equals the dimension of the rectangular box in the corresponding direction, $k_{b_{1,x}} = \frac{b_1 \pi}{L_x}$ and $k_{b_{2,y}} = \frac{b_2 \pi}{L_y}$. The other wavenumber components are calculated from the bending wave number k_b corresponding to the frequency of analysis,

$$k_{b_{1,y}} = \begin{cases} \pm\sqrt{k_b^2 - k_{b_{1,x}}^2} \\ \pm i\sqrt{k_b^2 + k_{b_{1,x}}^2} \end{cases} \quad \text{and} \quad k_{b_{2,x}} = \begin{cases} \pm\sqrt{k_b^2 - k_{b_{2,y}}^2} \\ \pm i\sqrt{k_b^2 + k_{b_{2,y}}^2} \end{cases} .$$

The number of bending wave functions included in the field variable expansion (6), is related to the frequency and the dimension of the enclosing bounding box,

$$n_b = 4(n_{b_1} + 1) + 4(n_{b_2} + 1) \tag{9}$$

where $n_{b_1}/L_x \approx n_{b_2}/L_y \geq T k_b/\pi$, with n_{b_1} and n_{b_2} integer truncation numbers and T, a user defined truncation parameter.

The boundary conditions are enforced by using a weighted residual formulation. The residues on the boundary are orthogonalized with respect to a weighting function $\tilde{w}$ or its derivative, which are expanded in terms of the same set of wave functions used in the field variable expression (6). This results in a system of n_b equations in n_b unknowns,

$$[\mathbf{A}]\{\mathbf{c}\} = \{\mathbf{f}\}, \tag{10}$$

where the matrix $\mathbf{A}$ is complex, fully populated and frequency dependent and the right-hand side (RHS) represents the forcing term, $\mathbf{f}$. By solving the system of equations, the contribution factors $\mathbf{c}$ can be evaluated and substituted in the field expansion (6) to compute the system response at point $\mathbf{x}$.

For the sake of brevity in this paper, the WBM formulation is presented only for a simply supported plate. The reader is referred to [7, 9, 10, 11] for more complete and general formulations in acoustics and structural dynamics.

3.1 Extension of the WBM to point connections

When a one-dimensional resonator is attached to the plate at point $\mathbf{x}_{pc}$, expansion (6) is supplemented with the force-displacement relation

$$\hat{w}_{pc}(\mathbf{x}) = -\frac{if_{pc}(\omega)w_{pc}}{8k_b^2 D}[H_0^{(2)}(k_b r_{pc}) - H_0^{(2)}(-ik_b r_{pc})]. \tag{11}$$

Using n_{pc} DVAs leads to the introduction of the particular solution (11) into the residual formulation and to a system matrix of dimension $(n_b) \times (n_b + n_{pc})$. To solve the problem, n_{pc} auxiliary equations

$$w_{pc}(\mathbf{x}_{pc,j}) = \mathbf{\Psi}(\mathbf{x}_{pc,j}) \cdot \mathbf{c} + \hat{w}_f(\mathbf{x}_{pc,j}) + \sum_{j=p}^{n_{pc}} \hat{w}_{pc,p}(\mathbf{x}_{pc,p}), \tag{12}$$

are added. The system of equations now has the form

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{c} \\ \mathbf{w}_{pc} \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{f} \\ \mathbf{f}' \end{array} \right\}, \tag{13}$$

where the vector $\mathbf{w}_{pc}$ is composed of the displacements at each point connection location. For the sake of brevity, matrices $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and vector $\mathbf{f}'$ are not reported here. The reader is referred to [10] for their complete expressions.

After having clarified the general procedure to include n_{pc} point connections, it is worth noting that their addition does not influence the matrix of the main system, $\mathbf{A}$, and only requires the addition of n_{pc} columns and n_{pc} rows to the system. As a consequence, if the point connection has a different location for different configurations, there is no need for recomputing $\mathbf{A}$, but only the additional terms have to be reevaluated.

4 MINIMIZATION OF THE INPUT POWER

When dealing with the optimization of the dynamic behaviour of a structure, the choice of the objective function plays a crucial role and computational aspects have to be taken into account. For example, using the acoustic radiated power, or any other acoustic-related quantity as an objective function can be computationally demanding, since both structural and acoustic problems have to be solved at each iteration.
In these regards, Jog [5] suggests to reduce the overall vibration of the structure by minimizing the input power. As a consequence, structural resonances are drifted away from the target frequency and a reduction of the radiated acoustic power is achieved. Moreover, when the vibro-acoustic model is uncoupled, the optimization is performed on the bare structure, reducing the computational cost.
Another delicate aspect to consider when designing a mechanical component is the possibility to have sources acting over a wide frequency band. In this situation a robust design is always preferred. Traditional quadrature techniques can be used to estimate the band-averaged input power but their computational cost is strictly related to the oscillatory behaviour of the input mobility and the width of the frequency band [2]. This can dramatically slow down the efficiency of the optimization procedure. Furthermore, if the band-averaged value is not accurately evaluated, the identification of the optimal configuration might be jeopardized.
In this context, the result presented in the next section provides a powerful tool to perform accurate band evaluations at a reduced computational cost. As described in Section 4.2, this can be embedded in an optimization procedure and increases the computational performance.

4.1 Evaluation of the band-averaged input power

The input power averaged over the band $[\omega_0 - \omega_C, \omega_0 + \omega_C]$ can be evaluated as follows,

$$\langle P_{in} \rangle_W = \frac{1}{2} \int_{-\infty}^{+\infty} W(\omega) \Re \left\{ i\omega w(\mathbf{x_f}, \omega) F^* \right\} \, d\omega, \tag{14}$$

where $W(\omega)$ is defined as a rectangular window centered at ω_0 and of half-width ω_C. Integral (14) can be estimated by using classic quadrature techniques, although a refined sampling scheme is required to obtain accurate approximations. Recently, the residue theorem has been used as an alternative to numerical quadrature for computing band averaged input power [1]. Instead of using a rectangular window, the square magnitude of a Butterworth filter,

$$B(\omega) = \frac{1}{1 + (\frac{\omega - \omega_0}{\omega_C})^{2n}}, \tag{15}$$

weighs the input power in Eq. (14). For a filter of order n, $2n$ poles are located in the complex z plane at the following positions,

$$z_k = \omega_0 + \omega_C e^{-i\theta_k}, \tag{16}$$

where

$$\theta_k = \frac{\pi}{2n}(1 + 2k). \tag{17}$$

The value k is an integer which defines the position of a pole. When $0 \leq k \leq n - 1$, the k-th pole is located in the lower complex half plane; when $n \leq k \leq 2n - 1$, the k-th pole is located in the upper complex half plane. With increasing n, the shape of the filter becomes closer to the shape of a rectangular window.

The use of the residue theorem allows for computing integral (14) in a very straightforward manner and, as a result, the Butterworth-weighted frequency averaged input power into a structure can be evaluated by means of the following expression,

$$\langle P_{in} \rangle_B = -\frac{1}{2} \frac{\pi \omega_C}{n} \Re \left\{ F^* \sum_{k=0}^{n-1} z_k w(\mathbf{x_f}, z_k) e^{-i\theta_k} \right\}, \tag{18}$$

which is exact when the poles of the mechanical system are confined to the upper half plane. This is not the case for hysteretic damping models and Eq. (18) results into an approximation rather than an exact result. Nevertheless, the error quickly decreases when increasing the order of the filter and it has been proven to be negligible with respect to the error made by approximating a rectangular window by using a Butterworth filter [1]. When $n = 1$, the filter degenerates into a Lorentzian function [12].

It is worth underlining the potential of the result in Eq. (18). To evaluate the Butterworth-weighted frequency average, system displacements have to be computed at complex frequencies, which correspond to the positions of the poles of the Butterworth filter and no modification to the underlying numerical strategy is implemented. Moreover, for methods which require the solution of a system of complex matrices, i.e. the WBM, this does not constitute an additional cost with respect to the evaluation at a real frequency. Finally, there is no dependence on the bandwidth over which the averaging is carried out. This means that the same number of poles can be used regardless of the bandwidth, unlike numerical quadrature whose order has to be chosen as function of the interval of integration.

4.2 Summary of the optimization strategy

Among all the properties of the WB modelling for vibration problems, two are of particular interest for this specific application. First of all, the WBM is a meshless approach. Consequently it allows locating the resonator at any point over the plate and it does not require any re-meshing procedure. Secondly, it shows a higher convergence rate than element-based approaches and a lower number of degrees of freedom is needed.
To evaluate the input power over a band, only a few computations at complex frequencies are required. So, the small matrices of the bare system can be computed at the aforementioned complex frequencies and stored in memory. At each iteration they are recalled and complemented with the point connection contributions. The problem is solved for the wave contribution factors and the input power is computed.
GAs are used to solve the following optimization problem, that is formulated as an unconstrained minimization of Eq. (18). Unlike gradient-based optimization procedures, GAs are inspired by the natural selection process, where the best individuals dominate the population [6]. However, the present strategy can be extended to gradient-based techniques [4].
The following procedure is performed in four steps, which are now summarized:

1. Problem definition. Data related to the problem geometry, material, boundary conditions, excitation etc. are defined. Parameters of the GA are specified, i.e. tolerance, population size, stochastic parameters, etc.

2. Determination of filter poles. The poles of the Butterworth filter are determined according to the order of the filter and to Eq. (16).

3. Evaluation and storage of system matrices. The WB matrices and RHS vectors for the bare system are computed at each complex frequency corresponding to a filter pole. Since the number of poles is commonly small and the WB matrices have a reduced number of degrees of freedom, storing the matrices does not require large memory consumption.

4. Optimization process. At each iteration, system matrices are recalled and complemented with a number of columns and rows equal to the number of point connections considered in the problem. Also the RHS dimensions are increased. Successively, the system of equations is solved and the input power is evaluated. This process is repeated until convergence is reached and function (18) is minimized.

5 APPLICATION CASE

The plate under consideration is simply supported and rectangular of dimensions 1x0.6 m, 1 mm thick. The material is steel of Young modulus 210 GPa, density 7800 kg/m^3 and Poisson ratio 0.3. The damping mechanism is hysteretic with constant loss factor equal to 0.01. The plate is unbaffled and radiates in air characterized by a density of 1.225 kg/m^3, and a speed of sound of 340 m/s. The structure is excited by a point force located at (0.85, 0.15)m.
The input power is minimized over a band which sweeps frequencies from 58 Hz to 79 Hz. In that region, four modes are contributing to the response which are located at 64.1 Hz, 66.9 Hz, 68.5 Hz and 71.6 Hz. The first and the third are odd modes and strongly contribute to the radiated acoustic power, as can be seen in Figure 1.
The flexural behaviour is predicted by means of the WBM with truncation factor $T = 2$. To quantify the global vibration of the structure, displacements are averaged over 1421 points randomly distributed over the plate. Moreover, velocities over a mesh consisting of 480 linear

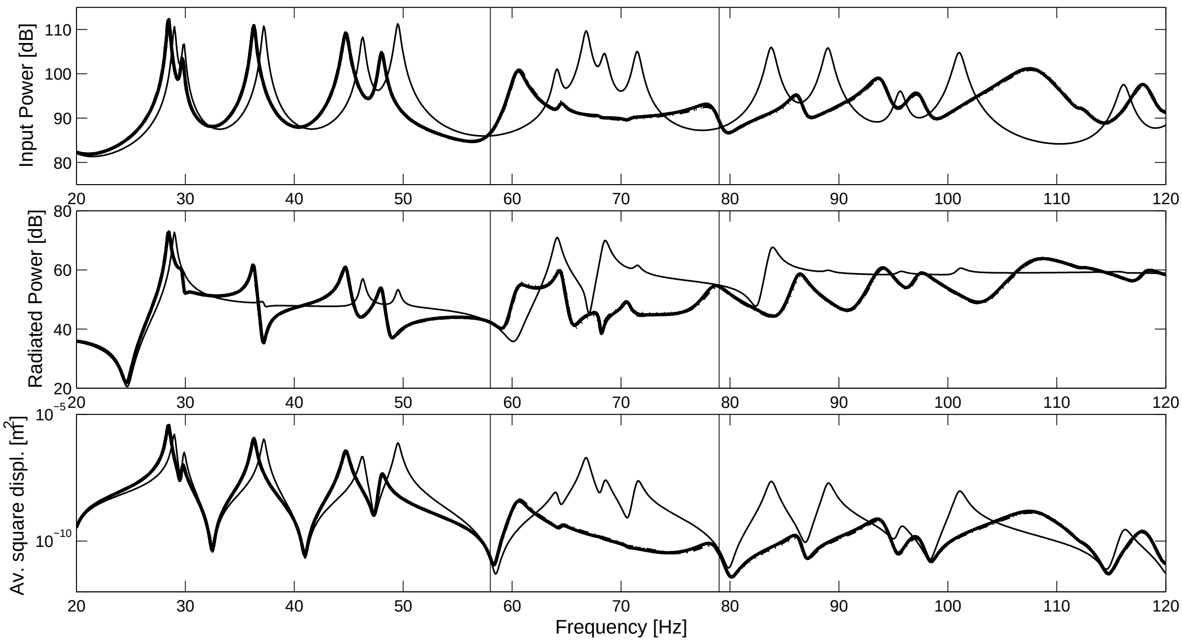

Figure 1: Initial configuration (thin solid line), optimized B4 (thick solid line), optimized B8 (dashed line), optimized B20 (mixed line) and optimized T1.0 (dotted line).

triangular elements and 273 nodes, are used to compute the radiated power by means of the indirect BEM.

One DVA is used to reduce the vibration of the plate. Its mass is equal to 2% of the mass of the bare plate, while the damping ratio is 0.1. The coordinates and the resonance frequency of the resonator are determined by means of the optimization process. Thus, the problem has 3 design variables. The GA implemented in Matlab 2010 is used as single objective optimizer and the population consists of 30 individuals.

To benchmark the proposed optimization strategy, the objective function is evaluated in different ways. Butterworth filters are used with order 4 (B4), 8 (B8) and 20 (B20). For each iteration, the system response needs to be computed 4, 8 and 20 times, respectively, to evaluate the band-averaged value. Alternatively, a quadrature scheme is used to evaluate the average. The trapezoidal rule is used with 1 Hz (T1.0), 0.5 Hz (T0.5) and 0.2 Hz (T0.2) step. It is clear that using classic quadrature schemes is, in general, more computationally demanding. Moreover, the inaccuracy due to the rough integration step, also influences the convergence of the optimization algorithm.

To quantify the reduction with respect to the initial design, a reduction coefficient, $r_{(\bullet)}$, is used and is computed by means of the following expression

$$r_{(\bullet)} = 100 \cdot \frac{\bullet_{ini} - \bullet_{opt}}{\bullet_{ini}}, \tag{19}$$

where $\bullet_{ini}$ stands for the initial band value, while $\bullet_{opt}$ for the optimized one. Reduction coefficients in Table 1 refer to the band-averaged input power, acoustic radiated power and square averaged displacements. Band values are computed by using the trapezoidal rule with 0.1 Hz resolution.

In Table 1, the optimal configurations are reported and it is evident that all the optimization processes lead to similar designs. This is confirmed by Figure 1, in which the responses of optimized configurations are almost indistinguishable from each other.

Simulation ID	B4	B8	B20	T1.0	T0.5	T0.2
x_{pc} [m]	0.879	0.879	0.878	0.878	0.878	0.878
y_{pc} [m]	0.124	0.124	0.125	0.123	0.125	0.125
f_{res} [Hz]	85.9	85.9	86.1	86.7	86.0	86.1
t_{opt} [s]	198.0	406.0	1256.6	1590.7	2412.3	6059.9
Generations [-]	41	44	55	62	46	48
r_{pow} [-]	74.0	74.0	74.0	74.0	74.0	74.0
r_{rad} [-]	90.5	90.4	90.5	89.7	90.5	90.5
r_{disp} [-]	96.2	96.2	95.9	95.8	95.9	95.9

Table 1: Data of the optimal configurations.

If there was only one target resonance, the resonator would be placed such that it could reduce the vibration of the corresponding mode shape. Nevertheless, in this case there are four modes contributing to the objective function and, as it could be foreseen, the resonator is located close to the excitation point. However, the resonance frequency is above the band of interest. Although it is difficult to justify, it has been verified that this configuration minimizes the frequency averaged input power over that band.
All optimal designs present a noticeable improvement over the band of interest. The responses at frequencies below 58 Hz do not show a benefit from the optimization process. On the contrary, above 78 Hz the behaviour is, in general, slightly improved. This may be due to the resonance frequency and the damping coefficient of the DVA.
From a computational point of view, the proposed strategy yields a tremendous advantage with respect to classic quadrature techniques. In this regard, it is interesting to compare calculation times (t_{opt}) in Table 1. Although configuration B20 and T0.2 provide the same optimum, the difference in computation times is quite large, especially considering the number of generations to converge.
For all cases, convergence is reached when the average change in the fitness function is below 10^{-7} for 10 consecutive generations. This justifies the relatively large number of generations to convergence. Using a larger tolerance value would not hamper the convergence to the optimal value and its choice is user- and case-dependent.

6 CONCLUSIONS

This paper presents a strategy to reduce the global vibration and radiated acoustic power over a frequency band. The main feature of the present approach is the efficiency achieved by exploiting the residue theorem. In contrast to classic quadrature, a few computations at complex frequencies allow the band value evaluation regardless of the bandwidth.
In combination with the efficient WB modelling and GAs, the proposed strategy has been proven to be very effective in optimizing the properties of a DVA to minimize the input power into a plate over a frequency band. Indirect reduction of vibration and radiated acoustic power have been achieved by applying the aforementioned procedure.
The general idea can be easily extended to more complex application cases, i.e. complex geometries and different types of excitation, and to situations where robust design is required.

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