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NONLINEAR CONTROL OF VEHICULAR VIBRATIONS

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Abstract. *For passenger ride comfort, durability of the vehicle and also to safeguard the commercial products inside the vehicle, vibration of the vehicle suspension traversing on the rough road have to be controlled. In this paper, performance of MR damper as a semi active suspension using modified Bouc-wen model is studied, as MR damper are hysteretic devices and there additions make the vehicular model nonlinear and therefore nonlinear control algorithms are best suited to cater this situations. The study considers integral backstepping technique to develop the control algorithm for MR damper so that we can monitor the applied current directly using system feedback. A quarter car model with bounce (heave) motion is considered. The model is tested on three different excitations to the bump input, sudden ramp input and sinusoidal input. Results are reported for semi-active control, passive control and open loop systems. A comparative study is conducted*

1 INTRODUCTION

Due to road undulations, braking forces, aerodynamics forces and cornering forces, a vehicle traversing a rough road is subjected to vibrations, which causes discomfort to the drivers and influences the manoeuvrability. To reduce the vibrations, good suspension systems is to be developed. Vehicle suspension systems are broadly classified as passive, active and semi-active systems. In passive systems, stiffness and damping parameters are fixed and cannot be varied for different road conditions, it does not have an external power source to act as per the varying road condition. To overcome this, active suspension which has a capability of adapting to varying road conditions by the use of an actuator have been considered by many authors [1, 2, 3]. Advantage of using an active suspension over passive suspension had been outlined in [4]. Semi-active suspension provide the robustness of passive systems with wide range of working frequencies like active suspensions, semi-active systems works on battery power. In this, variable control force can be achieved by means of rapidly adjustable damping, it replaces the actuator that provides the control force in active suspension. Karnoop was amongst the earliest to propose semi active systems as an alternate to active systems [5]. Semi active systems are used in many applications to control vibrations like in structures and building [6] and vehicular applications [7]. The reliability of semi active suspensions makes it a possible choice for vibration control. Most of the previous research in semi active suspension uses mechanical components (like valves etc.) to vary the damping force, but it is very noisy and its maintenance is costly and also having slow response time. Taking this into considerations a smart fluid with adjustable viscosity to control the damping force has been considered. MR damper is one of these smart fluids whose viscosity can be changed with changing its input voltage and thereby and thereby damping force change. Due to low power consumptions, quick response time of MR damper has attracted many researchers to use it as a semi active control systems [8]. Applications of MR damper in automobile suspension systems as semi active devices have been considered by many authors [10, 11]. Backstepping technique is been used to control the vibration of active suspension systems [12]. Most previous studies has used linear control algorithms and linear relations to map damper current to force. In this paper we are using integral backstepping control to monitor MR damper current input directly from systems feedback.

In this paper, mathematical model of MR damper using modified Bouc wen model to characterize the hysteretic behaviour is been adopted [13]. Based on quarter car model with two degree of freedom, an Integral Backstepping technique is used to develop a control algorithm that will monitor the applied current directly from the system feedback. The car model is tested on three road excitations i.e. bump input, sudden ramp input and sinusoid input. Finally comparison with open loop systems, passive off system with MR Damper as well as passive on with MR Damper is reported.

2 TWO DOF QUARTER CAR MODEL

For the purpose of simulation and analysis, a two DOF quarter car model is used. Mass of the vehicle is represented as sprung mass (m_s), mass of the wheel and associated components are represented as unsprung mass (m_u), suspension damping is represented as (c_s), suspension stiffness is represented as (k_s) and tyre stiffness is represented as (k_t), h is the road input given to the systems.

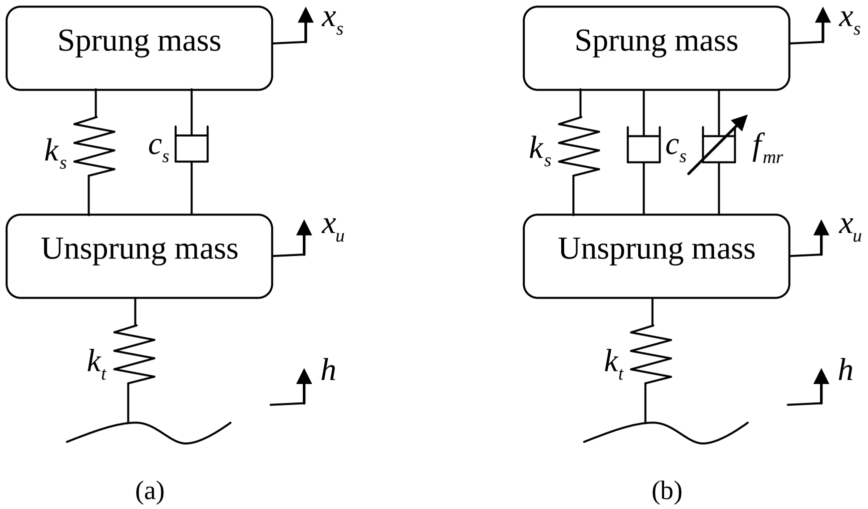

Figure 1: (a) Quarter car model with conventional damper, (b) Quarter car model with MR damper.

Equation of motion derived using newton second law for quarter car with conventional damper is given by (see Figure 1 a.)

$$\begin{aligned} m_s\ddot{x}_s + k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) = 0 \\ m_u\ddot{x}_u + k_s(x_u - x_s) + c_s(\dot{x}_u - \dot{x}_s) + k_t(x_u - h) = 0 \end{aligned} \quad (1)$$

Similarly, Equation of motion derived using newton second law for quarter car with semi active MR damper is given by (see Figure 1 b.)

$$\begin{aligned} m_s\ddot{x}_s + k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) + f_{mr} = 0 \\ m_u\ddot{x}_u + k_s(x_u - x_s) + c_s(\dot{x}_u - \dot{x}_s) + k_t(x_u - h) - f_{mr} = 0 \end{aligned} \quad (2)$$

where f_{mr} is MR Damper force and is given by (see Figure 2.)

$$f_{mr} = k_0(x_s - x_u) + c_0(\dot{x}_s - \dot{x}_u) + \alpha z \quad (3)$$

And z is Modified Bouc wen evolutionary variable and is given by

$$\dot{z} = -\gamma|\dot{x}_s - \dot{x}_u|z|z|^{n-1} - \beta(\dot{x}_s - \dot{x}_u)|z|^n + A(\dot{x}_s - \dot{x}_u) \quad (4)$$

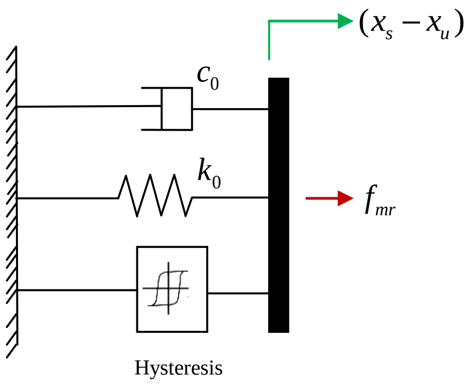

Figure 2: Modified Bouc wen model

Here β , γ , A, n are the parameters controlling the linearity in unloading from pre yield to post--yield region and, k_0 , c_0 and α are the internal parameters of the MR damper and are the functions of current supplied and its relations is given by,

$$k_0 = k_{oa} + k_{ob} i_c \quad \alpha = \alpha_a + \alpha_b i_c \quad c_0 = c_{oa} + c_{ob} i_c \tag{5}$$

Parameter	Values
k_{oa}	3610 N/m
k_{ob}	3610 N/m
c_{oa}	784 Ns/m
c_{ob}	1803 Ns/m
α_a	12441 N/m
α_b	38430 N/V m
β	2059020/m2
γ	136320/m2
n	1
A	58

Table 1: Parameters for MR Damper

System parameter		Parameter values	
Sprung mass	m_s	650	Kg
Unsprung mass	m_u	45	Kg
Suspension damping	c_s	350	Ns/m
Suspension stiffness	k_s	30	kN/m
Tyre stiffness	k_t	300	kN/m

Table 2: Parameters for Quarter car model

3 CONTROLLER DESIGN USING INTEGRAL BACKSTEPPING TECHNIQUE

By substituting Eqs. (3-4) back to Eq. (2) and then rewriting the closed loop system dynamics in the form of state space form one gets,

$$\begin{aligned} \dot{X} &= F_1(t, X) + G_1(t, X) i_c \\ \dot{i}_c &= F_2(t, X, i_c) + G_2(t, X, i_c) i_a \end{aligned} \tag{6}$$

where

$$X = [x_1, x_2, x_3, x_4, x_5]^T$$

$$F_1 = \begin{bmatrix} x_2 \\ -\dfrac{1}{m_s}(k_s(x_1 - x_3) + c_s(x_2 - x_4) + k_{oa}(x_1 - x_3) + c_{oa}(x_2 - x_4) + \alpha_a x_5) \\ x_3 \\ -\dfrac{1}{m_u}(k_s(x_3 - x_1) + c_s(x_4 - x_2) + k_t(x_3 - h) - k_{oa}(x_1 - x_3) - c_{oa}(x_2 - x_4) - \alpha_a x_5) \\ -\gamma |x_2 - x_4| x_5 |x_5|^{n-1} - \beta(x_2 - x_4)|x_5|^n + A(x_2 - x_4) \end{bmatrix}$$

$$G_1 = \left[0, -\frac{1}{m_s}(k_{ob}(x_1 - x_3) + c_{ob}(x_2 - x_4) + \alpha_b x_5), 0, \frac{1}{m_u}(k_{ob}(x_1 - x_3) + c_{ob}(x_2 - x_4) + \alpha_b x_5), 0 \right]^T$$

$$F_2 = -\eta i_c \qquad G_2 = \eta$$

Eq. (6) is in the strict feedback form, but to apply integrator backstepping we have to convert the equation. Let us introduce variable i_{dummy} such that,

$$\begin{aligned} \dot{X} &= F_1(t, X) + G_1(t, X) i_c \\ \dot{i}_c &= i_{dummy} \end{aligned} \tag{7}$$

Where

$$i_{dummy} = F_2(t, X, i_c) + G_2(t, X, i_c) i_a$$

Now, we have to choose the candidate Lyapunov function as

$$V_1 = \frac{1}{2} X^T P X \tag{8}$$

where, P is a diagonal matrix of size 5*5,

Taking the derivative of Eq. (8) we get,

$$\dot{V}_1 = \frac{1}{2}(\dot{X}^T P X + X^T P \dot{X}) \tag{9}$$

The Lyapunov time derivative $\dot{V}_1$ should be made negative definite so as to get stable closed loop response, so we have to select i_{cdes} such that derivative should be negative, following i_{cdes} is selected,

$$i_{cdes} = \frac{(-hk_t x_4 - 2x_2 x_4 - c_s x_4{}^2 + k_{oa} x_1 x_2 - k_{oa} x_1 x_4 - k_{oa} x_3 x_2 - k_s x_1 x_4 - k_s x_3 x_2 + \frac{\beta x_2 x_5 \alpha_a |x_5|}{A} - \frac{\beta x_4 x_5 \alpha_a |x_5|}{A})}{(2c_{ob} x_2 x_4 - k_{ob} x_1 x_2 + k_{ob} x_2 x_3 + k_{ob} x_1 x_4 - k_{ob} x_3 x_4 - \alpha_b x_2 x_5 + \alpha_b x_4 x_5)}$$

So if we put i_{cdes} in Eq. (9) and then solving it we get our desired derivative of Lyapunov function to be negative

$$\dot{V}_1 \le 0$$

Nevertheless i_c is a state variable and perfect tracking is to be required. Therefore again introducing an error variable such that

$$\dot{e} = \dot{i}_c - \dot{i}_{cdes} \tag{10}$$

Then the error dynamics will be

$$\dot{e} = \dot{i}_c - \dot{i}_{cdes} = i_{dummy} - i_{cdes,X} \dot{X} \tag{11}$$

Now again we have to choose the second Lyapunov function i.e.

$$V_2 = V_1 + \frac{1}{2}e^2$$

By solving above equations, one can show that systems defined in Eq. (7) become asymptotically stable [14],

$$i_{dummy} = i_{cdes,X}\,(F1(t,X) + G1(t,X)V_2 - V_{1,x}G1(t,X) - K(i_c - i_{cdes}) \tag{13}$$

Here K is the controller gain which can be chosen by the designer; here for simulation purpose we had taken the value to be 1.

4 RESULT AND DISCUSSIONS

To evaluate the performance of different suspension systems, here sprung mass displacement and velocity of the quarter car model is considered as a comfort criteria, for this simulations is carried out using three different road profile i.e. bump, ramp and sinusoid excitation respectively.

4.1 ROAD PROFILE MODELING

A. Road Bump input: This type of the road profile is very common and most encountered in reality, so mathematical model for a road bump is formulated as (see Figure 3.). Here r is the road bump height.

$$h = \begin{cases} r(1-\cos(8\pi t)), & 0.5 \le t \le 1 \\ 0, & otherwise \end{cases} \tag{14}$$

B. Road Ramp input: This type of the road profile comes into pictures when there is sudden elevation in road profile example when you are parking your car in garage, so mathematical model for a road ramp is formulated as (see Figure 4.)

$$h = \begin{cases} 0, & 0 \le t \le 1 \\ 10r(t-1), & 1 \le \mathrm{t} \le 1.1 \\ r & \text{otherwise} \end{cases} \tag{15}$$

C. Road Sinusoidal input: Mathematical model for a road sinusoidal input is formulated as (see Figure 5.)

$$h = \frac{r}{2}(1 + \sin(\pi t - \frac{\pi}{2})) \tag{16}$$

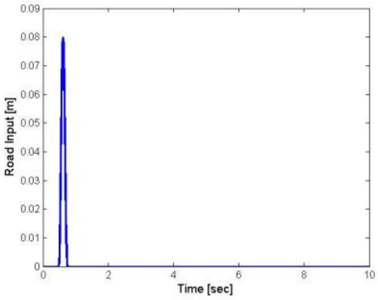

Figure 3: Bump input

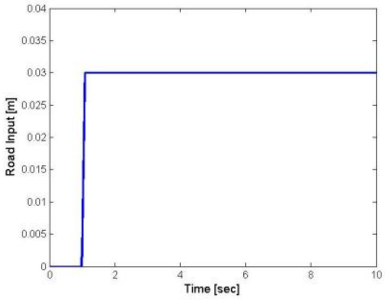

Figure 4: Ramp input

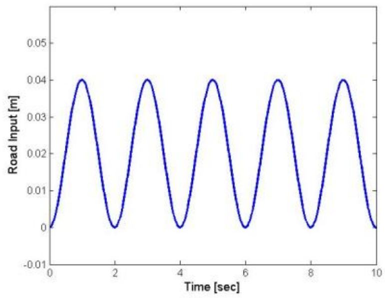

Figure 5: Sinusoid input

To evaluate the performance of the suspension systems, the sprung mass displacement and velocity of the quarter car model are considered as comfort criteria. We simulated for three different road input; first on bump input with a bump height of 0.08m, second on limited ramp with height of 0.03m and then for sinusoid input with an amplitude of 0.04m respectively. Parameters of quarter car are adapted from [15] shown in Table 2,also parameters for MR damper is adapted from [16] and shown in Table 1, is being used for simulation purpose.

Figure (6-7) shows the results due to bump input, Figure (10-11) for limited ramp input and Figure (14-15) for sinusoidal input, the response of a semi-active suspension with MR damper using Integral Backstepping method (i.e. variable current input) is far better than those with passive suspensions (i.e. MR damper passive on with current equals to 2A, MR damper passive off with current equals to 0A) and open loop suspension (i.e. normal conventional damper) and as far as current that is supplied to the damper is concerned Figure (8) and Figure (12) shows actual current supply to MR damper and one can say that it is exponential comes to zero, it means that we require more current when a vehicle hit a bump and then it gradually goes to zero, and the maximum current that is required by the damper is 2 A, and Figure (9), Figure (13), and Figure (17) shows the force that is been developed by the MR damper .

A. For Bump input:

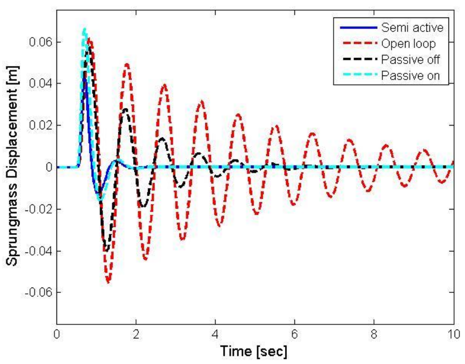

Figure 6: Sprung mass vertical displacement

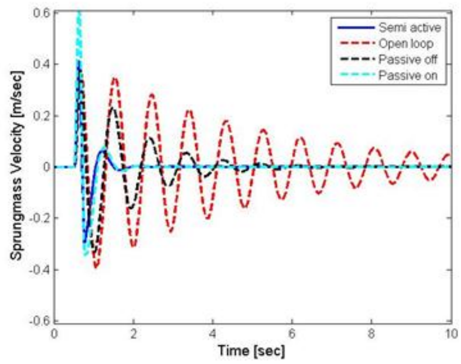

Figure 7: Sprung mass vertical velocity

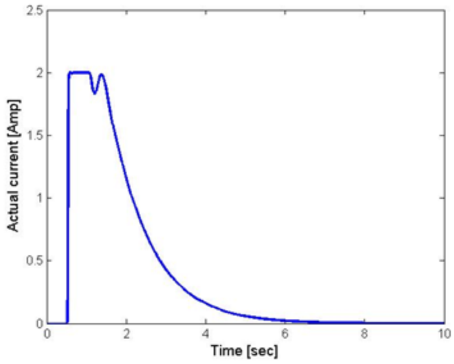

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Figure 8: Actual current input to damper

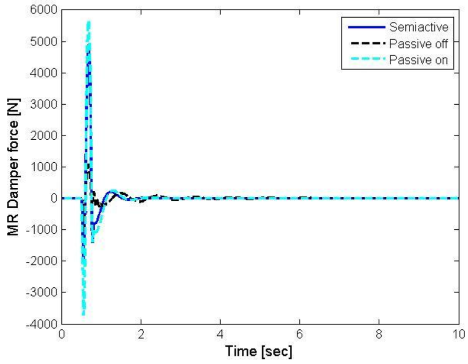

Figure 9: MR damper control force input

B. For Ramp input:

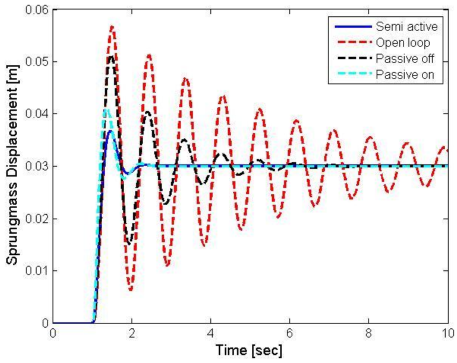

Figure 10: Sprung mass vertical displacement

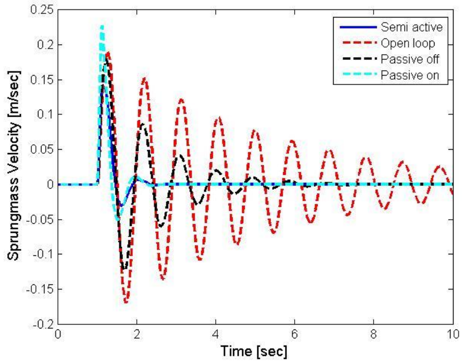

Figure 11: Sprung mass vertical velocity

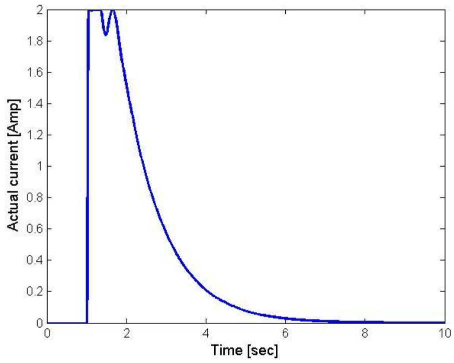

Figure 12: Actual current input to damper

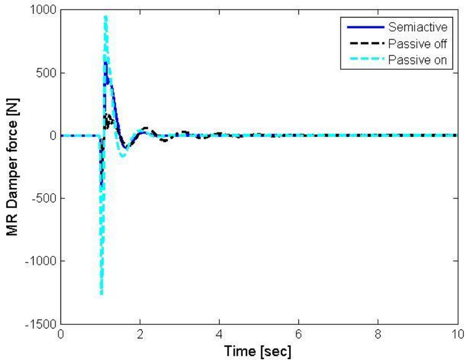

Figure 13: MR damper control force input

C. For Sinusoidal input:

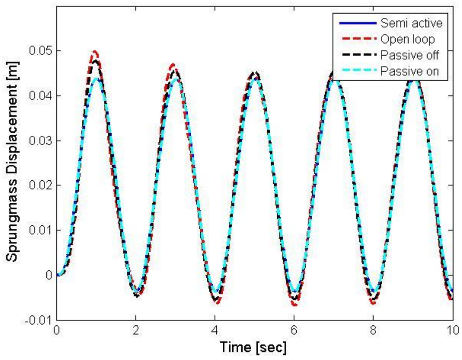

Figure 14: Sprung mass vertical displacement

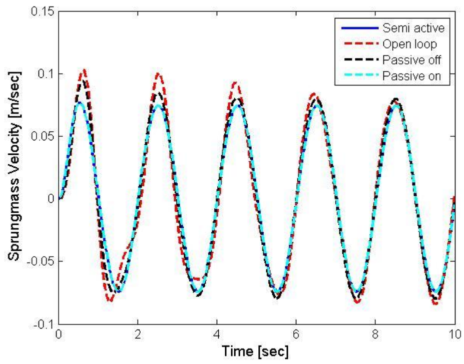

Figure 15: Sprung mass vertical velocity

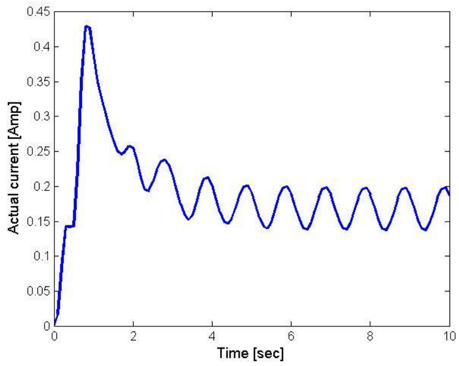

Figure 16: Actual current input to damper

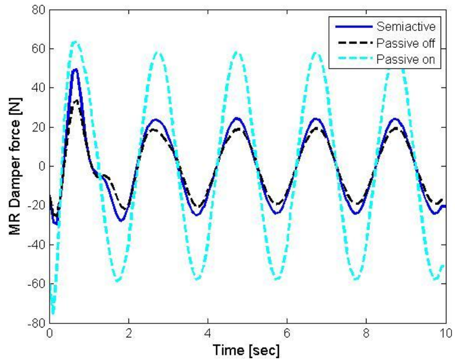

Figure 17: MR damper control force input

5 CONCLUSION

A quarter car suspension system with MR damper has been investigated and compared with passive suspensions (i.e. MR damper passive on with current equals to 2A, MR damper passive off with current equals to 0A) and open loop suspension (i.e. normal conventional damper).Vibration controller is designed using Integral Backstepping technique in such a way that it can monitor the current that is supplied to the damper directly from system feedback. So based on the result we say that semi-active suspension systems with MR damper is far more superior to normal conventional damper as well as MR damper passive cases.

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