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HOW FRICTION INFLUENCES REGENERATIVE CHATTER

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Abstract. *Cutting process is still one of the most popular methods of manufacturing in spite of the fact that, it naturally generates unwanted vibrations called chatter. Chatter vibrations are mainly caused either by regenerative or frictional effect Usually in the literature, regenerative and frictional chatter are investigated separately although from our point of view these types of chatter mechanisms exist together. Therefore, this work focuses on the interplay between regenerative and frictional mechanisms. To examine nonlinear phenomena the one degree of freedom model is developed. Next, the mathematical model is solved analytically by means of the time multiple scales method. The results show possibility of two solutions: trivial and non-trivial which are sensitive on system parameters. An interaction between frictional and regenerative chatter can lead to stabilizing cutting process.*

1 INTRODUCTION

Cutting process is still one of the most popular methods of manufacturing. In spite of the fact that, it naturally generates unwanted self-excited vibrations called chatter. Chatter can reduce the volumetric efficiency, increase the tool wear, and decrease the geometric accuracy. Therefore, understanding the origin of these vibrations is very important. Generally, there are four types of chatter mechanism: regenerative, frictional, thermo-mechanical and mode-coupling [1]. The regenerative or frictional chatter are the most important in practice. The former exists when flexible cutting tool or workpiece starts to vibrate due to the regeneration of the cutting force. A wave appearing on the workpiece surface generates the relative motion between the tool and the workpiece. The resulting modulations cause variations in chip thickness that in turn affects variations of the cutting force and as a consequence harmful vibrations called regenerative chatter. The latter - frictional mechanism is bases on dry friction phenomenon between the tool and the workpiece and the chip and the tool. Generally, the regenerative effect is more meaningful for high speed machining (HSM) On the contrary, the frictional effects are dominant for low and moderate cutting speeds. Usually in the literature, regenerative and frictional chatter are investigated separately. From our point of view the regenerative and the frictional mechanisms of chatter exist together. Therefore, the interplay between them should be examined carefully.

Historically, the first model of cutting forces was proposed by Taylor [2], Piispanen [3] and Merchant [4] basing on the orthogonal metal cutting. From that time a lot of the scientists have examined the dynamical behaviour of a cutting system. In the 60's, Tobias [5] and Tlusty [6] explained the machine-tool chatter as regeneration of waviness. They developed mathematical description in the form of delay-differential equations. These fundamental studies are applicable to orthogonal cutting where the direction of cutting force, chip thickness and system dynamics do not change with time. On the other hand, the stability of milling is complicated due to the rotating tool, multiple cutting teeth, periodical cutting forces and chip load directions, and multi-degree of freedom structural dynamics. Later works treat not only the orthogonal cutting but the case of milling when the direction of the cutting force changes with time rotation and cutting is interrupted as each tooth enters and leaves the workpiece [7-10]. The dynamic cutting process and chatter is analysed and modelled in the frequency domain or in the time domain. The frequency domain analysis leads to the identification of chatter free cutting conditions in plane of parameters such as spindle speed and depth of cut (axial or radial) presented as stability lobs diagram [11]. Analytical investigations have predicted the occurrence of bifurcation phenomena in interrupted cutting processes. Hopf bifurcations, period doubling bifurcations have been analytically predicted by Davies et al. [12], Insperger and Stépán [8], Bayly et al. [13], Mann [9], Fofana [14], Nayfeh [15] and confirmed experimentally by Davies et al. [12], Bayly [13] and Mann [16] as well.

Simultaneously, the frictional model of cutting process were developed, but only Grabec et al. [17] inspired researches to penetrating analysis of cutting process with friction phenomenon. On the basis of the frictional model, they have proved the possibility of occurrence of chaotic vibrations. Chaotic motions was observed in the responses of chattering machine-tool systems also by Wiercigroch [18], Wiercigroch and Cheng [19], Warminski et al. [20], Kalmar-Nagy and Pratt [21], and Wiercigroch and Krivtsov [22]. These authors examined the dynamics of several different variations on the basic tool-to-work-piece configuration. The general conclusion to be drawn from their work is that intermittency is consistently seen to be responsible for instabilities and bifurcations.

The stability lobs diagram, predicted on the basis of regenerative cutting is usually hard to confirm in a real manufacturing process. From our point of view, the interplay between the regenerative and the frictional mechanisms of chatter is very relevant to practical machining operations and can fully explain the chatter phenomenon. Therefore, we focus on obtaining a better understanding of the various aspects of frictional and regenerative chatter. In this aim the one degree of freedom nonlinear model is developed, which is next solved analytically by means of the multiple scales method.

2 MODEL OF ORTHOGONAL CUTTING PROCESS

The orthogonal cutting is represented as a single degree of freedom system (Fig. 1) with an equivalent mass of the tool (*m*), viscoelastic properties represented by the dash-pot (*c*) and the spring (*k*). Classically, the machine tool is modelled as a cantilever beam vibrating only in the direction normal to the machined surface (x direction in Fig. 1). We assumed that the tool is in permanent contact with the workpiece, the chip thickness is the difference between the current positions of the cutting tool x(t) and the position of the cutting tool one revolution earlier x(t-τ) (in case of turning). This introduces a time delay part $\tau = 2\pi / \Omega$, where Ω is the spindle speed. Then, the instantaneous chip thickness h(t) is given by

$$h(t) = h_o - x(t) + x(t-\tau)\,, \tag{1}$$

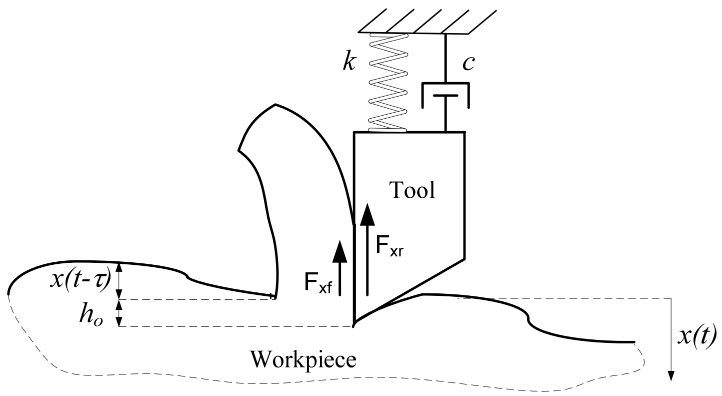


Figure 1: Single degree of freedom model of orthogonal cutting.

where h_o is the initial (the desired steady-state) chip thickness. The cutting force acting on the tool in x direction has two components. The one resulting from regeneration of waviness (F_{xr}) and the second comes from friction between the tool face and the chip (F_{xf}). Thus, in the direction perpendicular to the workpiece the equation of motion is

$$mx''(t) + cx'(t) + kx(t) = K_r\left(h_o - x(t) + x(t-\tau)\right) + K_f\left(\mathrm{sgn}(v_{rel}) - a_r v_r + b_r v_{rel}^3\right), \tag{2}$$

where: K_r and K_t mean the specific cutting force of regenerative and frictional component respectively, v_{rel} denotes relative velocity between the tool (x') and the chip (v_o) and is defined as follows

$$v_{rel} = v_o - x'\,. \tag{3}$$

The regenerative (F_{xr}) and frictional (F_{xf}) component of the cutting force are presented:

$$F_{xr} = K_r\left(h_o - x(t) + x(t-\tau)\right),$$
$$F_{xf} = K_f\left(\mathrm{sgn}(v_{rel}) - a_r v_{rel} + b_r v_{rel}^3\right). \tag{4}$$

The characteristic of the friction force (F_{xf}) is nonlinear for the sake of velocity and used in the literature to model dry friction [23-25].

Assuming, a real amplitude (0.1μm) and frequency (1kHz) of chatter vibrations, the velocity of chatter (x') is about 6m/min. Hence, for the cutting velocity v_c>6m/min, sgn(v_r) can be replaced by 1. This operation is fully justified because the cutting velocity is often greater than 100m/min. Then, the differential equation of motion with shifted argument (Eq. (2)) can be transformed to a dimensionless form

$$\ddot{x}(t) + \delta\dot{x}(t) + \omega_o^2 x(t) = \alpha x(t-\tau) + \mu\left(1 - a_r v_r + b_r v_r^3\right), \tag{5}$$

in which, the constant factor is omitted because it does not influence the system dynamics. δ is the damping coefficient, ω_o is the natural frequency (equals 1). The cutting forces are represented by the right side of the Eq. (5) where, α is the regenerative force factor (known in control theory as a feedback gain), μ is friction force factor, τ is time delay the relative velocity

$$v_r = v_c - \dot{x}(t), \tag{6}$$

where: v_c is the equivalent of dimensionless cutting velocity. Next, this model is analysed theoretically in order to find interaction between regenerative and frictional chatter.

3 ANALYTICAL SOLUTION

The system described by Eq.(5) is solved analytically with the help of the multiple time-scales method (MTSM). We assume two scales (fast and slow) expansion of the solution. A fast scale T_o and slow scale T_1 are described by eq.(7)

$$T_0 = t, T_1 = \varepsilon t. \tag{7}$$

Then a solution in the first order approximation is in the form:

$$x(t) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1),$$
$$x(t-\tau) = x_\tau = x_{0\tau}(T_0,T_1) + \varepsilon x_{1\tau}(T_0,T_1). \tag{8}$$

It is assumed that

$$\omega_0^2 = \omega^2 + \varepsilon\sigma, \delta = \varepsilon\hat{\delta}, \alpha = \varepsilon\hat{\alpha}, \mu = \varepsilon\hat{\mu}, \tag{9}$$

where: ε is a formal small parameter and σ is detuning parameter. Next, in order to facilitate notation, the tilde is omitted. By using the chain rule, the time derivative is transformed according to the expressions:

$$\frac{d}{dt} = \frac{\partial}{\partial T_0} + \varepsilon\frac{\partial}{\partial T_1}$$
$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial T_0^2} + \varepsilon\frac{\partial^2}{\partial T_0 \partial T_1} + \varepsilon\frac{\partial^2}{\partial T_1 \partial T_0} + \ldots = \frac{\partial^2}{\partial T_0^2} + 2\varepsilon\frac{\partial^2}{\partial T_0 \partial T_1} + \ldots \tag{10}$$

Substituting Eqs. (7)-(10) into (5) yield

$$\frac{\partial^2 x(t)}{\partial T_0^2} + 2\varepsilon \frac{\partial^2 x(t)}{\partial T_0 \partial T_1} + \delta\left[\frac{\partial x(t)}{\partial T_0} + \varepsilon \frac{\partial x(t)}{\partial T_1}\right] + \omega_0^2\left(x_0 + \varepsilon x_1\right) =$$
$$\alpha\left(x_{0\tau} + \varepsilon x_{1\tau}\right) + \mu\left[1 - a_r\left(v_c - \frac{\partial x(t)}{\partial T_0} - \varepsilon \frac{\partial x(t)}{\partial T_1}\right) + b_r\left(v_c - \frac{\partial x(t)}{\partial T_0} - \varepsilon \frac{\partial x(t)}{\partial T_1}\right)^3\right] \tag{11}$$

Expanding derivatives of the equation (11) we obtain:

$$\begin{aligned} \frac{\partial x(t)}{\partial T_0} &= \frac{\partial x_0}{\partial T_0} + \varepsilon \frac{\partial x_1}{\partial T_0} \\ \frac{\partial^2 x(t)}{\partial T_0^2} &= \frac{\partial^2 x_0}{\partial T_0^2} + \varepsilon \frac{\partial^2 x_1}{\partial T_0^2} \\ \frac{\partial^2 x(t)}{\partial T_0 \partial T_1} &= \frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \varepsilon \frac{\partial^2 x_1}{\partial T_0 \partial T_1} \end{aligned} \tag{12}$$

$$\begin{aligned} &\varepsilon \frac{\partial^2 x_1}{\partial T_0^2} + \frac{\partial^2 x_0}{\partial T_0^2} + 2\varepsilon \frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \varepsilon\mu b_r \left(\frac{\partial x_0}{\partial T_0}\right)^3 - 3\varepsilon\mu b_r v_c \left(\frac{\partial x_0}{\partial T_0}\right)^2 - \varepsilon\alpha x_{0\tau} + \varepsilon\omega^2 x_1 + \\ &\varepsilon\left(3\mu b_r v_c^2 - \mu a_r + \delta\right)\frac{\partial x_0}{\partial T_0} + \omega^2 x_0 + \varepsilon\sigma x_0 - \varepsilon\mu b_r v_c^3 + \varepsilon\mu a_r v_c - \varepsilon\mu = 0 \end{aligned} \tag{13}$$

Equating coefficients of powers of ε^0 and ε^1, we obtain:

$$\begin{aligned} &\varepsilon^0 \Rightarrow \frac{\partial^2 x_0}{\partial T_0^2} + \omega^2 x_0 = 0 \\ &\varepsilon^1 \Rightarrow \frac{\partial^2 x_1}{\partial T_0^2} + 2\frac{\partial^2 x_0}{\partial T_0 \partial T_1} + \mu b_r \left(\frac{\partial x_0}{\partial T_0}\right)^3 - 3\mu b_r v_c \left(\frac{\partial x_0}{\partial T_0}\right)^2 + \left(3\mu b_r v_c^2 - \mu a_r + \delta\right)\frac{\partial x_0}{\partial T_0} - \\ &\alpha x_{0\tau} + \omega^2 x_1 + \sigma x_0 + \mu\left(-b_r v_c^3 + a_r v_c - 1\right) = 0 \end{aligned} \tag{14}$$

It is convenient to express the solution of first equation (14) in the complex form:

$$\begin{aligned} x_0(T_0, T_1) &= A(T_1)e^{iT_0} + \overline{A}(T_1)e^{-iT_0} \\ x_{0\tau}(T_0, T_1) &= A(T_1)e^{i(T_0-\tau)} + \overline{A}(T_1)e^{-i(T_0-\tau)} \end{aligned} \tag{15}$$

where: $\overline{A}$ is the complex conjugate of A, which is an arbitrary complex function of T_1. Substituting equations (15) into second equation (14) and expanding derivatives we get:

$$\begin{aligned} \frac{\partial x_0}{\partial T_0} &= A(T_1)ie^{iT_0} - \overline{A}(T_1)ie^{-iT_0} \\ \frac{\partial^2 x_0}{\partial T_0 \partial T_1} &= A'(T_1)ie^{iT_0} - \overline{A}'(T_1)ie^{-iT_0} \end{aligned} \tag{16}$$

and then the following equation is obtained:

$$\begin{aligned}
&\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+2\left(A'(T_1)ie^{iT_0}-\overline{A}'(T_1)ie^{-iT_0}\right)+\mu b_r\left(A(T_1)ie^{iT_0}-\overline{A}(T_1)ie^{-iT_0}\right)^3-\\
&3\mu b_r v_c\left(A(T_1)ie^{iT_0}-\overline{A}(T_1)ie^{-iT_0}\right)^2+\left(3\mu b_r v_c^2-\mu a_r+\delta\right)\left(A(T_1)ie^{iT_0}-\overline{A}(T_1)ie^{-iT_0}\right)-\\
&\alpha\left(A(T_1)e^{i(T_0-\tau)}+\overline{A}(T_1)e^{-i(T_0-\tau)}\right)+\sigma\left(A(T_1)e^{iT_0}+\overline{A}(T_1)e^{-iT_0}\right)+\mu\left(-b_r v_c^3+a_r v_c-1\right)=0
\end{aligned} \tag{17}$$

Ordering equation (17) we get its final form

$$\begin{aligned}
&\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+\mu\left[-1+a_r v_c-b_r v_c^3-6b_r v_c\overline{A}(T_1)A(T_1)\right]+\\
&\begin{bmatrix}\sigma A(T_1)-\alpha A(T_1)e^{-i\tau}+i\delta A(T_1)-i\mu a_r A(T_1)+3i\mu b_r v_c^2 A(T_1)+\\ 3i\mu b_r A(T_1)^2\,\overline{A}(T_1)+2iA'(T_1)\end{bmatrix}e^{iT_0}+\\
&\begin{bmatrix}\sigma\overline{A}(T_1)-\alpha\overline{A}(T_1)e^{-i\tau}-i\delta\overline{A}(T_1)+i\mu a_r\overline{A}(T_1)-3i\mu b_r v_c^2\overline{A}(T_1)-\\ 3i\mu b_r\overline{A}(T_1)^2\,A(T_1)-2i\overline{A}'(T_1)\end{bmatrix}e^{-iT_0}-\\
&i\mu b_r A(T_1)^3 e^{3iT_0}+i\mu b_r\overline{A}(T_1)^3 e^{-3iT_0}+3\mu b_r v_c A(T_1)^2 e^{2iT_0}+3\mu b_r v_c\overline{A}(T_1)^2 e^{-2iT_0}=0
\end{aligned} \tag{18}$$

The secular term of equation (18) vanishes if and only if:

$$ST_1 e^{iT_0}=0,\ ST_2 e^{-iT_0}=0 \tag{19}$$

where: ST_1 and ST_2 are secular generating terms. This leads to the equations:

$$\begin{aligned}
&\sigma A(T_1)-\alpha A(T_1)e^{-i\tau}+\\
&i\delta A(T_1)-i\mu a_r A(T_1)+3i\mu b_r v_c^2 A(T_1)+3i\mu b_r A(T_1)^2\,\overline{A}(T_1)+2iA'(T_1)=0\\
&\sigma\overline{A}(T_1)-\alpha\overline{A}(T_1)e^{-i\tau}-\\
&i\delta\overline{A}(T_1)+i\mu a_r\overline{A}(T_1)-3i\mu b_r v_c^2\overline{A}(T_1)-3i\mu b_r\overline{A}(T_1)^2\,A(T_1)-2i\overline{A}'(T_1)=0
\end{aligned} \tag{20}$$

Eliminating from equation (18) the secular generating terms we have:

$$\begin{aligned}
&\frac{\partial^2 x_1}{\partial T_0^2}+\omega^2 x_1+\mu\left[-1+a_r v_c-b_r v_c^3-6b_r v_c\overline{A}(T_1)A(T_1)\right]-\\
&i\mu b_r A(T_1)^3 e^{3iT_0}+i\mu b_r\overline{A}(T_1)^3 e^{-3iT_0}+3\mu b_r v_c A(T_1)^2 e^{2iT_0}+3\mu b_r v_c\overline{A}(T_1)^2 e^{-2iT_0}=0
\end{aligned} \tag{21}$$

Solving (21) for:

$$\begin{aligned}
x_1(T_0,T_1)&=B(T_1)e^{3iT_0}+\overline{B}(T_1)e^{-3iT_0}\\
x_{1\tau}(T_0,T_1)&=B(T_1)e^{3i(T_0-\tau)}+\overline{B}(T_1)e^{-3i(T_0-\tau)}
\end{aligned} \tag{22}$$

where:

$$B(T_1)=\frac{i\mu b_r A(T_1)^3}{\omega^2-9}$$
$$\overline{B}(T_1)=-\frac{i\mu b_r \overline{A}(T_1)^3}{\omega^2-9} \tag{23}$$

we obtain:

$$x_1(T_0,T_1)=\frac{i\mu b_r A(T_1)^3}{\omega^2-9}e^{3iT_0}-\frac{i\mu b_r \overline{A}(T_1)^3}{\omega^2-9}e^{-3iT_0}$$
$$x_{1\tau}(T_0,T_1)=\frac{i\mu b_r A(T_1)^3}{\omega^2-9}e^{3i(T_0-\tau)}-\frac{i\mu b_r \overline{A}(T_1)^3}{\omega^2-9}e^{-3i(T_0-\tau)} \tag{24}$$

Substitution into Eq. (20) the polar form of the complex amplitude:

$$A(T_1)=\frac{1}{2}a(T_1)e^{i\beta(T_1)}$$
$$A'(T_1)=\frac{1}{2}a'(T_1)e^{i\beta(T_1)}+\frac{1}{2}ia(T_1)\beta'(T_1)e^{i\beta(T_1)}$$
$$\overline{A}(T_1)=\frac{1}{2}a(T_1)e^{-i\beta(T_1)}$$
$$\overline{A}'(T_1)=\frac{1}{2}a'(T_1)e^{-i\beta(T_1)}-\frac{1}{2}ia(T_1)\beta'(T_1)e^{-i\beta(T_1)} \tag{25}$$

results in:

$$-\frac{1}{2}\alpha a(T_1)e^{-i\tau+i\beta(T_1)}+\frac{1}{2}i\delta a(T_1)e^{i\beta(T_1)}+\frac{1}{2}\sigma a(T_1)e^{i\beta(T_1)}-\frac{1}{2}i\mu a_r a(T_1)e^{i\beta(T_1)}+$$
$$\frac{3}{8}i\mu b_r a(T_1)^3e^{i\beta(T_1)}+\frac{3}{2}i\mu b_r v_c^2 a(T_1)e^{i\beta(T_1)}+2i\left[\frac{1}{2}a'(T_1)e^{i\beta(T_1)}+\frac{1}{2}ia(T_1)\beta'(T_1)e^{i\beta(T_1)}\right]=0$$
$$-\frac{1}{2}\alpha a(T_1)e^{i\tau-i\beta(T_1)}-\frac{1}{2}i\delta a(T_1)e^{-i\beta(T_1)}+\frac{1}{2}\sigma a(T_1)e^{-i\beta(T_1)}+\frac{1}{2}i\mu a_r a(T_1)e^{-i\beta(T_1)}-$$
$$\frac{3}{8}i\mu b_r a(T_1)^3e^{-i\beta(T_1)}-\frac{3}{2}i\mu b_r v_c^2 a(T_1)e^{-i\beta(T_1)}-2i\left[\frac{1}{2}a'(T_1)e^{-i\beta(T_1)}-\frac{1}{2}ia(T_1)\beta'(T_1)e^{-i\beta(T_1)}\right]=0 \tag{26}$$

After transformations first equation (26) we obtain:

$$-\frac{1}{2}\alpha a(T_1)e^{-i\tau}+\frac{1}{2}i\delta a(T_1)+\frac{1}{2}\sigma a(T_1)-\frac{1}{2}i\mu a_r a(T_1)+$$
$$\frac{3}{8}i\mu b_r a(T_1)^3+\frac{3}{2}i\mu b_r v_c^2 a(T_1)+ia'(T_1)-a(T_1)\beta'(T_1)=0 \tag{27}$$

Then recalling

$$e^{-i\tau}=\cos\tau-i\sin\tau \tag{28}$$

The normal form is obtained:

$$
\begin{aligned}
&-\frac{1}{2}\alpha a(T_1)\cos\tau+\frac{1}{2}i\alpha a(T_1)\sin\tau+\frac{1}{2}i\delta a(T_1)+\frac{1}{2}\sigma a(T_1)-\frac{1}{2}i\mu a_r a(T_1)+\\
&\frac{3}{8}i\mu b_r a(T_1)^3+\frac{3}{2}i\mu b_r v_c^2 a(T_1)+ia'(T_1)-a(T_1)\beta'(T_1)=0
\end{aligned}
\tag{29}
$$

Separating real and imaginary parts, the two, so called, modulation equations are found:

$$
\begin{aligned}
&\frac{1}{2}\alpha a(T_1)\sin\tau+\frac{1}{2}\delta a(T_1)-\frac{1}{2}\mu a_r a(T_1)+\frac{3}{8}\mu b_r a(T_1)^3+\frac{3}{2}\mu b_r v_c^2 a(T_1)+a'(T_1)=0\\
&-\frac{1}{2}\alpha a(T_1)\cos\tau+\frac{1}{2}\sigma a(T_1)-a(T_1)\beta'(T_1)=0
\end{aligned}
\tag{30}
$$

Transforming, we obtain the modulation equations in the form (31).

$$
\begin{aligned}
&a'(T_1)=\frac{1}{8}\left(-4\alpha a(T_1)\sin\tau-4\delta a(T_1)+4\mu a_r a(T_1)-3\mu b_r a(T_1)^3-12\mu b_r v_c^2 a(T_1)\right)\\
&\beta'(T_1)=\frac{1}{2}\left(\sigma-\alpha\cos\tau\right)
\end{aligned}
\tag{31}
$$

For the steady state solution $a'(T_1)=0$ and $\beta'(T_1)=0$, next taking into account Eq. (9) the amplitude has two solutions: the trivial and the nontrivial one:

$$
a_1=0 \qquad a_2=\frac{2\sqrt{\mu a_r-\delta-3\mu b_r v_c^2-\alpha\sin\tau}}{\sqrt{3\mu b_r}}
\tag{32}
$$

The non-trivial solution (a_2) exists if:

$$
\mu a_r-\delta-3\mu b_r v_c^2-\alpha\sin\tau>0
\tag{33}
$$

The solutions are stable when eigenvalues of the Jacobian matrix are negative. This leads to the conditions:

$$
4\mu a_r-4\delta-9\mu b_r a^2-12\mu b_r v_c^2-4\alpha\sin\tau<0
\tag{34}
$$

Substituting the solutions (32) into (34) we get the two conditions, for stability of the trivial and the non-trivial solutions, respectively:

$$
\begin{aligned}
&\mu a_r-\delta-3\mu b_r v_c^2-\alpha\sin\tau<0\\
&-\mu a_r+\delta+3\mu b_r v_c^2+\alpha\sin\tau<0
\end{aligned}
\tag{35}
$$

Comparing Eqs. (33) and (35) one can draw out the conclusion that, if only the non-trivial solution exist it is always stable regardless the system parameters. An influence of the model parameters on chatter vibrations is presented in the next section.

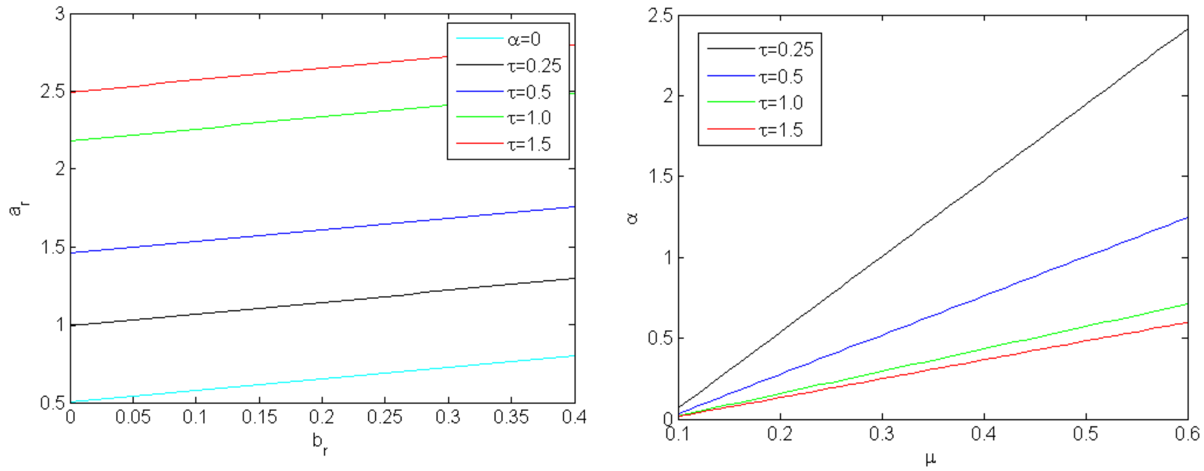
4 FRICTIONAL AND REGENERATIVE CHATTER

An influence of frictional chatter on regenerative one and vice versa is investigated on the basis of Eq. (33). Generally, three cases can be analysed here:

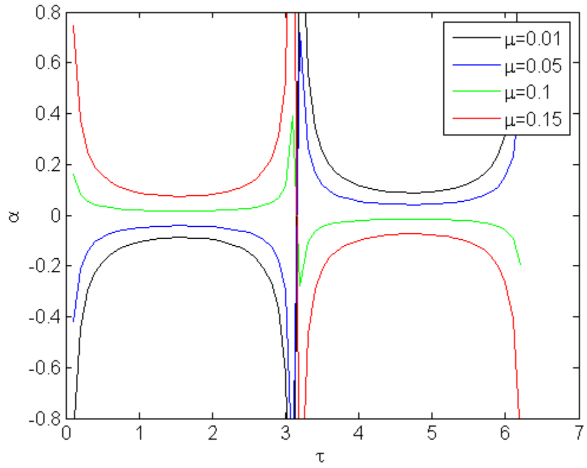
- when only regenerative effect exists (μ=0),
- when only frictional effect exists (α=0),
- when regenerative and frictional effects exist ($\alpha \neq 0$ and $\mu \neq 0$).

The first case is well-known because leads to the linear problem with time delay. Then, the exact analytical solution can be found by means of the Laplace transform method [15, 26, 27]. The second case shows self-excited vibrations generated by the nonlinear friction force. The third case presents chatter vibrations produced by friction force and regenerative effect simultaneously. Further analysis is conducted with parameters: μ=0.2, α=0.4, τ=0.25, δ=0.1, v_c=0.5, a_r=1.2, b_r=0.05. Usually, the change of cutting velocity can be realized by change of the angular velocity $2\pi/\tau$ and/or the diameter of the tool in milling or workpiece in turning. Here it is assume that v_c is independent on τ. The curves presented in Figure 2 partition the regions where the trivial and non-trivial (chatter) solutions exist for various delay τ (Figure 2a, b) and μ (Figure 2c, d). Chatter vibrations occurs: when a_r is above the lines shown in Figure 2a, below the lines $\alpha(\mu)$ (Figure 2b), below the curve $\alpha(\tau)$ in the range $0<\tau<\pi$ or above for $\pi<\tau<2\pi$ (Figure 2c) and below the curve $v_c(\tau)$ (Figure 2d). Figure 2b tell us how to select a_r and b_r to obtain trivial or non-trivial solution.

a) b)



c)



d)

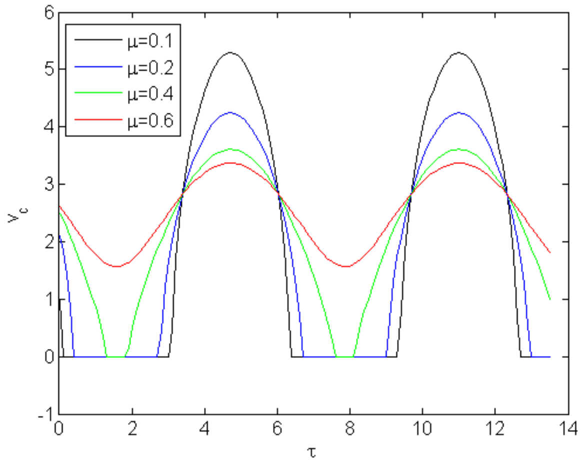


Figure 2: The regions of the non-trivial solution existence, above the curve (a) and below the curves (b,c,d)

Strength of frictional chatter is represented by coefficient μ. If μ grows the coefficient showing strength of regenerative effect (α) rises as well (Figure 2b). For small τ the region of non-trivial solution (chatter) is bigger. An increase of μ also enlarges the chatter region. The limiting value of α depends also on the time delay (Figure 2c). For μ=0.01 the black curve is very close to the stability lobs obtained for regenerative cutting (μ=0). The limiting v_c changes periodically with τ as well (Figure 2d). From practical point of view the trivial solution, which appears above the limiting curves, is important. Interestingly, if velocity v_c is big enough (e.g. v_c =4, μ=0.4 and μ=0.6) there is no non-trivial solution. That means the process is chatter free regardless the time delay (τ). In the identical system without frictional effect chatter emerges in unstable lobs which exist periodically for the sake of τ.

5 CONCLUSIONS

Regenerative chatter in the linear system manifest itself as unlimited increase of vibrations for the time delay which corresponds to the natural frequency of a system. In case when nonlinear self-excited component is introduce into the system chatter vibrations amplitude is limited by limit cycle but on the other hand additional regions of chatter vibrations appear, that do not exist in pure regenerative system. However, the possibility of avoiding chatter in case of big velocity v_c and strong frictional effect measured by coefficient μ is the most interesting form technological point of view. That could be applied in practice but the results need to be carefully checked by experiment and numerical simulations.

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