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DYNAMIC ANALYSIS OF THE SOIL-FOUNDATION-TANK-FLUID INTERACTION

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Abstract. *Ground-supported tanks are used to store a variety of liquids. During the earthquake activity the liquid exerts impulsive and convective pressure (sloshing) acting simultaneously onto the walls and the bottom of the rectangular tank. This paper provides theoretical background for investigation of hydrodynamic pressure that is being developing during an earthquake in the liquid storage ground-supported rectangular container – the endlessly long shipping channel, is grounded on hard soil or sub-soil 30 MNm^{-3}. The hydrodynamic pressure obtained by the analytical method is compared with results based on the numerical modeling of the problem by using ALE FSI FEM formulation in program ADINA.*

1 INTRODUCTION

Large-capacity ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes [2 - 7].

The seismic analysis and design of liquid storage tanks are complicated by many numbers of problems, for examples: dynamic interaction between contained fluid and tank which is called fluid-structure interaction; sloshing motion of the contained fluid; and dynamic interaction between tank and sub-soil, tank-soil interaction could under specific conditions have a significant effect on seismic response of the tank.

The knowledge of forces and pressure acting on the walls and the bottom of containers during an earthquake plays essential role in reliable and durable design of earthquake resistance structure/facility – tanks.

2 HYDRODYNAMIC PRESSURES ON THE WALL OF TANK

Seismic design of liquid storage tanks has been adopted in [2-4, 6-7, 8]. When a tank containing liquid vibrates, the liquid exerts impulsive and convective hydrodynamic pressure on the tank wall and the tank base, in addition to the hydrostatic pressure.

The objective for investigations presented in this paper is the rectangular container partially filled with fluid while opened free surface of the fluid. And let the walls of the container have horizontal acceleration $\ddot{u}_o$ in the x – direction, [1, 3]. Due to the acceleration $\ddot{u}_o$ the unknown pressure profile is generated over the walls of the container. Let the fluid have a depth H, a length $2L$ and a unit thickness, Figure 1a. It is obvious that that the behavior of the fluid is similar to the case which would be obtained assuming that the horizontal component of fluid velocity $\dot{u}$, is independent on y coordinate. This situation looks like as the fluid constrained by thin, massles, vertical membranes that are free to move in the x – direction. Let the distance between adjacent membranes is dx (see Figure 1).

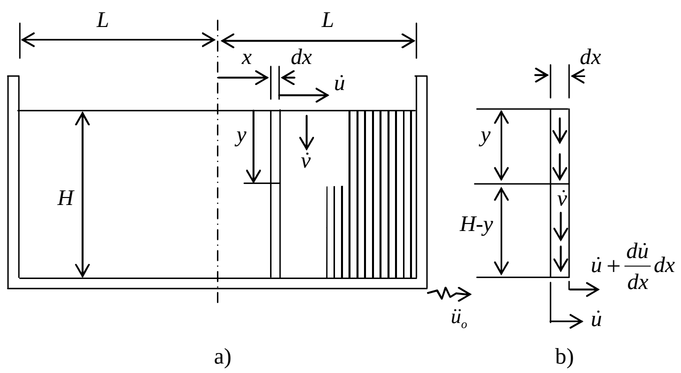


Figure 1: Rectangular tank is filled with water

In the case the walls of the container are under known acceleration the membranes do accelerate proportionally to the fluid while fluid does at the same time squeeze along membranes vertically along y-direction. As shown in Figure 1b, since the fluid is restrained between two adjacent membranes, the vertical velocity $\dot{v}$ is dependent on the horizontal velocity $\dot{u}$ according to

$$\dot{v} = (H - y)\frac{d\dot{u}}{dx}. \tag{1}$$

This is the equation specifying the constraint of the fluid flow. As the fluid is considered incompressible, it follows that the acceleration $\ddot{v}$ is proportional to the velocity $\dot{v}$ and the acceleration $\ddot{u}$ is proportional to the velocity $\dot{u}$, and the pressure in the fluid between two membranes is given by the standard hydrodynamic equation:

$$\frac{\partial p}{\partial y} = -\rho\ddot{v}, \tag{2}$$

where ρ is mas-density of the fluid.

2.1 Hydrodynamic impulsive pressures

The acceleration $\ddot{u}_o$ thus produces an increase of the pressure acting onto one wall and the decrease of the pressure acting onto the opposite wall according to the nondimensional coordinate $\xi = z/H$, where z is the distance-height of the free surface measured from the bottom of tank to the free surface of the fluid $z = H - y$. The formula for the pressure generated on the membrane's wall is given by

$$p_{HDIw}(\xi) = C_{iw}(\xi)\rho H A_g(t); \tag{3}$$

Where the free-field motion of the ground is here represented by the peak value of $Ag(t)$, ρ is mass density of liquid. The distribution of the hydrodynamic pressure $p_{HDIw}(\xi)$, acting onto the wall of the tank along its high, is given by eq. (3), where the function $C_{iw}(\xi)$ (shown in Figure 3a) does depend on slimness of the tank γ. The shape function $C_{iw}(\xi)$ prescribes the distribution of the impulse pressure along the high of the wall. Tank's slimness is given by the relation $\gamma = H/R$, where H is the height of the fluid filled container, and L is half of width of the container.

2.2 Hydrodynamic convective pressures

The effect of the impulsive pressure tends to generation of oscillation of the fluid. To examine the fundamental mode of vibration, consider the fluid to be constrained between rigid membranes that are free to rotate as shown in Figure 2.

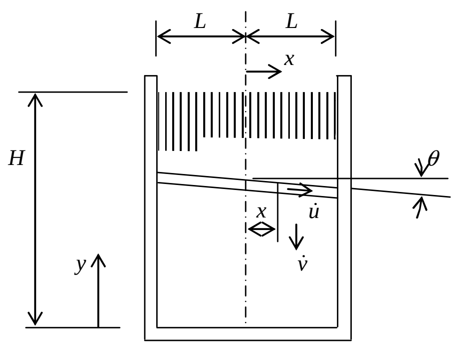


Figure 2: Rectangular tank is filled with water

The constraint is described by the following equations:

$$\dot{u} = \frac{L^2 - x^2}{2} \frac{d\dot{\theta}}{dy}, \tag{4}$$

$$\dot{v} = \dot{\theta} z. \tag{5}$$

The pressure in the fluid is given by

$$\frac{\partial p}{\partial x} = -\rho \ddot{u}. \tag{6}$$

The hydrodynamic pressure is given in dependence on $\xi = z / H$, where distance z is distance (height) from the bottom of tank to the free surface of the liquid $z = y$, the convective hydrodynamic pressure is given by a summation of modal terms (sloshing modes), each one having a different variation with time.

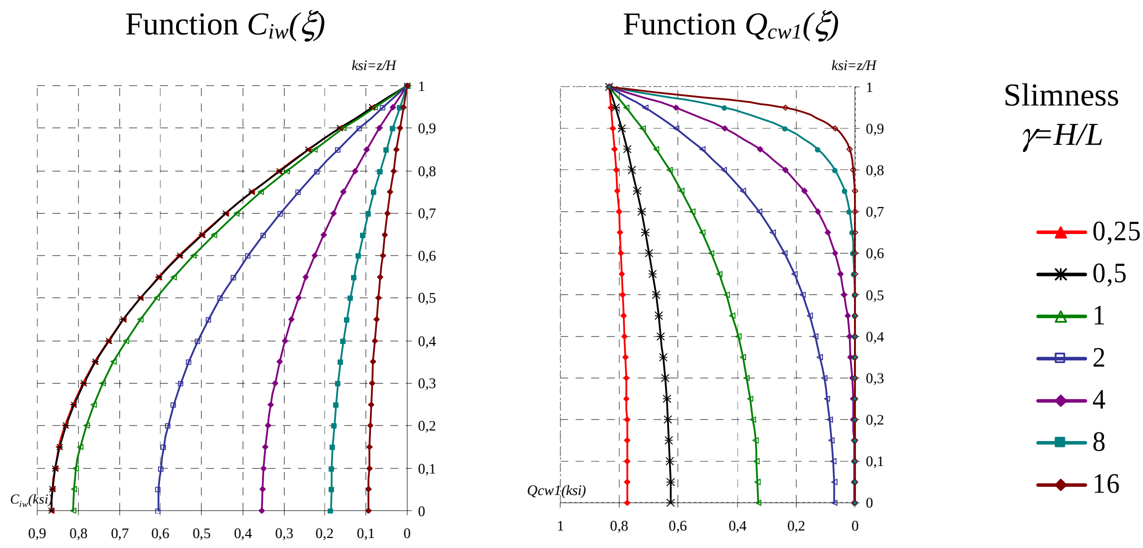
$$p_{HDCwn}(\xi) = \sum_{n=1}^{\infty} Q_{cwn}(\xi) \rho L A_n(f_{cn}). \tag{7}$$

As for rectangular tanks, the dominant contribution is that of the fundamental mode, that is:

$$p_{HDCw1}(\xi) = Q_{cw1}(\xi) \rho L A_1(f_{c1}); \tag{8}$$

the free-field motion of the ground is here represented by the peak value of $A_1(t)$ that is the acceleration response function of a simple oscillator having the frequency of the first mode, the appropriate value of the damping, and subjected to an input acceleration $A_g(t)$. ρ is mass density of liquid. The distribution of hydrodynamic pressure along the high of the wall of the tank $p_{HDCw1}(\xi)$ is given by eq. (8), where the function $Q_{cw1}(\xi)$ is described in Figure 3b with respect to the slimness of the tank γ. The shape function $Q_{cw1}(\xi)$ prescribes the distribution of the convective pressure along the high of the wall. Tank's slimness is given by the relation $\gamma = H/R$, where H is the height of the fluid in the tank and L is the half of the width of the container.

2.3 Hydrodynamic pressures



a) b)

Figure 3: Functions $C_{iw}(\xi)$ of partitions of the impulse pressure acting onto the wall of the tank in dependence on slimness of the tank γ. Functions $Q_{cw1}(\xi)$ of partitions of the impulse pressure acting onto the wall of the tank, in dependence on slimness of tank γ.

For those of tanks, whose walls can be assumed as rigid, a solution of the Laplace equation relating to the horizontal excitation can be obtained in a close form, so that the total pressure is again given by the sum of impulsive and convective pressures by use of absolute summation rule [9]:

$$p_{HDw}(\xi) = p_{HDIw}(\xi) + p_{HDCw1}(\xi)\,. \tag{9}$$

3 FLUID-STRUCTURE INTERACTION BY USING FEM

For the fluid-structure interaction analysis, there are three different finite element approaches to represent fluid motion; i.e., Eulerian, Lagrangian and mixed methods. In the Eulerian approach, velocity potential (or pressure) is used to describe the behavior of the fluid, whereas the displacement field is used in the Lagrangian approach. In the mixed approaches, both the pressure and displacement fields are included in the element formulation.

In fluid-structure interaction analyses, fluid forces are applied into the solid and the solid deformation changes the fluid domain. For most interaction problems, the computational domain is divided into the fluid domain and solid domain, where a fluid model and a solid model are defined respectively, through their material data, boundary conditions, etc. The interaction occurs along the interface of the two domains. Having the two models coupled, we can perform simulations and predictions of many physical phenomena.

In general the fluid model is defined in the fluid domain with wall boundary conditions, prescribed velocity at the inlet, zero distributed normal-traction at the outlet and, most importantly, the fluid-structure interface condition. The solid model is defined in the structural domain, where its bottom is fixed and its bottom and two sides is the fluid-structure interface. The two models must have also been discretized using the elements that are available in ADINA Solids & Structures and ADINA-F. The typical task of an analysis of a fluid-structure model is to obtain the fluid and structure response through the coupled solution. The structural model is based on a Lagrangian coordinate system and the displacements are the primary unknowns. A pure fluid model is always analyzed using a Eulerian coordinate system. However, for fluid-structure interaction problems, the fluid model must be based on an arbitrary-Lagrangian-Eulerian coordinate system, since the fluid-structure interface is deformable. Therefore, the solution variables of the fluid flow include the usual fluid variables (pressure, velocity, etc.), as well as displacements, [3-7, 11].

The ADINA system has in respect of the dynamics used equation

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{d}_k a_k(t)\,. \tag{10}$$

4 SOLUTION, RESULTS AND DISCUSSION

As an example case we will assume the ground supported rectangular endlessly long shipping channel, with the length $L = 5$ m and the height $H_w = 3$ m. Chanel surrounding walls have the uniform thickness of 0.25 m. The base slab of the channel is $h = 0.4$ m thick. Shipping channel is filled with water up to the height of 2.5 m. There is no roof slab structure covering the channel. This water filled tank is grounded on hard soil or sub-soil 30 $\mathrm{MNm^{-3}}$, Figure 4. As the excitation input we consider horizontal earthquake load response given by the accelerogram of the earthquake in Loma Prieta, California (18.10.1989), Figure 5. In the analysis we use just the accelerogram for the seismic excitation in *x*-direction. In the numerical analysis earthquake is simulated by using of input horizontal displacement of Loma Prieta in California in *x*-direction. The seismic excitation starts after 10 seconds, the first 10 seconds ask only hydrostatical loading. During first 10 seconds only hydrostatic load is applied.

The first numerical simulation of the earthquake problem was performed by application of Finite Element Method (FEM) utilizing software ADINA. Arbitrary-Lagrangian-Eulerian formulation is used for the problems including free surface of the fluid. Two way Fluid-Structure Interaction (FSI) techniques were used for simulation of the interaction between structural deformation and inertial forces of the fluid at the common boundary. The solid walls and base of the shipping channel was modeled by using 2D SOLID finite elements or 3D SOLID finite elements. The fluid inside the shipping channel was modeled by using 2D FLUID finite elements or 3D FLUID finite elements. The structural response 2D SOLID finite elements were based on the plain strain conditions and 2D FLUID finite elements.

The second numerical simulation of the earthquake problem was performed by application of Finite Element Method (FEM) utilizing software ADINA as well. The solid walls and base of the shipping channel was modeled by using 2D SOLID finite elements or 3D SOLID finite elements. The hydrostatic and hydrodynamic pressures were used as loading on the walls and bottom of tank, given by procedure in code EC8-4 [9, 10].

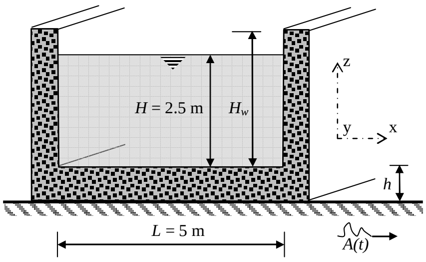


Figure 4: Details of tank geometry

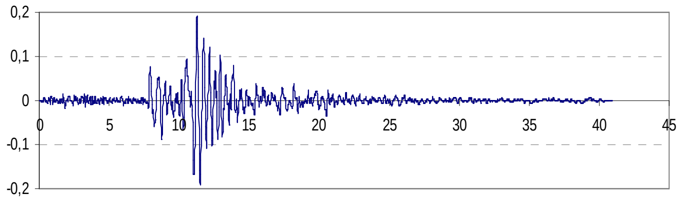


Figure 5: Akcelerogram Loma Prieta, California

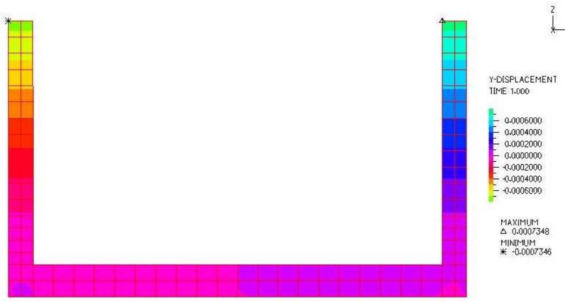


Figure 6: Horizontal displacement given from hand made hydrostatic pressures

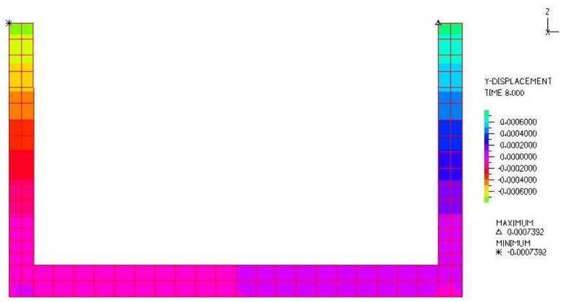


Figure 7: Horizontal displacement by using FEM ALE FSI in time t = 8 s (only hydrostatic effect)

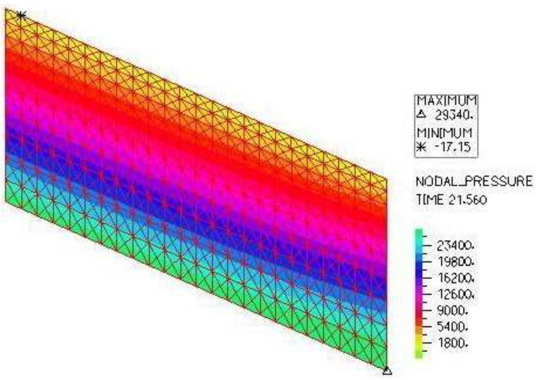


Figure 8: Pressure of fluid in time t = 21.56 s

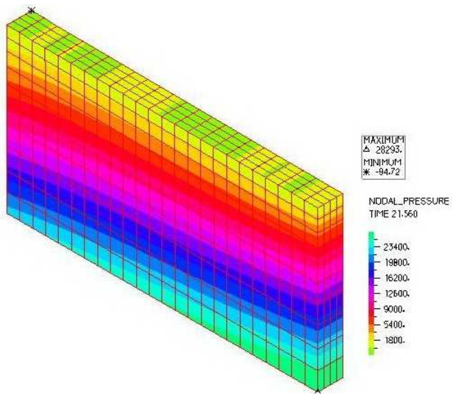


Figure 9: Pressure of fluid in time t = 21.56 s

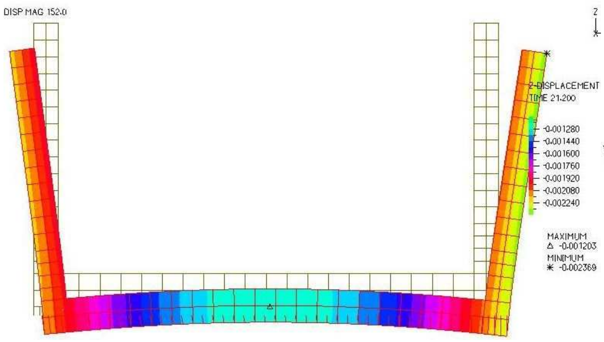


Figure 10: Displacement of shipping channel on sub-soil 30 MNm^{-3} in time t = 21.20 s

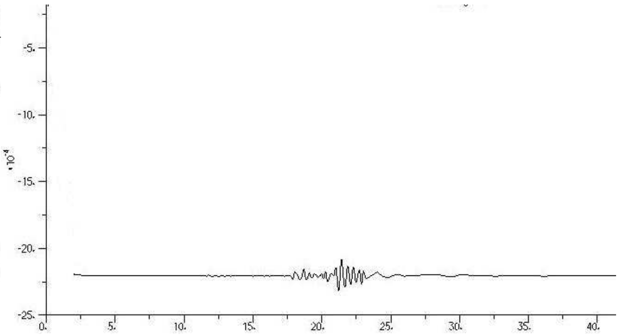


Figure 11: Time dependent response of the vertical displacement at the bottom right point of the shipping channel on sub-soil 30 MNm^{-3} in [m].

	The tank is located on hard soil		**The tank is located on soil 30 MNm^{-3}**	
	EC8	**FEM**	**EC8**	**FEM**
Maximal horizontal displacement at reservoirs [mm]	1.072	0.841	3.340	2.677
Maximal Von Mises stress in reservoirs [MPa]	2.555	2.258	2.478	1.987
Maximal stress in sub-soil [MPa]	-	-	0.0626	0.0464
Maximal pressure of fluid [kPa]	30.590	29.340	30.590	29.354
Maximal height of wave [mm]	50.0	19.6	50.0	19.6

Table1: Comparison of results by using simple procedure in EC8-4 and by modeling FEM FSI ALE (2D Fluid)

5 CONCLUSION

Ground supported rectangular endlessly long shipping channel having the length of L = 5 m and the height H_w = 3 m was analyzed. The channel is partially filled with the water up to the height 2.5m. No upper roof slab above the channel is considered. This water filled tank is grounded in hard soil or sub-soil 30 MNm^{-3}. As the excitation input we consider horizontal earthquake load of the earthquake Loma Prieta in California. Focusing on dynamic response of the structure due to the earthquake excitation the analytical method (procedure by EC8-4 [9]) together with numerical method (FEM ALE FSI) was successfully applied in the complex analysis. The comparison of computational methods the representative measures (pressure, stress) correspondent very well.

The measures of responses of the interest were: pressure in the fluid, displacement of the free surface of the fluid, horizontal and vertical displacements and structural stress of the tank. Time dependent responses correlate with excitation input while the oscillating components do not cause any dramatic behavior of the structure.

From the Figures 6, 7 is seen, that maximal horizontal displacement of the walls of the shipping channel is 0.7392 mm (from hydrostatic pressure), and 0.7346 mm (by using numerical simulation ALE FSI FEM), given from program ADINA.

The maximum value of hydrodynamic pressure of fluid at the wall is at the bottom of the wall of the shipping channel in node at the right bottom very near the wall location, it is 20.34 kPa by using numerical simulation ALE FSI FEM, using 2D Fluid finite elements, Figure 8. The maximum value of pressure of fluid at the wall at the right bottom of the wall of the shipping channel is 28.21 kPa by using numerical simulation ALE FSI FEM, using 3D Fluid, Figure 9.

From the Figure 10 is seen displacement of tank on sub-soil 30 MNm^{-3} in time t = 21.20 s and from the Figure 11 is seen time dependent response of the vertical displacement at the bottom right point of the shipping channel in [m].

The maximal horizontal displacement and maximal Von Mises stress in the reservoir, maximal stress in the sub-soil, the maximal pressure of fluid and the maximal height of the wave of fluid (behavior of the free surface of the fluid) are listed in Table 1 for two types od sub-soils. “FEM” results (numerical solutions - FEM ALE FSI formulation using of 2D SOLID finite elements of the shipping channel and 2D FLUID finite elements of the fluids filling) are listed in the Table 1. “EC4” results are given from the analytical solution using of hydrostatic and hydrodynamic pressure, given by procedure in code EC8-4.

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