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STUDY OF PRACTICAL LIMITATIONS ON THE USE OF ELECTRODYNAMIC ACTUATORS IN THE CONTROL OF MECHANICAL IMPEDANCE AND TRANSMISSIBILITY

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Abstract. *In achieving high-precision positioning systems is necessary to have mechanical systems able to control the mechanical impedance, mobility or transmissibility. A general aspect is that conventional mechanical solutions have limitations, mainly in ensuring adaptability to the changing operating conditions. This paper addresses the issues of mentioned control parameters for 1-DOF systems by including in their structure of an electrodynamic actuator used as bidirectional converter of mechanical energy and electrical energy and some additional external electrical impedance, focusing on establishing practical limitations of using electrodynamic actuators in controlling mechanical impedance and transmissibility.*

1 INTRODUCTION

Many technical domains, where the precision of positioning is essential, requires mechanical systems where is possible the control of impedance and vibration transmissibility.

Conventional mechanical solutions [1] have restrictions or limitations, especially when they function in conditions of frequent change of parameters (frequency limits, amplitudes, spectrum or tones). An optimal solution is their transformation from mechanical systems, designed and engineered for specific operating modes or conditions, in to electro-mechanical systems, with a greater ability of adaptation to a variety of situations and modes of operation, adaptability characteristic of electric domain.

2 GENERAL CONSIDERATIONS

Vibration or vibration effects control of machines or mechanical structures is a very important process. There are three main ways[2] in which it can achieve better vibration isolation effect, namely:

- significant reduction of the number of sources of vibrations in the system or structure;
- introduction on the routes of vibration transmission of isolators or change of these routes in order to reduce the influence of vibrations;
- the change of system parameters on which acts vibration (the receiver system) or the change of radiating elements (which transmit and amplify the received vibrations).

Regardless of the method used, in all three situations can be used passive means and ways of control, with good results only in well-defined cases, for which were initially set the functioning parameters, or in limited areas of variation of vibration frequency.

If the desired goal is to isolate a system from the effect of vibrations in a narrow frequency range can be used protective methods that are based on insulators or dynamic absorber.

The subject of the paper is the theoretically determination and verification of the influence of the introduction of an electrodynamic actuator in the structure of a mechanical system with one degree of freedom, with the estimation of control limits of mechanical impedance and transmissibility determined by the electrical parameters of the actuator.

3 MATHEMATICAL MODEL OF THE SYSTEM

3.1 The main electro-mechanical system

The studied system is a simple mechanical system Figure 1, with one degree of freedom, for isolation from external influences of a body of mass ***m***. The system integrates mechanical components for passive isolation (a spring that has elastic constant ***k*** and a viscous damper with viscous damping constant ***c***), and an electrodynamic actuator for active and semi-active control of the system.

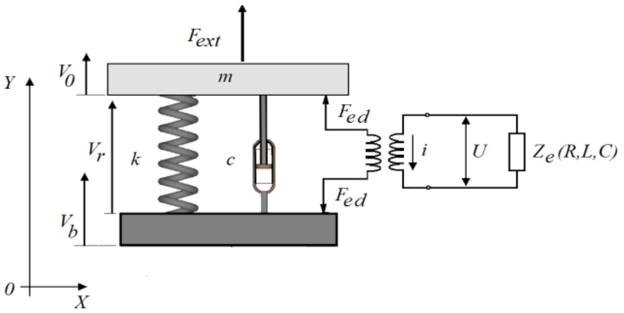

Figure 1: The main electro-mechanical system.

The electrodynamic actuator can develop on the mechanical elements of the system an electrodynamic force $\boldsymbol{F_{ed}}$ at actuator terminals, being attached an external electric impedance $\boldsymbol{Z_e}$ including elements such an electrical resistance ***R***, an inductance ***L*** and a capacitance ***C***.

The principle of operation is simple: when on the base of the main system is applied an exterior disturbing movement, the control system must ensure the cancellation effects in such a way that the position of the object of mass ***m*** must be a constant and its velocity $\boldsymbol{v_t}$ to be null from the external coordinate system (XOY) Eq.(1).

$$v_0 = v_r + v_b \tag{1}$$

In Eq.(1) $\boldsymbol{v_0}$ is the velocity of mass ***m***, $\boldsymbol{v_r}$ is the relative velocity of mass ***m*** from the base and $\boldsymbol{v_b}$ is the velocity of the base.

The electrodynamic actuator is the active element for the control of system characteristics and can be considered as a semi-active component when is used as a converter of mechanical energy into electrical energy that can be dissipated later by an external impedance (Z_e).

For the study is used a loudspeaker [3] to test the possibility of using the electrodynamic actuator for vibration control. The main parameters of the system and of the loudspeaker, both mechanical and electrical are the following:

- ***B*** – [N/A] – magnetic induction;
- ***c*** – [m/N] – mechanical compliance;
- ***i*** – [A] – the control current applied to the coil;
- ***k*** =2452,5 N/m – the elastic constant of spring;
- $\boldsymbol{k_m}$ =4 - mechanical constant of system- adimensional***;***
- $\boldsymbol{k_e}$=4 - electrical constant of system - adimensional***;***
- ***l -*** [m] – the length of the coil wire;
- $\boldsymbol{L_b}$ =0,00056 H – coil inductance;
- ***m*** = 0,0434 kg – the mass of the stabilized system;
- $\boldsymbol{R_b}$ =8,00 Ω – the coil resistance;
- ***U*** – [V] – the control voltage of the coil;
- $\boldsymbol{U_{em}}$ – [V] – the electromotive force voltage induced in the coil;
- $\boldsymbol{v_r}$ – [m/s] – the speed of movement of the mass ***m*** relative to the base;
- $\boldsymbol{Z_e}$ – [Ω] – the external impedance.

The numerical values of enumerated mechanical and electrical parameters is used to simulate the operation of the system and for the determination of the characteristics of the main electro-mechanical system.

3.2 The mathematical model of the main electro-mechanical system

The mechanical equilibrium equation of the system is:

$$m \cdot \frac{dV_0}{dt} = F_{ext} + k_m \cdot i - c \cdot V_r - k \cdot X_r \tag{2}$$

To determine the equation describing the operation of the electrodynamic actuator control is used the scheme presented in Figure 1.

Electric circuit equation is:

$$k_e \cdot V_r + L_b \cdot \frac{di}{dt} + \left(R_b + Z_{ext}\right) \cdot i = U \tag{3}$$

As can be seen, the system of equations, Eq.(2) and (3) are written in the time domain.

For the modeling of the electromechanical active control system and his operational performance [4] with the help of Matlab-Simulink software package, is especially useful the application of Laplace transform to the system of equations, conducting to the equations:

$$\begin{cases} m \cdot s \cdot V_0 + c \cdot (V_0 - V_b) + \dfrac{k}{s} \cdot (V_0 - V_b) = F_{ext} + k_m \cdot i(s) \\ U(s) - k_e \cdot (V_0 - V_b) = R_b \cdot i(s) + L_b \cdot s \cdot i(s) + Z_{ext} \cdot i(s) \end{cases} \tag{4}$$

respectively:

$$\begin{cases} (m \cdot s + c + k/s) \cdot V_r + m \cdot s \cdot V_b = F_{ext} + k_m \cdot i(s) \\ k_e \cdot V_r + (R + L \cdot s + Z_{ext}) \cdot i(s) = U(s) \end{cases} \tag{5}$$

The mechanical impedance of the system is:

$$Z_m = m \cdot s + c + k/s \tag{6}$$

and electrical impedance is:

$$Z_e = R + L \cdot s + Z_{ext} \tag{7}$$

From equations system Eq. (5), using relatively simple operations, result the relations for the calculation of current and velocity. For mobility and impedance control is assumed a null external voltage U=0, and also a null velocity of the base V_b=0, conducting to:

$$i = -[k_e/(Z_e \cdot Z_m)] \cdot F_{ext} / [1 + (k_e \cdot k_m)/(Z_e \cdot Z_m)] \tag{8}$$

$$V_r = F_{ext} / [Z_m + (k_e \cdot k_m)/Z_e] = F_{ext} / \{Z_m \cdot [1 + (k_e \cdot k_m)/(Z_e \cdot Z_m)]\} \tag{9}$$

Further in the study is considered an equivalent mechanical impedance:

$$Z_m^* = Z_m \cdot [1 + (k_e \cdot k_m)/(Z_e \cdot Z_m)] \tag{10}$$

Few observations on the extreme values of terms and their consequences, from previous relationships are necessary.

So, if: $(k_e \cdot k_m)/(Z_e \cdot Z_m) << 1$ then $Z_m^* \cong Z_m$ and results:

$$V_r \cong F_{ext}/Z_m \,; \tag{11}$$

$$i = [-k_e/(Z_e \cdot Z_m)] \cdot F_{ext} \tag{12}$$

which represents the transfer function of a force transducer with output in current.

If: $(k_e \cdot k_m)/(Z_e \cdot Z_m) >> 1$ then $Z_m^* \cong (k_e \cdot k_m)/Z_e$ and results:

$$V_r \cong F_{ext} / [(k_e \cdot k_m)/Z_e] \tag{13}$$

$$i = (-1/k_m) \cdot F_{ext} \tag{14}$$

which represents also the transfer function of a force transducer with output in current.

3.3 The study of system impedance when the additional external impedance is null

If U=0 and Z_{ext}=0 then $Z_e = R + s \cdot L$.

One can distinguish two situations, namely:

a) If $\omega >> \omega_{el} = R/L$ then $Z_e \cong s \cdot L$ and results:

$$Z_m + (k_m \cdot k_e) / Z_e = m \cdot s + c + k / s + (k_m \cdot k_e) / (s \cdot L) \tag{15}$$

determining a behaviour as if the spring damping coefficient ***k*** is increased.

b) If $\omega << \omega_{el} = R / L$ then $Z_e \cong R$ and results:

$$Z_m + (k_m \cdot k_e) / Z_e = m \cdot s + c + k / s + (k_m \cdot k_e) / R \tag{16}$$

determining a behaviour as if the viscous damping coefficient ***c*** is increased.

If: $\omega_{rez} = \sqrt{k / m} << R / L$ then, when the system reaches the mechanical resonance, will dominate the influence of the electrical resistance ***R***:

$$V_r = F_{ext} / [c \cdot (1 + (k_m \cdot k_e) / (c \cdot R))] = F_{ext} / (c + k_m \cdot k_e / R) = F_{ext} / c^* \tag{17}$$

and it can be observed that is changing only the value of the relative speed (lower mobility peak) and does not change also the value of the resonant frequency of the system.

If: $\omega_{rez} = \sqrt{k / m} >> R / L$ then, when the system reaches the mechanical resonance, will dominate the influence of the electrical inductance ***L***:

$$V_r = F_{ext} / [c + (k_m \cdot k_e) / (s \cdot L)] \tag{18}$$

and it can be observed that is changing the value of the relative speed (lower mobility peak) and also the value of the resonant frequency of the system (higher resonance frequency) .

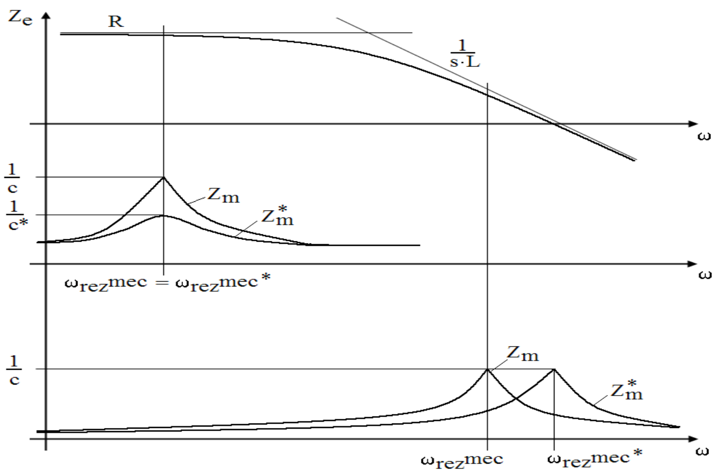

Figure 2: Influence of electrical impedance in general changing of mechanical impedance of the system.

If ***U=0*** and ***Z_{ext}=1/(C·s)***, then:

$$Z_m^* = Z_m + (k_m \cdot k_e) / Z_e = m \cdot s + c + k / s + (k_m \cdot k_e) / (R + L \cdot s + Z_{ext}) \tag{19}$$

The electrical impedance can be calculated using the relationship:

$$Z_e = R + L \cdot s + 1 / (C \cdot s) \tag{20}$$

and has the resonant frequency ***ω_e***:

$$\omega_e = 1 / (\sqrt{L \cdot C}) \tag{21}$$

The changes of the impedance of the system, due to the variation of the electrical parameters R, L and C is shown in the Figure 3.

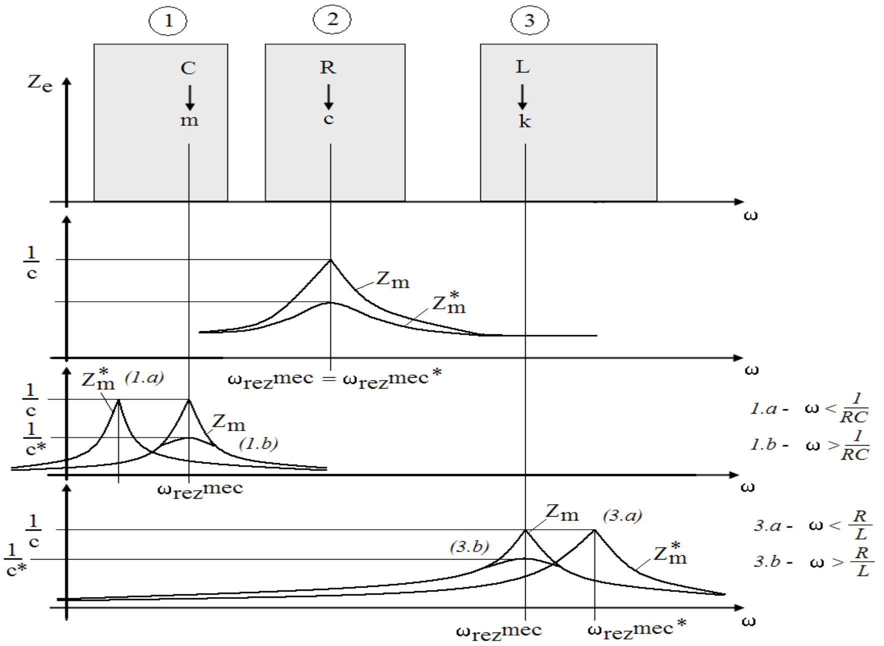

Figure 3 : The influence of electrical impedance on the mechanical impedance of the system and areas of electrical impedance approximation by simple components.

From the analysis of how changes the value of system impedance, can be identified three distinct areas (Figure 3):

1) when $\omega \ll \omega_e$ then $Z_e \cong R + 1/(C \cdot s)$ and two situations can be distinguished:

- **1.a)** when $\omega < (1/R \cdot C)$ in which case: $Z_m = m \cdot s + c + k/s + k_m \cdot k_e \cdot C \cdot s$, aspect that causes a decrease of mechanical resonance frequency, an effect similar to that produced by changing the mass ***m*** of the system (Figure 3, zone 1, pct.1.a) with $m^* = m + k_m \cdot k_e \cdot C$.
- **1.b)** when $\omega > (1/R \cdot C)$ in which case: $Z_m = m \cdot s + c + k/s + (k_m \cdot k_e)/R$, aspect that causes a modification of amplitude of mechanical resonance frequency, an effect similar to that produced by changing the value of viscous damping coefficient ***c*** of the system (Figure 3, zone 1, pct.1.b) with $c^* = c + k_m \cdot k_e / R$.

2) when $\omega \approx \omega_e$ in which case: $Z_m = m \cdot s + c + k/s + (k_m \cdot k_e)/R$, which does not cause the change of mechanical resonance frequency, if $\omega = \omega_{rez\ mech} = \sqrt{k/m}$, but occurs change in value at resonance, similar to the influence of changes in viscous damping coefficient ***c*** (Figure 3, zone 2).

3) when $\omega \gg \omega_e$ in which case: $Z_e \cong R + L \cdot s$ and two situations can be distinguished:

- **3.a)** when $\omega > (R/L)$ in which case: $Z^*_m = m \cdot s + c + k/s + (k_m \cdot k_e)/(L \cdot s)$, aspect that causes a increase of mechanical resonance frequency, an effect similar to that produced by changing the spring damping coefficient ***k*** of the system (Figure 3, zone 3, pct.3.a) with $k^* = k + (k_m \cdot k_e)/\mathrm{L}$.
- **3.b)** when $\omega < (R/L)$ in which case: $Z_m = m \cdot s + c + k/s + (k_m \cdot k_e)/R$, aspect that, if $\omega = \omega_{rez\ mech} = \sqrt{k/m}$, not cause the change of mechanical resonance frequency but

occurs change in value at resonance, similar to the influence of changes in viscous damping coefficient ***c*** (Figure 3, zone 3, pct.3.b) with $c^* = c + (k_m \cdot k_e) / \mathrm{R}$.

3.4 Study of influence of variation of parameters R, L and C on the system mobility

Based on Matlab software application possibilities to simulate the operation of the systems and on the real values of the mechanical and electrical parameters of system, was developed a simulation program in order to investigate the influences of variation of electrical parameters on the system mobility.

For simulation of the mobility are represented two functions, the mobility of the pure mechanical system H and the mobility of the system with electrodynamic actuator G:

```
H=1/( m*s+c+k/s);
G=(Rez+L*s+Ze)/((Rez+L*s+Ze)*(m*s+c+k/s)+Ke*Km);
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Further are presented the simulations carried out for different values of electrical inductance, total electrical resistance at circuit terminals and the electrical capacity.

The values of inductance have been selected after examining the range of variation of inductance for electromagnetic actuators existing on the market. For systems with low inductance, with the order of magnitude of hundreds of µH, was selected a inductance of 560 µH.

The variation of system mobility for a null terminal capacity and variable resistance between 0.01Ω - 20Ω , with step 0.01Ω, is shown in Figure 4.

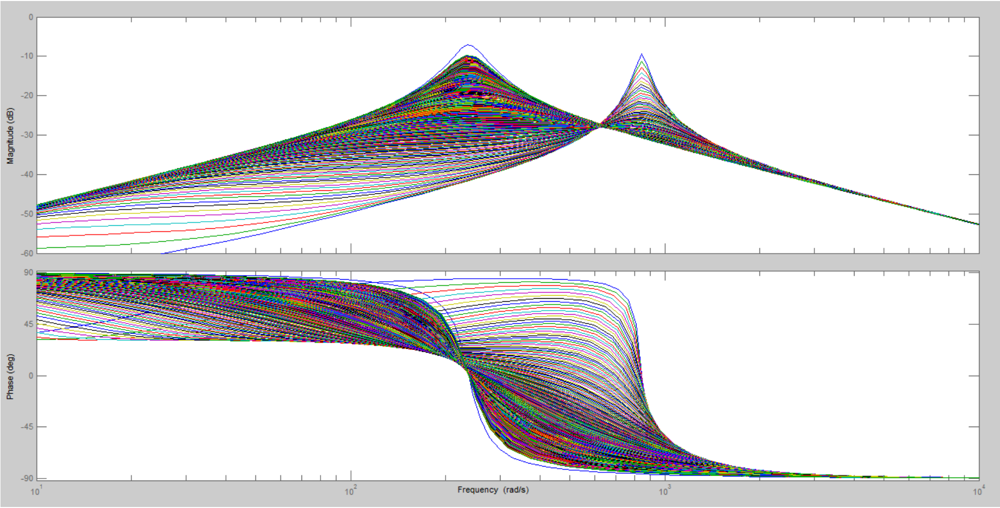


Figure 4 The mobility of the system when C=0 F; L=560µH and R varies from 0.01Ω to 20Ω with step 0.01Ω

On the Figure 4 can be identified three major zones, according with the values of electrical resistance:

- if $0\Omega \leq R \geq 0.5\Omega$ then the dominant behavior of electric circuit is inductive, resulting in an increase of equivalent mechanical stiffness and consequently of the resonant frequency of the system. Operation of the system in this area leads to high attenuation in the vicinity of purely mechanical resonant frequency but is dangerous because at very low values of electric resistance significantly increases the maximum of electro-mechanical resonance;
- if $0.5\Omega \leq R \geq 2.0\Omega$ the values of electrical resistance lead to an answer almost flat with a loss of more than 20dB in relation to the maximum value of the curve just for mechanical system. With increasing of values of electrical resistance decreases the importance of electri-

cal inductance, which leads to a decrease of the electromechanical resonance but without practical relevance due to the shape of curves. This is the zone most convenient for use;

- if $2.0\Omega \leq R \geq 20.0\Omega$ then the electrical resistance contributes only to increasing the equivalent damping coefficient (higher resistance is producing lower attenuation).

An increase of the value of electrical inductance, with the preservation of mechanical parameters lead to a much lower differentiation between the three areas, as can be seen in Figure 5 and Figure 6.

For the parameters used in figure 5 the introduction of the electrical resistance to increase the attenuation and decrease the maximum leads to frequency shift towards higher values of associated maximum but the maximum attenuation of mechanical resonance frequency is much lower than in first cases. The appearance of electro-mechanical resonance is achieved for values of electrical resistance less than 1Ω.

For the parameters used in figure 6 the usefulness of the actuator is null.

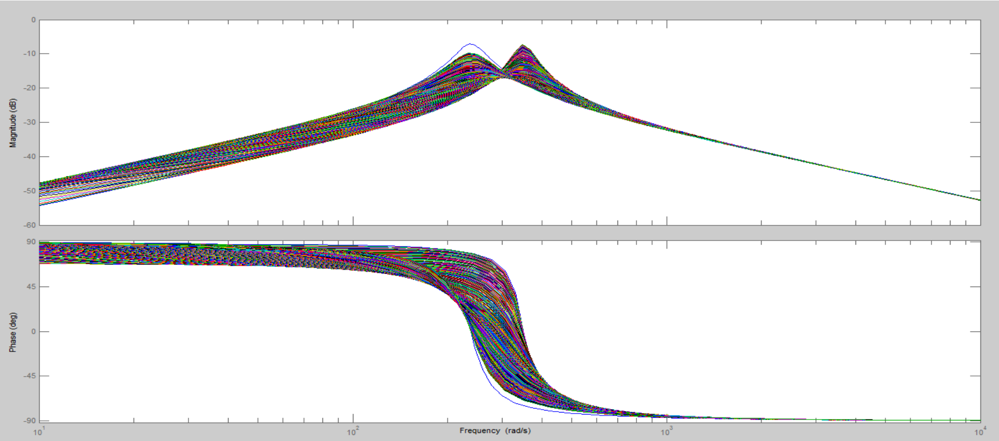


Figure 5 The mobility of the system when C=0 F; L=5600μH and R varies from 0.01Ω to 20Ω with step 0.01Ω

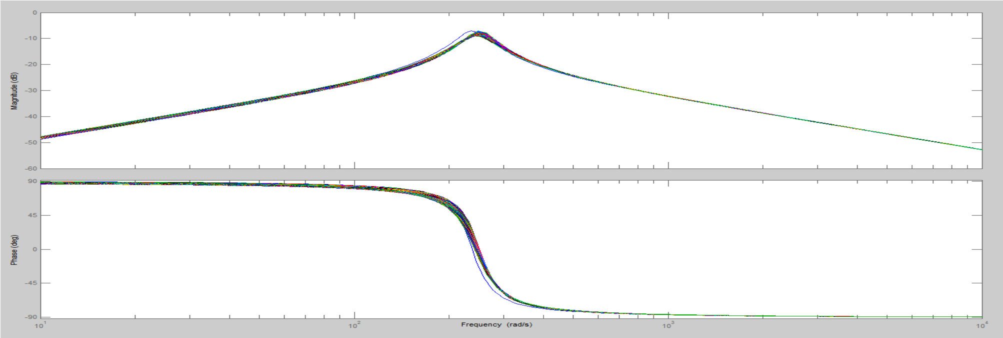


Figure 6 The mobility of the system when C=0 F; L=0.056 H and R varies from 0.01Ω to 20Ω with step 0.01Ω

An important change in the behavior of the system occurs if is changed the value of the electric capacity C, as shown in the picture from Table 1. For high values of electrical resistance is obtained a reduce of maximum value at mechanical resonance. If the resistance is reduced the use of electrical capacitance C determine a new resonance behavior at higher frequencies, due to electric resonance (L-C). This is convenient for a wide frequency range excitation. Increasing the capacitor value leads to a greater decrease in the frequency of mechanical resonance and an increase of the maximum at this frequency and a decrease in the frequency of electrical resonance simultaneously with an increase in the maximum value at electrical resonance. This is dangerous for a wide frequency range excitation but is convenient for an excitation with frequencies at the mechanical resonance and higher values. The maximum attenuation is obtained for a high capacity but only in a small frequency range.

It is possible to choose the electrical parameter for obtaining a band stop mechanical filter with a narrow band but a very high attenuation.

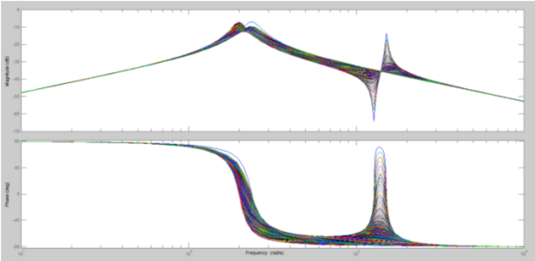
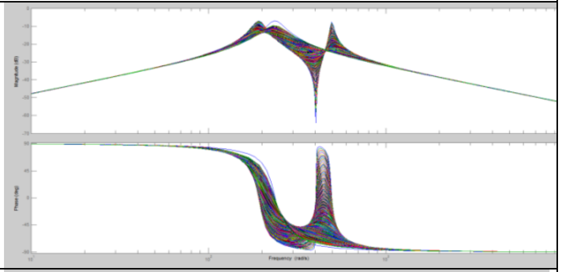
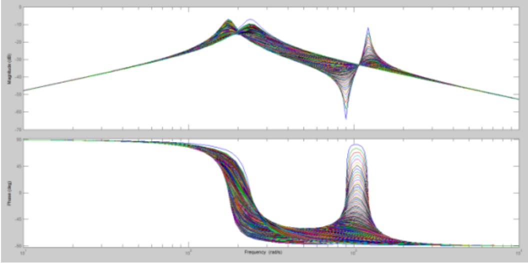
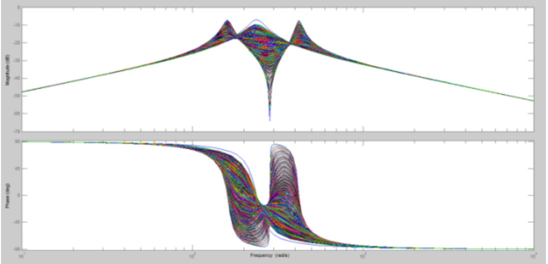
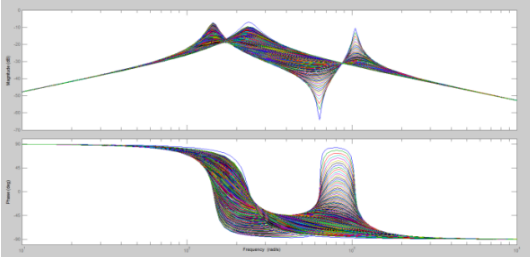
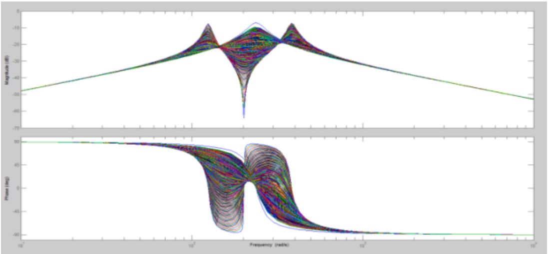
C	**R** = 0.01Ω ... 20Ω **L** = 0.00056H	**R** = 0.01Ω ... 20Ω **L** = 0.0056H
1.1 mF		
2.2 mF		
4.4 mF		

Table 1: The mobility of the system when C; R and L varies.

If electrical inductance is much higher, electric resonance occurs at frequencies lower than the mechanical resonance and mechanical resonance frequency increases as those common inductance dominates, leading to an increase in the elastic constant.

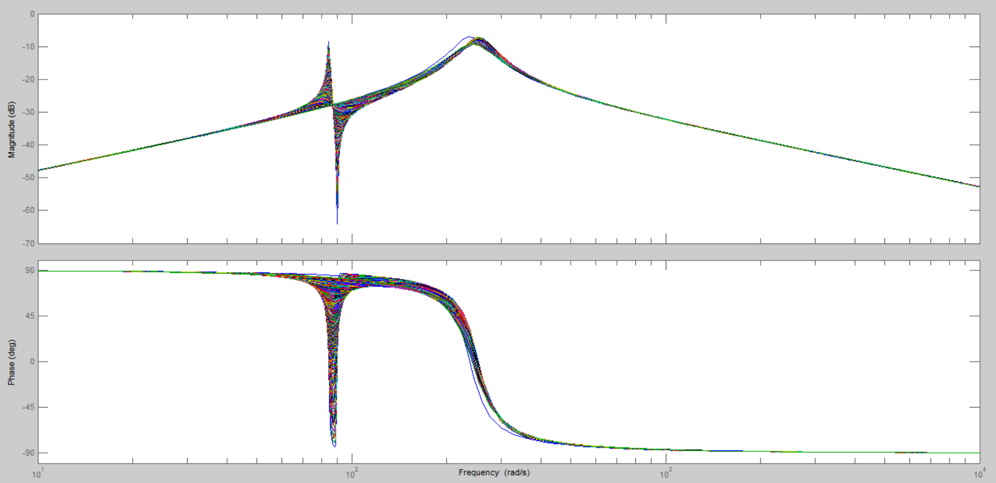


Figure 7 The mobility of the system when C=2.2 mF; L=0.056 H and R varies from 0.01Ω to 20Ω

Similar conclusions could be obtained for transmissibility of vibration.

The performance analysis of electromagnetic actuators, commercially available, has led to a number of conclusions:

- **for the control of circuit dynamics by R-L**: the pulsation of passing between the resistive and inductive zone is approximately 1400-2200 rad/s which can be extended to values of 3000 rad/s or exceptionally at 5500-6000 rad/s, equivalent to common 220-350 Hz frequencies, with the possible extension to 875-950 Hz. Achieving a high ratio is associated usually with a high electrical resistance (5-7Ω) which significantly reduces of usefulness of the actuator for increasing amortization in the resistive zone, or is associated with an electrical resistance of the order of 1-2 Ω but with a lower value of the electromagnetic constant, having a similar effect, because the additional depreciation is given by $\Delta c = k_m \cdot k_e / R$. In the low frequencies zone, the usual electrodynamic actuators can be used only on the resistive domain and can obtain an additional amortization of tens to hundreds, leading to a very flat response of mobility curve, with an reduced extension of the curve toward higher frequencies. The control of movement for the peak resonant, in the inductive zone, is difficult and is possible only at high frequency and eventually by adding an additional inductance;
- **for the control of circuit dynamics by R-L-C**: to obtain a dominant behavior with RC components impose the use of capacitive components of much higher capacity than 100-300μF, leading to pulsations in which dominates RC circuit, with values well below 1500-3000 rad/s and frequencies of 240-480 Hz. Consequently, the R-C control is possible only at low frequencies. Obtaining a dominant pure capacitive impedance can be achieved only with capacities of tens of mF, which lowers the frequency range in which dominates RC circuit under 25Hz, with capacitive dominating factor in zone under 5-8Hz, so it's applicable only at very low frequencies, for example, the vibrations of buildings.

4 CONCLUSIONS

The study prove the possibility of using the commercial electro-dynamic actuators for control of the impedance and mobility of a simple mechanical system, 1DOF. Also by changing the type of actuator and addition of electrical components at the electrical connections of the actuator is possible to tune the overall response. The study also indicated the limitations of using the commercial actuator regarding the frequency domain of excitation and the resonance parameters of the system supposed to control.

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