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MODAL SCALING OF A SYMMETRIC SCALE MODEL OF A TWO STORY BUILIDING

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Abstract. *It is well known that mode shapes cannot be mass normalized when operational modal analysis (OMA) is used to estimate the modal parameters. In the last few years, some equations have been proposed by different authors to scale mode shapes using the mass-change method. Some of the proposed equations are approximate because the change of the mode shapes of the modified structure is not properly taken into account whereas other equations can be considered exact because they are derived from the theory of structural modification. In this paper, the scaling factors of a symmetric scale model of a two story building, presenting repeated modes, have been determined by the mass change method. The results are compared with those predicted by a finite element model set up using the commercial FEA software ANSYS. The floors have been modelled using shell elements whereas 2 node beam elements have been chosen for the columns.*

1 INTRODUCTION

Operational modal analysis (OMA) is a technique that, in the last years, has been used for modal identification of civil structures [1, 2] and mechanical systems [3, 4]. Modal scaling is one of the most important drawbacks in OMA since the forces are unknown and the mode shapes cannot be directly mass normalized, i.e. only the un-scaled mode shapes can be identified for each mode [5, 6, 7, 8]. Consequently, an additional procedure to calculate the scaling factors is needed.

Several approaches for scaling have been proposed recently based on changing the mass of the structure [6, 7, 9, 10, 11, 1]. The method consists of attaching masses to the points of the structure where the mode shapes of the unmodified structure are known and then perform operational modal analysis on both the unperturbed and the perturbed structures. In order to facilitate the process, lumped masses are often used, in which case the mass-change matrix [Δm] becomes diagonal.

In this paper, the scaling factors of a symmetric scale model of a two story building, presenting repeated modes, have been determined by the mass change method. The modal parameters of both the unperturbed and the perturbed structure have been identified with Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic SSI method using ARTeMIS Extractor software. The scaling factors estimated by the mass change method are compared with those predicted by a finite element model set up using the commercial FEA software ANSYS. The floors have been modelled using shell elements whereas 2 node beam elements have been chosen for the columns.

2 MODAL SCALING IN OMA

In the case of no damping, the equation of motion of a structure provides the eigenvalue equation:

$$m \cdot \phi_0 \cdot \omega_0^2 = k \cdot \phi_0, \tag{1}$$

Where ϕ_0 and ω_0 are the mass normalized mode shape and the natural frequency, respectively, and the subscript ‘0’ indicates the unperturbed or unmodified structure.

If a dynamic modification given by the mass Δm is undertaken, the new equation of motion provides the following eigenvalue equation for the i-th mode:

$$\omega_{Ii}^2 \cdot (m + \Delta m) \cdot \phi_{Ii} = k \cdot \phi_{Ii}\,, \tag{2}$$

where ω_{Ii} and ϕ_{Ii} are the natural frequency and the eigenvector, respectively, of the modified i-th mode.

The mode shapes of the modified structure ϕ_I are related to those of the original structure by:

$$\phi_I = \phi_0 \cdot A\,, \tag{3}$$

From Eq. (3), it follows that the modified mode shapes ϕ_I are expressed as a linear combination of the unmodified mode shapes ϕ_0.

The unscaled ψ_{0i} and the scaled or mass-normalized ϕ_{0i} mode-shape vectors, corresponding to i-th mode, are related by the expression:

$$\phi_{0i} = \alpha_{0i} \cdot \psi_{0i} \,, \tag{4}$$

where the coefficient α_{0i} is the scaling factor of i-th unmodified mode. The corresponding equation for the modified structure is given by:

$$\phi_{Ii} = \alpha_{Ii} \cdot \psi_{Ii} \,, \tag{5}$$

if Eqs. (4) and (5) are replaced in Eq. 3, the result is:

$$\psi_I = \psi_0 \cdot B, \tag{6}$$

which relates the unscaled mode shapes of both systems.From Eqs. (3) and (6), it can be inferred that each term B_{ji} of matrix B is related to the corresponding term A_{ji} of matrix A by:

$$B_{ji} = \frac{\alpha_{0j} \cdot A_{ji}}{\alpha_{Ii}} \tag{7}$$

If Eq. (2) is pre-multiplied with the un-perturbed mass normalized mode shape vector ϕ_{0j}^{T}, it results in:

$$\phi_{0j}^T \cdot (m \cdot \phi_0 + \Delta m \cdot \phi_0) \cdot A_i \cdot \omega_{Ii}^2 = \phi_0^T \cdot (k \cdot \phi_0) \cdot A_i \tag{8}$$

Taking into account the orthogonality properties given by Eq. (1) and the projection of the perturbed mode shapes on the unperturbed ones given by Eq. (3), Eq. (8) can be expressed as:

$$\left(\omega_{0j}^2 - \omega_{Ii}^2\right) \cdot A_{ji} = \phi_{0j}^T \cdot (\omega_{Ii}^2 \cdot \Delta m) \cdot \phi_{Ii} \,, \tag{9}$$

where ω_{0j} is the j-th natural frequency of the unperturbed system. If Eq. (7) is substituted in Eq. (9), a closed form expression for the j-th scaling factor is derived:

$$\alpha_{0j}^2 = \frac{\left(\omega_{0j}^2 - \omega_{Ii}^2\right) \cdot B_{ji}}{\omega_{Ii}^2 \cdot \psi_{0j}^T \cdot \Delta m \cdot \psi_{Ii}} \tag{10}$$

Eq. (10) is a set of N_m (number of modes) equations which have to be fulfilled for any value of i, i.e. there are as many expressions for the scaling factor of mode j as number of modes considered in the analysis. In recent years, different authors have proposed equations to estimate the scaling factors by the mass change which are shown in Table 1. The different expressions can be derived from Eq. (10) together with the assumptions made in the existing equations.

Author	Equation	Assumptions
Bernal [11]	$\alpha_{0i}^2 = \frac{(\omega_{0i}^2 - \omega_{Ii}^2)\cdot B_{ii}}{\omega_{Ii}^2 \cdot \psi_{0i}^T \cdot \Delta m \cdot \psi_{Ii}}$	$j = i$ B: identity matrix
Bernal [12]	$\begin{pmatrix}\psi_{I1}\\ \psi_{I2}\\ \cdots\\ \psi_{Ii}\\ \cdots\end{pmatrix} = \begin{pmatrix}\sum_{j=1}^{Nm}\psi_{0j}\cdot\beta_{j1}\cdot\alpha_{0j}^2\\ \sum_{j=1}^{Nm}\psi_{0j}\cdot\beta_{j2}\cdot\alpha_{0j}^2\\ \cdots\\ \sum_{j=1}^{Nm}\psi_{0j}\cdot\beta_{ji}\cdot\alpha_{0j}^2\\ \cdots\end{pmatrix}$ $where$ $\beta_{ji} = \frac{\psi_{0j}^T\cdot(\omega_{Ii}^2\cdot\Delta m - \Delta k)\cdot\psi_{Ii}}{(\omega_{0j}^2 - \omega_{Ii}^2)}$	This is a system of equations that considers simultaneously the set of equations given in Eq (10).
Brincker and Andersen [7]	$\alpha_{0i}^2 = \frac{(\omega_{0i}^2 - \omega_{Ii}^2)}{\omega_{Ii}^2 \cdot \psi_{0i}^T \cdot \Delta m \cdot \psi_{0i}}$	$j = i$ $\psi_{Ii} \cong \psi_{0i}$
Aenlle et al. [8]	$\alpha_{0i}^2 = \frac{(\omega_{0i}^2 - \omega_{Ii}^2)}{\omega_{Ii}^2 \cdot \psi_{0i}^T \cdot \Delta m \cdot \psi_{Ii}}$	$j = i$
Parloo [5]	$\alpha_{0i}^2 = \frac{2(\omega_{0i} - \omega_{Ii})}{\omega_{0i} \cdot \psi_{0i}^T \cdot \Delta m \cdot \psi_{0i}}$	$j = i$ $\frac{(\omega_0^2 - \omega_I^2)}{\omega_I^2} \approx \frac{2\,(\omega_0 - \omega_I)}{\omega}$

Table 1. Equations proposed to estimate the scaling factors.

3 NUMERICAL MODEL

In order to validate the experimental results, a 3d model has been set up using the commercial FEA software ANSYS. The floors have been modelled using 4 node shell elements of type SHELL181 whereas 2 node beam elements of type BEAM44 have been chosen for the columns. In order to account for eccentric connections between floors and columns, rigid links (CERIG) have been defined between the column nodes at floor level and the corresponding nodes of the floor. Linear spring elements have been used to model the elastic support condition that results from the connection between the RHS profiles at the base. The finite element model has 588 shell elements, 172 beam elements and 850 nodes. In Figure 2 some views of the FE model are displayed, the rigid links are marked in pink. All parts of the structure are made of the same type of steel, being the corresponding material properties: Young's modulus E = 210 GPa, Poisson's ratio $\nu = 0.3$ and density $\rho = 7850$ kg/m^3.

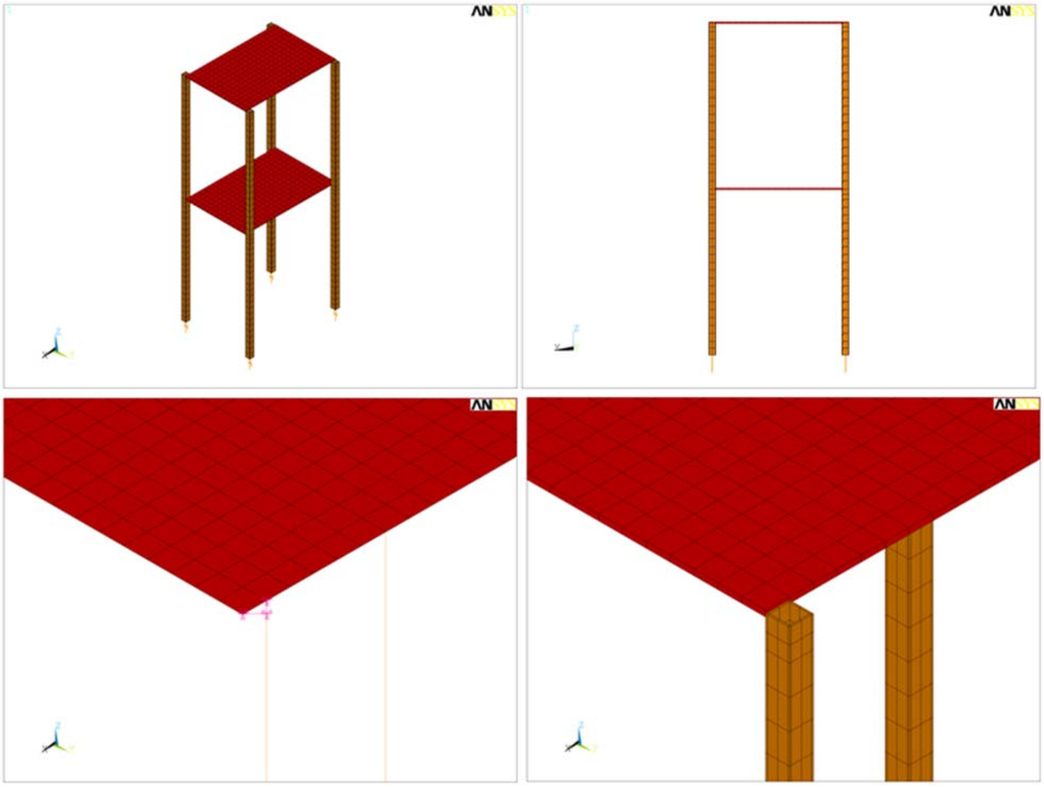

Figure 1. Views of the FE model.

4 EXPERIMENTAL TESTS

The structure tested was a scale model of symmetric two story building as shown in Figure 1 and operational modal analysis was applied to identify the modes. The scaling factor of the two first bending and torsional modes were estimated by the mass change method.

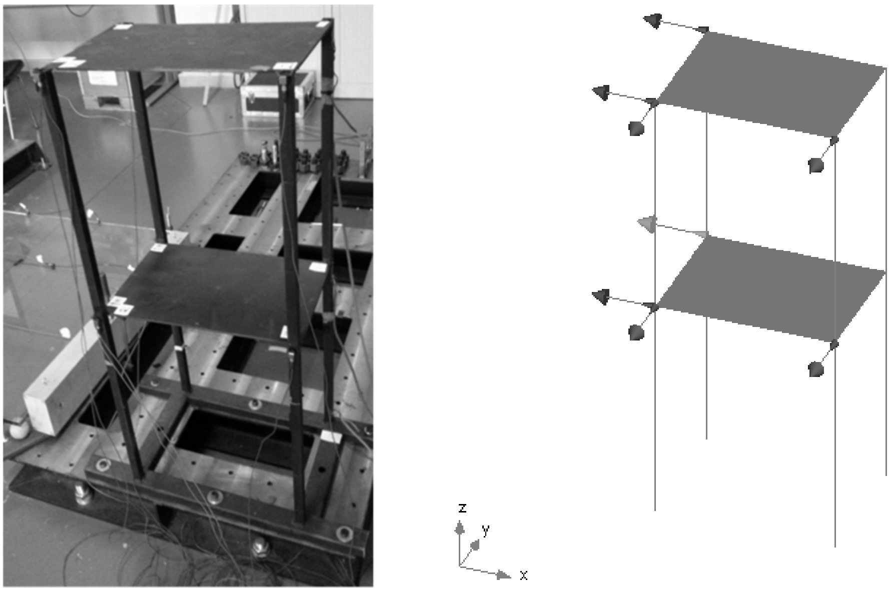

Figure 2. Location of accelerometers.

The structure was excited applying hits on the columns and the floors trying to reproduce a stationary broad band loading. The responses were measured using 8 accelerometers with a sensitivity of 100 mv/g, located as shown in Figure 2, and recorded with a data acquisition card (National Instruments PCI4472) controlled by LabVIEW. The tests were carried out at a sampling frequency of 2000 Hz. The natural responses were measured during a period of approximately 4 minutes.

The modal analysis was performed using operational modal analysis software (ARTeMIS Modal). The first six natural frequencies identified with the Stochastic Subspace Identification (SSI), together with those obtained from the numerical model, are shown in Table 2.

Mode	Natural Frequencies [Hz]		Mode shape
	Numerical	SSI method	
1	11.54	11.35	1º Bending X
2	12.05	11.61	1º Bending Y
3	26.36	26.04	1º Torsion
4	51.68	50.78	2º Bending X
5	52.05	52.22	2º Bending Y
6	90.01	86.75	2º Torsion

Table 2. Natural frequencies (Hz) identified by the SSI technique.

The dynamic behaviour of the structure was modified attaching steel lumped masses at the four corners of both floors. Two mass change configurations were used to modify the dynamic behavior of the structure. The location and magnitude, in grams, of the attached masses are shown in Table 3.

	Degree of freedom	Mass-change [g] -CONFIGURATION 1-	Mass-change [g] -CONFIGURATION 2-
First Floor	5	65	149
	6	64	146
	7	66	149
	8	66	148
Second Floor	9	67	151
	10	65	150
	11	65	146
	12	65	146
Total mass change		3.95%	8.94%

Table 3. Magnitude of the attached masses

A new operational modal testing and analysis was performed on the modified structures. The tests configuration was the same as that used in the unperturbed structure. The natural frequencies estimated with the SSI technique are presented in table 4, together with the frequency shifts.

Mode	Natural Frequency (SSI) -CONFIGURATION 1-		- CONFIGURATION 2-	
	f_1 [Hz]	% Δf_{0-1}	f_2 [Hz]	% Δf_{0-2}
1	11.08	2.38%	10.77	5.11%
2	11.32	2.50%	11.02	5.08%
3	24.81	4.72%	23.73	8.87%
4	49.6	2.32%	48.24	5.00%
5	50.95	2.43%	49.62	4.98%
6	82.6	4.78%	79.03	8.90%

Table 4. Natural frequencies (Hz) identified by SSI technique on the modified structure.

In Table 5 is presented the MAC between the original and the modified mode shapes. As the mass change is proportional to the mass of the structure, no significance discrepancies exist between the unperturbed and the perturbed configurations. The numerical mode shapes compared with the experimental ones also present a good correlation (see Table 5).

MODE	MAC		
	Original – Mass change 1	Original – Mass change 2	Original – Numerical
1	0.9994	0.9982	0.9926
2	0.9984	0.9995	0.9909
3	0.9999	0.9994	0.9918
4	0.9998	0.9989	0.9941
5	0.9998	0.9992	0.9925
6	0.9993	0.9956	0.9924

Table 5. MAC between the unperturbed and the perturbed configurations and with the numerical model.

The scaling factors were estimated using the equations proposed by Brincker and Andersen [7], Aenlle et al. [8] and by Bernal [11]. The results are shown in table 6 and correspond to mode shapes normalized to maximum component equal to unity.

Mode	$\left[\frac{(\omega_{0i}^2-\omega_{Ii}^2)}{\omega_{1i}^2\cdot\boldsymbol{\psi}_{0i}^T\cdot\boldsymbol{\Delta m}\cdot\boldsymbol{\psi}_{0i}}\right]^{\frac{1}{2}}$ Mass Configuration		$\left[\frac{(\omega_{0i}^2-\omega_{Ii}^2)\cdot B_{ii}}{\omega_{1i}^2\cdot\boldsymbol{\psi}_{0i}^T\cdot\boldsymbol{\Delta m}\cdot\boldsymbol{\psi}_{1i}}\right]^{\frac{1}{2}}$ Mass Configuration		$\left[\frac{(\omega_{0i}^2-\omega_{Ii}^2)}{\omega_{1i}^2\cdot\boldsymbol{\psi}_{0i}^T\cdot\boldsymbol{\Delta m}\cdot\boldsymbol{\psi}_{1i}}\right]^{\frac{1}{2}}$ Mass Configuration		Numerical
	1	2	1	2	1	2	
1	0.3949	0.3928	0.3951	0.3950	0.3945	0.3944	0.3889
2	0.4213	0.4134	0.4217	0.4147	0.4221	0.4149	0.3837
3	0.4655	0.4399	0.4664	0.4357	0.4663	0.4361	0.4466
4	0.3805	0.3779	0.3814	0.3830	0.3815	0.3831	0.3627
5	0.3958	0.3843	0.3964	0.3875	0.3965	0.3878	0.3587
6	0.4193	0.3929	0.4232	0.4024	0.4241	0.4042	0.3893

Table 6. Scaling factors estimated with the mass change method and from the finite element model.

5 DISCUSSION OF THE RESULTS

It is inferred from Table 6 that a reasonable good agreement exists between the scaling factors obtained from the numerical model and those estimated by the mass change method, being the larger errors ± 8% for modes 2 and 5. These two modes are bending modes in the “y” direction and the natural frequencies are slightly higher than those corresponding to the "x" direction, which explains why the scaling factors are not the same in both directions even though the structure is symmetric.

With respect to the mass change configuration, similar results have been obtained with a total mass change of 3.95% and with 8.94%. The largest mass magnitude configuration provides results slightly closer to the numerical ones; however this was expected because the accuracy of the mass change method improves as the mass magnitude increases [9].

6 CONCLUSSIONS

Symmetric real structures show repeated or closely spaced modes and this fact makes more difficult the modal analysis of this type of structures. In this paper, operational modal analysis has been applied successfully to identify the modes of a scale model of a symmetric two story building. A finite element model was also assembled in ANSYS. The maximum discrepancies in natural frequencies correspond to mode 2 (8%), the errors being less than 4% for the rest of modes. Moreover, the scaling factors of this structure have been estimated by the mass change method and from the finite element model, the discrepancies being less than 10%.

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