11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

A PRIORI DEFINITION OF THRESHOLD DETECTION OF DAMAGE OF MULTILAYER COMPOSITE STRUCTURES USING WAVELET PACKAGE DECOMPOSITION

Akli Boumrar, Amar Bouazzouni*

[1]Laboratoire de Mécanique, Structures et Energétique
Université Mouloud Mammeri de Tizi-Ouzou, BP17 RP,
Tizi-Ouzou 15000, Algérie
tak_gm@hotmail.fr, abouazzouni@yahoo.com

Keywords: Detection; Damage; Indicator; Wavelet Packet Decomposition; Energy.

Abstract. *In this work, we are interested in defining the threshold sensibility of a damage vibration indicator based on wavelet package decomposition of structural vibration responses before and after the occurrence of structural damage. Each of these structural vibration responses is decomposed to the jth order wavelet package sub-signals. The structure is subdivided into a certain number of finite elements. For a structure with one particular finite element perturbed to a certain rate, the damage indicator is then defined as the maximum of all energy variations of the wavelet package sub-signals of the structural vibration response before and after the occurrence of some structural damage. The indicator is called "maximum energy variation" (MEV). For the same mono-excitation, this indicator is then evaluated for all the elements of the discretization with the same perturbation rate. The values of this indicator are then represented on a graph in terms of the number of the finite element. Once we have determined the element whose damage indicator value is minimum, further trials are carried out in order to draw the curve representing the indicator in terms of the different rates of damage. From the graph, we determine the least detectable damage rate using the wavelet package decomposition of structural response. This threshold damage indicator may be used for detecting whether the structure is damaged at any other position along the beam. For the structure to be diagnosed, we evaluate its MEV and compare it with the threshold established beforehand to conclude whether it is or not effectively damaged. The sensitivity of this indicator is better if the frequencies of the mono-excitation components are taken close to the eigenfrequencies contained in the frequency band.*

1 Introduction

Many research works have dealt with the damage detection in composite structures but few of these have been concerned with the minimum of the magnitude of the damage that can be detected. Yam and al [1] use the wavelet packet analysis for the detection of damage in composite laminate plates. In their work, they define a damage indicator named “Maximum Energy Variation” (MEV). To establish this, they consider a structure with one particular finite element perturbed to a certain rate. Then, they calculate successively the energy of each sub-signal in the wavelet package decomposition of the dynamical response of the damaged structure as well as that of the corresponding sub-signal corresponding of the healthy structure. The difference of these respective energies is then calculated and divided by the total energy of the sub-signals of the considered level of wavelet decomposition. The maximum of these normalized variations constitutes the so-called MEV damage indicator.

For the same element, this above procedure is repeated for different damage rates and the value of this indicator is calculated. They then draw the graph of the MEV in terms of the damage rate. From this graph they determine the threshold of damage detectability. For a threshold of 20% they found that the deviation between the simulation and the experiment is less than 5%. As to the extent of the damage, their method may be much more sensitive than any other existing one. However, it should be noted that the effectiveness of this method depends on several factors such as the choice of the sensor positions, the choice of the exciting force, etc.

In our work, we are interested in using this damage indicator in the case of a layered beam structure. For the purpose of analysis, a finite element model of this structure is built. For the same excitation and damage rate, a graph representing the indicator in terms of the perturbed element number is drawn to determine the element for which this has the lowest value. Different graphs of the MEV indicator in terms of damage rate may be drawn for different damaged element positions along the structure, and a damage detection threshold is established. Our contribution consists in finding a global threshold damage indicator rather than a particular one for a defined position of the damaged element in the structure as did Yam and al.

2 Description the method

2.1 Finite element SI20

The finite element model we use in this study is based on the theory of the zigzag movement of the first order. The finite element [4] is composed of three layers in symmetrical stackings sequences. Thus, the displacement at an arbitrary point of the beam can be expressed by a longitudinal displacement u_1 (x) along the beam axis and a transverse u_3 (x) along the z axis and a rotation γ_x (x) characterizing the rotation about the y axis (see FIG. 1).

Displacements are given by:

$$\left.\begin{aligned} u_1(x) &= u(x) + \gamma_x(x) \\ u_3(x) &= w(x) \end{aligned}\right\} \tag{1}$$

where u is the longitudinal displacement at z = 0 and w is the deflection of the beam axis.

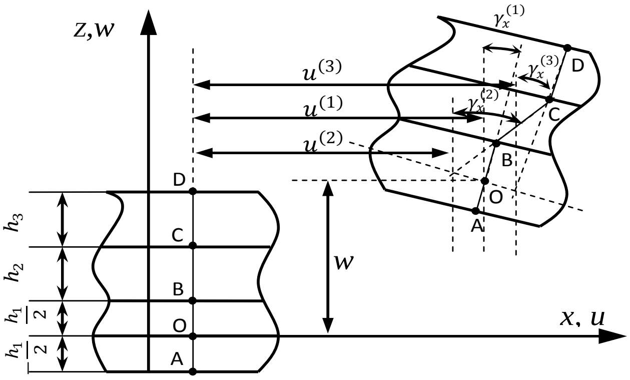


FIG. 1− Initial and deformed SI20 element

The rigidity matrix is obtained from the strain energy of the element:

$$U_e = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \varepsilon^{(k)T} D^{(k)} \varepsilon^{(k)} dx = \sum_{k=1}^{3} \frac{1}{2} \int_0^l v_e^{(k)T} B^{(k)T} D^{(k)} B^{(k)} v_e^{(k)} dx \tag{2}$$

where k : layer number. k = 1, 2 et 3. l : length of the element. D : elasticity matrix of the element.

B : deformation matrix. $v_e = [u^{(k)}, w^{(k)}, \gamma_x^{(k)}]^T$: nodal displacement vector.

Similarly, we may write the kinetic energy of the element in order to derive the mass matrix:

$$T_e = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \dot{u}^{(k)T} R_0^{(k)} \dot{u}^{(k)} dx = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \dot{v}_e^{(k)T} N^T R_0^{(k)} N \dot{v}_e^{(k)} dx \tag{3}$$

where $\dot{v}_e$: nodal velocity vector; N : shape function matrix. R_0 : matrix of generalized densities.

$$R_0 = \begin{bmatrix} \rho_0 & 0 & \rho_1 \\ 0 & \rho_0 & 0 \\ \rho_1 & 0 & \rho_2 \end{bmatrix}$$

Generalized densities ρ_0, ρ_1 and ρ_2 are given by:

$$\rho_0 = b\sum_{k=1}^{K} \rho_k [z_k - z_{k-1}]; \quad \rho_1 = \frac{1}{2} b\sum_{k=1}^{K} \rho_k [z_k^2 - z_{k-1}^2]; \quad \rho_2 = \frac{1}{3} b\sum_{k=1}^{K} \rho_k [z_k^3 - z_{k-1}^3]$$

Here ρ_k is density of the k^{th} layer.

Consider a conservative beam structure subjected to a sinewave excitation. The equation of forced motion writes as:

$$[M]\{\ddot{y}\} + [K]\{y\} = \{A_0\} \sin(\Omega t) \tag{4}$$

where $[K]$: global rigidity matrix. $[M]$: global mass matrix. $\{A_0\}$: maximum excitation amplitude vector.

The solution of equation (4) is of the following form:

$$y(t) = \{A\} \sin(\Omega t) \tag{5}$$

Substituting equation (5) into equation (4) leads:

$$y(t) = [K - \Omega^2 M]^{-1}\{A_0\}\, sin(\Omega t) \tag{6}$$

The frequency response of the beam is:

$$Y(\omega) = [K - \omega^2 M]^{-1}\{A_0\} \tag{7}$$

2.2 Wavelet analysis

Analysis of a function $y(t)$ by continuous wavelets consists in calculating the decomposition coefficients $C_w(a,b)$ defined by equation (8):

$$C_w(a,b) = \int_{-\infty}^{+\infty} y(t)\psi_{a,b}^*(t)dt \tag{8}$$

where : $\psi_{a,b}^*$ represents the complex conjugate of $\psi_{(a,b)}$.

The wavelets family $\psi_{a,b}(t)$ generated by the mother wavelet $\psi(t)$ by varying the scale and translation parameters a and b in a continuous domain.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \tag{9}$$

The reconstruction of the signal $y(t)$ is calculated as follows:

$$y(t) = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_w(a,b)\psi\left(\frac{t-b}{a}\right)\frac{dadb}{a^2} \tag{10}$$

with $C_\psi = \int_{-\infty}^{+\infty} \frac{\left|\hat{\psi}_{a,b}(\omega)\right|^2}{\omega} d\omega$ where $\hat{\psi}_{a,b}(\omega)$ is the Fourier transform of $\psi_{a,b}(t)$.

The wavelet packets is a more developed version of wavelet decomposition that allows us to carry out a richer analysis. The wavelet function in wavelet packets is given by :

$$\psi_{k,l}^j(t) = 2^{k/2}\psi^k(2^k t - l) \quad j = 1, 2, 3,\ldots \tag{11}$$

where the integers j, k and l s are respectively the modulation, scale and translation parameters.

The coefficients $c_{k,l}^j(t)$ of wavelet packets decomposition applied to the signal $y(t)$, are calculated by the following equation:

$$c_{k,l}^j(t) = \int_{-\infty}^{+\infty} y(t)\psi_{k,l}^j(t)dt \tag{12}$$

Thus, the components $y_k^j(t)$ of the signal can be represented by a linear combination of the functions of wavelets packages $\psi_{k,l}^j(t)$ as follows:

$$y_k^j(t) = \sum_{l=-\infty}^{+\infty} c_{k,l}^j(t)\,\psi_{k,l}^j(t) \tag{13}$$

After the kth level of decomposition, the initial signal can be rebuilt using the following equation:

$$y(t) = \sum_{j=1}^{2^k} y_k^j(t) \tag{14}$$

2.3 Energy change Indicator

Let $y(t)$ the signal of the dynamic response of a structure. This signal is decomposed by wavelets packets in a sum of subsignals $y_j^i(t)$, at the ith level, as follows:

$$y(t) = \sum_{i=1}^{2^j} y_j^i\,(t) \tag{15}$$

The energy U stored in a sub-signal is given by:

$$U_j^i = \int_{-\infty}^{+\infty} y_j^i(t)^2 \, dt \tag{16}$$

Thus, the total energy U of the signal is defined as being the sum of the energies of these sub-signals

$$U = \sum_{i=1}^{2^j} U_j^i \tag{17}$$

We consider the responses of the two structures healthy indexed s and damaged indexed e. For each structure, we define respectively the vectors Vs and Ve whose components are each the ratio of the sub-signal energy to the total energy of the signal at the level selected of wavelet package decomposition.

$$Vs = \left\{Cs_j^1, Cs_j^2, Cs_j^3, \dots, Cs_j^{j^2}\right\} = \left\{\frac{Us_j^1}{Us}, \frac{Us_j^2}{Us}, \frac{Us_j^3}{Us}, \dots, \frac{Us_j^{2^j}}{Us}\right\} \tag{18}$$

$$Ve = \left\{Ce_j^1, Ce_j^2, Ce_j^3, \dots, Ce_j^{j^2}\right\} = \left\{\frac{Ue_j^1}{Ue}, \frac{Ue_j^2}{Ue}, \frac{Ue_j^3}{Ue}, \dots, \frac{Ue_j^{2^j}}{Ue}\right\} \tag{19}$$

From these last two vectors we define the energy variation vector as follows:

$$EV = \left\{\left(\frac{Cs_j^1 - Ce_j^1}{Cs_j^1}\right), \left(\frac{Cs_j^2 - Ce_j^2}{Cs_j^2}\right), \left(\frac{Cs_j^3 - Ce_j^3}{Cs_j^3}\right), \dots, \left(\frac{Cs_j^{2^j} - Ce_j^{2^j}}{Cs_j^{2^j}}\right)\right\} \times 100\% \tag{20}$$

The MEV is defined as the maximum of the absolute energy variations (EV).

3 Numerical simulation test cases

The composite beam structure represented in FIG.2 is composed of 3 layers glass/époxy, disposed in the following configuration $[0°/90°/0°]$. The beam is subdivided into 60 SI20 finite elements [4].

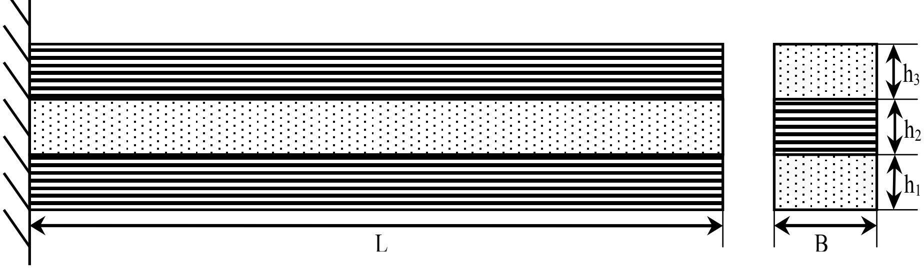


FIG. 2 – Stratified composite cantilever beam

The mechanical properties of the material are : E_1 = 47.518 GPa; E_2 = 4.588 GPa; G_{12} = 2.201 GPa; μ_{12} = 0.0419; μ_{21} = 0.434; ρ = 1850 kg/m^3. Its geometry is characterized by : L = 360 mm, $h_1 = h_2 = h_3$ = 4mm, B = 30 mm.

We consider two types of boundary conditions: a cantilever and a simply-simply supported beam. The damaged beam is simulated by reducing the longitudinal Young's modulus E_2 of the middle layer of the finite element.

For the first type of boundary conditions, the excitation is applied at its first node from the free end and for the second case of boundary conditions it is applied at the 25th node. The excitation force consists of three components and it is applied normally to the beam at the node previously indicated:

$$F_e(t) = A_0(\sin(\omega_1 t) + sin(\omega_2 t) + sin(\omega_3 t))$$

The measurement frequency band must contain as many eigenmodes as possible, and the exciting force must be chosen to excite the maximum number of these. The temporal responses signals of healthy and damaged structures are decomposed by «db4» wavelets packets to the 5th level.

3.1 Cantilever beam case

To begin, we damage by 40% successively each element of the structure and we draw the MEV curve in terms the element number of the structure (FIG. 3).

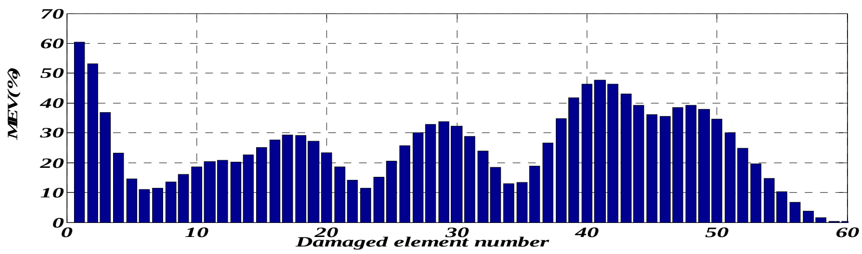


FIG. 3 – Cantilever beam : Histogram of MEV- number of the beam element damaged by 40%.

The sensitivity of this indicator is variable according to the location of the defect. We choose on the FIG.3 element 6 which has the lowest value of MEV. We damage it at various rates and we seek in the waveband a combination of components of the exiting force which gives us the curve representative of MEV according to the rate of damage. This shape of the curve enables us to fix a priori a threshold of detectability of damages at the selected position (FIG. 4). The threshold is fixed just under the point of inflection of the curve at 60% and the smallest rate of detectable damage of element 6 is thus approximately 22%.

We obtain the same shape of the MEV-damage rate curve each time we change the position of the damaged element, the structure being always subjected to the same excitation as in the case of the damage of element 6, while varying the extent of the damage. This makes it possible to fix a threshold for each selected position of the damaged element. We represent above for some elements the graphs of MEV according to the damage rate of the element.

We maintain the same threshold of 60% for all the damaged elements and we take note of the smallest rate of detectable damage (FIG.5.a). For element 12 for example, the smallest detectable rate is 20%; for element 18 it is 14%; for element 38 it is 12%.

3.2 Simply-simply supported beam case

Let us consider the case of the beam simply-simply supported. In the same manner as we proceeded in the preceding case, we start by damaging by 40% successively each element of the structure. The histogram representing the MEV according to the number of the damaged element is given by the FIG. 4.

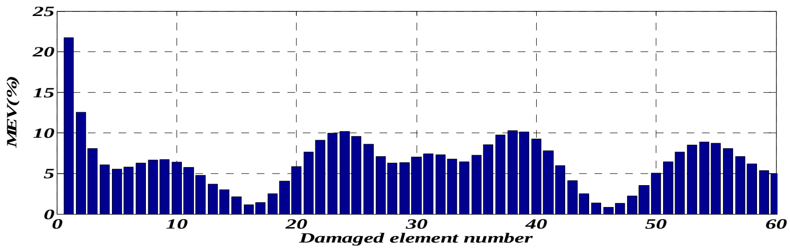
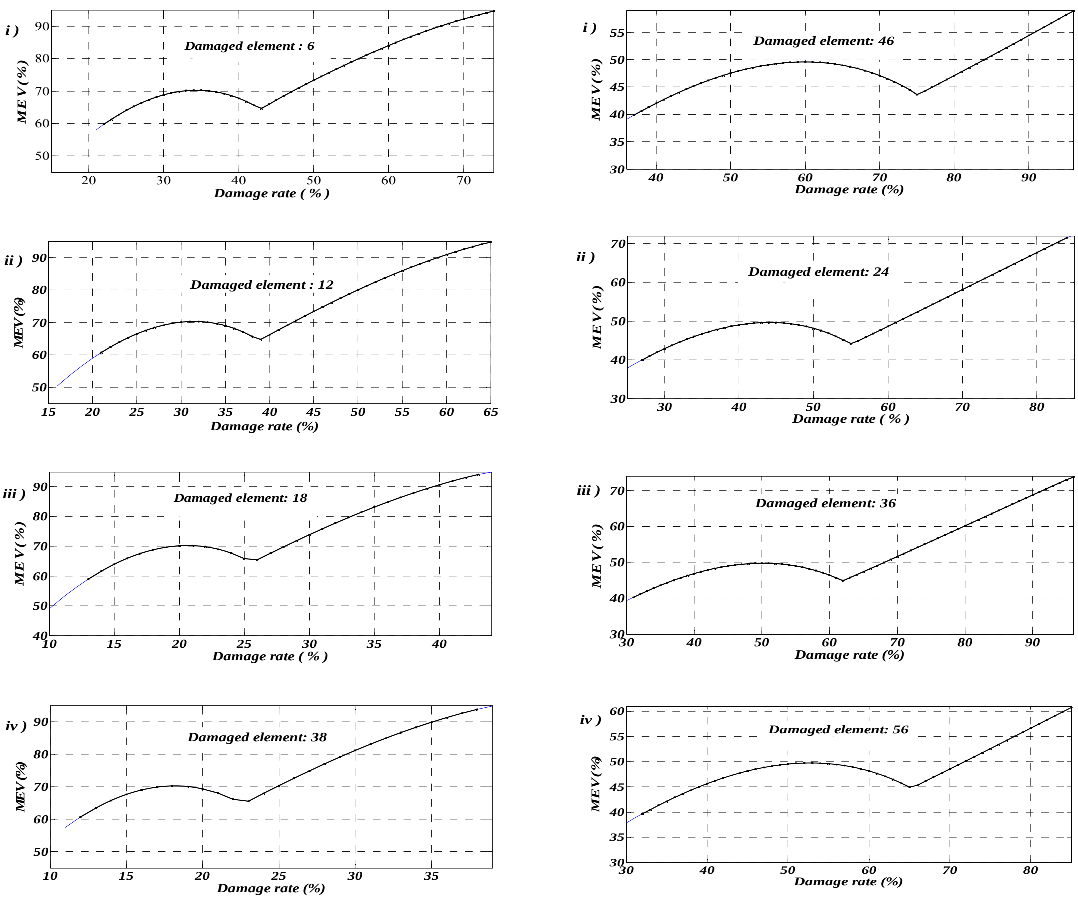


FIG. 4 – Simply-simply supported beam : MEV histogram in terms of 40% damaged beam element number

On FIG. 4 we choose element 46 and seek the good combination of the components of the exciting force giving the representative MEV curve. The threshold is fixed here at 40% and the smallest damage is located at approximately of 40%.

For all the structures the threshold of MEV is fixed at 40%. Of each curve we take note of the minimum detectable rate of damage. Some curves are presented in FIGS.5.b. The threshold of detectability is 35% for element 6; 38% for element 10; 44% for element 12 and 46 for the 56ème damaged element.



a) : Cantilever beam *b*) : Simply-simply supported beam

FIGS. 5 –Variation of MEV according to the damage rate.

FIGS.5 give us detailed information about the variation of MEV according to the size of the defect for each case of structure with different damaged element, and help us locate the threshold of detectability for each one of them. We notice a correspondence between the data of the histograms and those of FIGS.5, i.e. the elements having low values of MEV on the histograms have a detectability threshold higher and vice versa. For example, on histogram 3, the value of MEV for element 12 is 20% and for element 28 it is 35%. On FIGS.5, the damage is detectable from the value of MEV of 12% if the damaged element is the 38ème while it is detectable only from a MEV value of 20% for the élément12. The application of the method on the two types of boundary conditions gives us similar results.

4 Conclusion

In this work, we present an approach of numerical simulation by finite elements to detect damage in a laminated beam structure. This method uses the energy of the sub-signals of a certain level in the wavelet packages decomposition of the dynamic structural response. The method used enables us to estimate a threshold of damage detectability in some position on the structure before carrying out any experimentation. The proposed method is able to detect local damage at its earliest developmental stage. Our contribution consists in having contributed to determine a global a priori threshold damage indicator rather than one for a particular position of the damaged element in the structure.

REFERENCES

[1] Y. J. Yan, L. H. Yam. Detection of delamination damage in composite plates using energy spectrum of structural dynamic responses decomposed by wavelet analysis. [J] computers and structures 82 (2004) 347-358.

[2] Z. Wei, L. H. Yam. L. Cheng. Detection of internal delamination in multi-layer composites using wavelet packets combined with modal parameter analysis. [J] composite structures 64 (2004) 377-387.

[3] Y. J. Yan, L. H. Yam. Online detection of crack damage in composite plates using embedded piezoelectric actuators/sensors and wavelet analysis. [J] composite structures 58 (2002) 29-38.

[4] « Analysis of laminated structures course of lecture », Rolands Rikards Riga T. University 1999.