

## DESIGN OF SATELLITE ATTITUDE CONTROL SYSTEM CONSIDERING THE INTERACTION BETWEEN FUEL SLOSH AND FLEXIBLE DYNAMICS

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**Abstract.** *The design of the satellite Attitude Control System (ACS) becomes more complex when the satellite structure has different type of components like, flexible solar panels, antennas, mechanical manipulators and tanks with fuel, since the ACS performance and robustness will depend if the dynamics interaction effects between these components are considered in the satellite controller design. A crucial interaction can occur between the fuel slosh motion and the satellite rigid motion during translational and/or rotational maneuver since these interactions can change the satellite center of mass position damaging the ACS pointing accuracy. Although, a well-designed controller can suppress such disturbances quickly, the controller error pointing may be limited by the minimum time necessary to suppress such disturbances affecting thus the satellite attitude acquisition. It is known that one way to minimize such problems is to design controllers with a bandwidth below the lowest slosh and/or vibration mode which can result in slow maneuvers inconsistent with the space mission requirements. As a result, the design of the satellite controller needs to explore the limits between the conflicting requirements of performance and robustness. This paper investigates the effects of the interaction between the liquid motion (slosh) and the flexible satellite dynamics in order to predict what the damage to the controller performance and robustness is. The fuel slosh dynamics is modeled using its pendulum analogs mechanical system which parameters are identified using the Kalman filter technique. This information is used to designs and to compare the satellite attitude control system by the Linear Quadratic Gaussian (LQG) and H-infinity methods.*

## 1 INTRODUCTION

The problem of interaction between fluid and structure is important when one needs to study the dynamic behavior of offshore and marine structures, road and railroad containers partially filled with a fluid [1]. In space applications the problem appear with spinning spacecraft with liquid fuel, damp devices involving fluid as the damping material, fluid interaction with flexible manipulator [2]. An interesting approach to analyze a rigid container mounted on flexible springs interacting with a perfect fluid including sloshing effects can be found in [3]. Space mission's attitude control system (ACS) design the knowledge of the interaction between fluid motion (slosh) and structure dynamics is important because this interaction can damage the ACS pointing requirements. A space structure, like rockets, geosynchronous satellites and the space station usually contains liquid in tanks that can represent more than 40% of the initial mass of the system. As a result, the first step to design its ACS is to obtain a detailed dynamics model of the space structure. When the fuel tanks are only partially filled and suffer a transversal acceleration and/or rotational motion, large quantities of fuel moves uncontrollably inside the tanks and generate the sloshing effects. It has been shown in [4] that the dynamics interaction between the fuel motion and the rigid and/or flexible body dynamics can result in some kind of control instability. For minimizing these effects the ACS must be designed using a robust control method in order to assure stability and good performance to achieve the attitude control system requirement [5]. The dynamics of rigid-flexible satellite with fuel tanks when subject to large angle manoeuvre is only captured by complex non-linear mathematical model. Besides, the remaining flexible and/or liquid vibration can introduce a tracking error resulting in a minimum attitude acquisition time. A detailed investigation of the influence of the non-linearities introduced by the panel's flexibility into the ACS design can be found in [6]. It was shown that system parameters variation can degrade the control system performance, indicating the necessity to improve the ACS robustness. An experimental controller robustness and performance investigation has been done in [7], where the estimation of the platform inertia parameters was introduced as part of the platform ACS design. The problem of designing satellite non linear controller for rigid satellite has been done in [8] using the State Dependent Riccati Equation (SDRE) method which is able to deal with high non linear plants. Due to the complexity of modeling the fluid and/or flexible dynamic of the system it is common to use mechanical systems analogies that describe this dynamic. Besides, if one needs to know some physical parameters related with the slosh or the flexibility dynamics it is common to obtain then by experimental apparatus or some kind of estimating method such as Kalman filter [9]. In [10] a new technique to control the attitude of a rigid-flexible satellite has been developed where a reaction wheel was used for controlling the angular motion and the vibrations are damped by piezoelectric patches that are symmetrically bonded in the panel's surfaces. A multi-objective approach has been used in [11] to solve the problem of optimal solar sail trajectories control.

## 2 SATELLITE MODEL WITH SLOSHING

The phenomenon of sloshing is due to the movement of a free surface of a liquid that partially fills a compartment and this movement is oscillating. It depends on shape of the tank, the acceleration of gravity and of the axial/rotational acceleration of the tank. As representative of the behavior of the total weight of the system it is accepted that when the mass of the liquid oscillates the mass center of the rigid body also oscillates, thereby disturbing the rigid-flexible part of the vehicle under consideration. As an oscillating movement it is natural to consider the wave generated by the movement of the liquid as a stationary wave which all oscillation modes. Each mode of oscillation has a special feature of this phenomenon under

study, and one observes, in a quantitative sense, how much mass is displaced. Among all the modes that cause the greatest disruption in the system are the first and second modes. Despite the oscillation has lower frequency it is capable of resulting in violent shifting of the center of mass of the liquid creating an oscillation in the system as a role. The other oscillation modes act as a less aggressive and may not even vary the position of its center of mass due to the symmetry of the wave which on average causes no displacement. Due to its complexity, the sloshing dynamics is usually represented by mechanical equivalents that describes a similar and reproduce faithfully the actions and reactions due to forces and torques acting on the system. The main advantage of replacing the fluid model with an equivalent oscillating model is simplifying the analysis of motion in the rigid body dynamics, compared to the fluid dynamics equations. Due to the complexity of establishing an analytical model for the fluid moving freely within a closed tank, it is used a simplified system, taking into account the following criteria [5] : a) Small displacements, b) A rigid tank and c) No viscous, incompressible and homogeneous liquid. Under these conditions the dynamics of the sloshing can be approximated by mechanical system consisting of a mass-spring or pendulum. Consider a rigid spacecraft moving in a fixed plane, with a spherical fuel tank and including the lowest frequency slosh mode. Based on the Lagrange equation and the Rayleigh dissipation function one can model systems using the mechanical mass-spring and pendulum type system, respectively. Figure 1 shows a satellite model where slosh dynamics is represented by its pendulum analogous mechanical system.

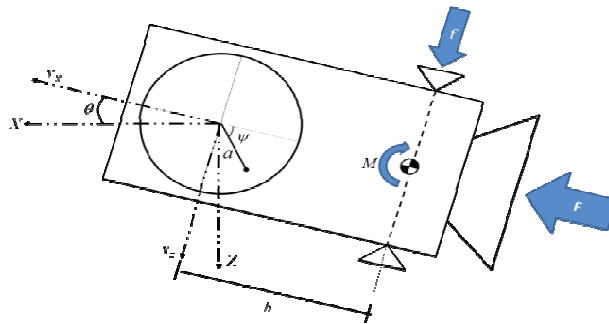


Figure 1: Satellite model with slosh dynamics pendulum analogous mechanical system.

The mass of the satellite and the moment of inertia, regardless of the fuel, are given by  $m$  and  $I$  respectively and the mass equivalent of fuel and its inertia moment is given by  $M_f$  and  $I_f$  respectively. It is assumed a transverse force  $f$  and a pitching moment  $M$ . A thrust  $F$  is assumed to act on the spacecraft longitudinal axis. Also it is given the velocity of the center of the fuel tank  $v_x$ ,  $v_z$  and the attitude angle  $\theta$  of the spacecraft with respect to a fixed reference  $(X, Y, Z)$ . Besides, one assumes as generalized coordinates:  $v$  representing the linear velocity,  $\omega$  representing the angular velocity of the rigid body,  $a$  is the length of the pendulum rod,  $b$  is the distance from satellite center of mass to the pendulum connected point,  $\psi$  is the angle of the pendulum with respect to the spacecraft longitudinal axis, which is assumed in the equilibrium position  $\psi = 0$  about the reference axis. The parameters  $m_f$ ,  $\psi$  and  $a$  depend on the shape of the tank, chemical-physical characteristics of the fuel and the fill ratio of the fuel tank.

The satellite equations of motion for the satellite with sloshing can be derived using the Lagrange equations given by

$$\begin{aligned}
 \frac{d}{dt} \left( \frac{\partial L}{\partial V} \right) + \omega^\times \frac{\partial L}{\partial V} &= \tau_t \\
 \frac{d}{dt} \left( \frac{\partial L}{\partial \omega} \right) + \omega^\times \frac{\partial L}{\partial \omega} + V^\times \frac{\partial L}{\partial V} &= \tau_r \\
 \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} + \frac{\partial R}{\partial \dot{\psi}} &= 0
 \end{aligned} \tag{1}$$

where  $L$  is the Lagrangian of the system,  $R$  is the Rayleigh dissipation function,  $\tau_r$  is the internal torque and  $\tau_t$  is the external torque. Assuming that  $R$ ,  $\tau_r$ ,  $\tau_t$ ,  $\omega$ ,  $V$  are given by

$$R = \frac{1}{2} \varepsilon \dot{\psi}^2; V = \begin{bmatrix} v_x \\ 0 \\ v_z \end{bmatrix}; \omega = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}; \tau_t = \begin{bmatrix} F \\ 0 \\ f \end{bmatrix}; \tau_r = \begin{bmatrix} 0 \\ M + fb \\ 0 \end{bmatrix} \tag{2}$$

The position vector of the satellite mass center with respect to the inertial system is

$$\vec{r} = (x-b)\hat{i} + z\hat{k} \tag{3}$$

assuming the relations  $v_x = \dot{x} + z\dot{\theta}$  and  $v_z = \dot{z} - x\dot{\theta}$  the satellite velocity is given by

$$\dot{\vec{r}} = v_x \hat{i} + (v_z + b\dot{\theta}) \hat{k} \tag{4}$$

The position of the mass of fuel is given by

$$\vec{r}_f = (x - a \cos(\psi)) \hat{i} + (z + a \sin(\psi)) \hat{k} \tag{5}$$

as a result, the velocity of the fuel mass is

$$\dot{\vec{r}}_f = (v_x + a \sin(\psi)(\dot{\theta} + \dot{\psi})) \hat{i} + (v_z + a \cos(\psi)(\dot{\theta} + \dot{\psi})) \hat{k} \tag{6}$$

The Lagrangian of the entire system is given by

$$L = \frac{1}{2} m \dot{\vec{r}}^2 + \frac{1}{2} m_f \dot{\vec{r}}_f^2 + \frac{1}{2} I_f (\dot{\theta} + \dot{\psi})^2 + \frac{1}{2} I \dot{\theta}^2 \tag{7}$$

Substituting the Eqs. (4-6) into Eq. (7), using the relations given by Eq. (2) and performing the derivations of Eq. (1), one obtains the satellite equations of motion given by

$$(m + m_f)(\dot{v}_x + v_z \dot{\theta}) + mb\dot{\theta} + m_f a(\ddot{\psi} + \ddot{\theta}) \sin(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \cos(\psi) = F \tag{8}$$

$$(m + m_f)(\dot{v}_z - v_x \dot{\theta}) + m_f a(\ddot{\theta} + \ddot{\psi}) \cos(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \sin(\psi) - mb\ddot{\theta} = f \tag{9}$$

$$(I_f + mb^2)\ddot{\theta} + mb(\dot{v}_z - v_x \dot{\theta}) - \varepsilon \dot{\psi} = M + bf \tag{10}$$

$$(m_f a^2 + I_f)(\ddot{\theta} + \ddot{\psi}) + m_f a((\dot{v}_x + v_z \dot{\theta}) \sin(\psi) + (\dot{v}_z - v_x \dot{\theta}) \cos(\psi)) + \varepsilon \dot{\psi} = 0 \tag{11}$$

Assuming the relations  $a_x = \dot{v}_x + v_z \dot{\theta}$ ,  $a_z = \dot{v}_z - v_x \dot{\theta}$  and substituting they into Eq.(8) and Eq.(9), one can isolate and obtain the satellite accelerations given by

$$a_x = \frac{F - mb\ddot{\theta} - m_f a(\ddot{\psi} + \ddot{\theta}) \text{sen}(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \cos(\psi)}{m + m_f} \quad (12)$$

$$a_z = \frac{f - m_f a(\ddot{\theta} + \ddot{\psi}) \cos(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \text{sen}(\psi) - mb\ddot{\theta}}{m + m_f} \quad (13)$$

All equations derived previously are no linear. However, in order to design a LQR and LQG controllers one has to get the linear set of equations of motion, which is obtained assuming that the system makes small movements around the zero point of equilibrium [12]. Now, substituting the Eqs (12-13) into Eqs (10-11) and assuming the linearization conditions, one has the satellite equation of motion given by

$$\ddot{\theta}(I_f + m^*(a^2 - ba)) + \psi(I_f + m^*a^2) + am_f^*F\psi + \varepsilon\dot{\psi} = -am_f^*f \quad (14)$$

$$\ddot{\theta}(I + m^*(b^2 - ba)) - m^*ab\psi - \varepsilon\dot{\psi} = M + b^*f \quad (15)$$

where  $b^* = \frac{bm_f}{m + m_f}$ ,  $m^* = \frac{mm_f}{m + m_f}$  and  $m_f^* = \frac{m_f}{m + m_f}$

### 3 SATELLITE MODEL WITH SLOSHING AND FLEXIBILITY

To derive the equations of motion for the satellite model with sloshing and flexibility one considers the same rigid satellite with tank partially filled plus a flexible appendage connected to the satellite as shown in Figure 2.

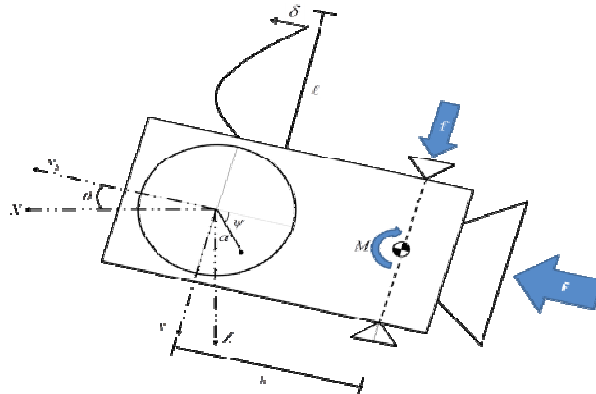


Figure 2 - Satellite model with slosh dynamics and a flexible panel.

The flexible appendage has mass  $m_p$  and length  $\ell$  and it has two motions, the angular motion  $\theta$ , resulting in a linear velocity  $\ell \dot{\theta}$  and a flexible deformation  $\delta$  with respect to the Z axis resulting in its variation given by  $\dot{\delta}$ . Thus, for small deformations, the panel velocity is given by

$$v_p = \dot{\delta} + \ell \dot{\theta} \quad (16)$$

The panel kinetic, potential energy and the dissipation function of energy D are given by

$$T_p = \frac{1}{2} m_p (\dot{\delta} + \ell \dot{\theta})^2 \quad (17)$$

$$E_{potencial} = \frac{\delta^2}{2} k \quad (18)$$

$$D = \frac{\dot{\delta}^2}{2} k_d \quad (19)$$

where  $k$  and  $k_d$  are the panel elastic constant and the dissipation constant.

Now the Lagrangian considering the slosh and the appendices' flexibility is given by

$$L_C = \frac{1}{2} \left( \begin{aligned} & (m+m_f)(v_x^2 + v_z^2) + m(2v_z b \dot{\theta} + b^2 \dot{\theta}^2) + m_f \left( a(\dot{\theta} + \dot{\psi}) \left( a(\dot{\theta} + \dot{\psi}) + 2(v_x \sin(\psi) + v_z \cos(\psi)) \right) \right) + \\ & + I \dot{\theta}^2 + I_f (\dot{\theta} + \dot{\psi})^2 + m_p (\dot{\delta}^2 + 2\ell \dot{\theta} \dot{\delta} + \ell^2 \dot{\theta}^2) - \delta^2 k \end{aligned} \right) \quad (20)$$

In order to obtain the equation of motion for the satellite with sloshing and flexible panel one uses Eq.(1) plus the Lagrange equation given by

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\delta}} \right) - \frac{\partial L}{\partial \delta} + \frac{\partial D}{\partial \dot{\delta}} = 0 \quad (21)$$

which after all derivation and performing similar previously linearization one obtains the linearized equations of motion given by

$$\ddot{\theta} (I + m_p \ell^2 + m^* (b^2 - ba)) + m_p \ell \ddot{\delta} - m^* ab \ddot{\psi} - \varepsilon \dot{\psi} = M + b^* f \quad (22)$$

$$\ddot{\theta} \left( \frac{m^* ba}{I_f + m^* a^2} - 1 \right) = \ddot{\psi} + \dot{\psi} \left( \frac{\varepsilon}{I_f + m^* a^2} \right) + \psi \left( \frac{a^* F}{I_f + m^* a^2} \right) + \frac{a^* f}{I_f + m^* a^2} \quad (23)$$

$$-\ddot{\theta} \ell = \ddot{\delta} + \dot{\delta} \frac{k_d}{m_p} + \delta \frac{k}{m_p} \quad (24)$$

where  $m^* = \frac{m_f m}{m + m_f}$ ,  $a^* = \frac{m_f a}{m + m_f}$ ,  $b^* = \frac{m_f b}{m + m_f}$

#### 4 LINEAR QUADRATIC REGULATOR - LQR

Assuming a plant described by the linear state equations given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad (25)$$

where  $\mathbf{x}$  represent the state vector,  $\mathbf{A}$  the state matrix,  $\mathbf{B}$  the input matrix and  $\mathbf{u}$  the control input. The LQR is an optimal control method that consists of minimizing the function given by

$$J = \frac{1}{2} \mathbf{x}'(t_f) \mathbf{H} \mathbf{x}(t_f) + \frac{1}{2} \int_{t_0}^{t_f} [\mathbf{x}'(t) \mathbf{Q}(t) \mathbf{x}(t) + \mathbf{u}'(t) \mathbf{R}(t) \mathbf{u}(t)] dt \quad (26)$$

where the final time  $t_f$  is fixed,  $\mathbf{Q}$  and  $\mathbf{H}$  are real positive semi-definite matrices, and  $\mathbf{R}$  is real symmetric positive definite matrix. The gain of the control law is obtained solving the Riccati equation [9] given by:

$$\dot{\mathbf{P}}(t) = -\mathbf{P}(t) \mathbf{A} - \mathbf{A}' \mathbf{P}(t) - \mathbf{Q} + \mathbf{P}(t) \mathbf{B} \mathbf{R}^{-1} \mathbf{B}' \mathbf{P}(t) \quad (27)$$

where  $\mathbf{P}$  is the symmetrical solution matrix of the differential Riccati equation. The optimal LQR control law can be written as

$$\mathbf{u}(t) = -\mathbf{R}^{-1} \mathbf{B}' \mathbf{P}(t) \mathbf{x}(t) \quad (28)$$

where the gain of the LQR control law is given by

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}' \mathbf{P}(t) \quad (29)$$

## 5 LINEAR QUADRATIC GAUSSIAN - LQG

The LQG method is the union of the LQR problem with the Kalman filter problem. However, if there is any state that is not available one uses the Kalman filter to estimate it in order to feedback. The separation principle [9] ensures that each problem can be solved independently of each other.

Assuming a plant described by the linear state equations given by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \Gamma w \\ \mathbf{y} &= \mathbf{C} \mathbf{x}(t) + v \end{aligned} \quad (30)$$

where  $\mathbf{x}$  is the state vector,  $\mathbf{A}$  is the state matrix,  $\mathbf{B}$  is the input matrix,  $\mathbf{y}$  is the output vector,  $\mathbf{C}$  is the output matrix,  $v$  and  $w$  are white noise and  $\mathbf{u}$  is the control input [13]. Following a similar approach described before, now the LQR gain is given by

$$\mathbf{K}_c = \mathbf{R}^{-1} \mathbf{B}' \mathbf{P}_c \quad (31)$$

Where  $\mathbf{R}$  is real symmetric positive definite matrix and  $\mathbf{P}_c$  is the symmetrical solution of the LQR Riccati equation given by

$$\mathbf{A}' \mathbf{P}_c + \mathbf{P}_c \mathbf{A} - \mathbf{P}_c \mathbf{B} \mathbf{R}^{-1} \mathbf{B}' \mathbf{P}_c + \mathbf{M}' \mathbf{Q} \mathbf{M} = \mathbf{0} \quad (32)$$

Similarly the Kalman filter gain now is given by

$$\mathbf{K}_f = \mathbf{P}_f \mathbf{C}' \mathbf{V}^{-1} \quad (33)$$

where  $\mathbf{V}$  is real symmetric positive definite matrix and  $\mathbf{P}_f$  is the symmetrical solution matrix of the KF Riccati equation given by.

$$\mathbf{P}_f \mathbf{A}^T + \mathbf{A} \mathbf{P}_f - \mathbf{P}_f \mathbf{C}^T \mathbf{V}^{-1} \mathbf{C} \mathbf{P}_f + \mathbf{\Gamma}^T \mathbf{W} \mathbf{\Gamma} = 0 \quad (34)$$

where  $\mathbf{P}_c = \mathbf{P}_c^T \geq 0$  and  $\mathbf{P}_f = \mathbf{P}_f^T \geq 0$  and  $\mathbf{Q}$ ,  $\mathbf{R}$ ,  $\mathbf{V}$  e  $\mathbf{W}$  are weights matrices which can be regarded as setting parameters ("tuning") that must be manipulated until they find one acceptable response to the system. The LQG method is more realistic than the LQR method, since it can estimate the states that are not available to be feedback and it allows to include the noise in the model which represents imperfections of the system.

## 6 SIMULATIONS RESULTS

The first simulation is the comparison between the LQR and LQG control law, for the satellite model with sloshing dynamics given by Eqs.(14-15). The parameters values used in the simulations are  $m=600\text{Kg}$ ,  $m_f = 100\text{Kg}$ ,  $I=720\text{Kg/m}$ ,  $I_f = 90\text{Kg/m}$ ,  $a=0.3\text{m}$ ,  $b=0.3\text{m}$ ,  $F=500\text{N}$ ,  $\varepsilon=0.19\text{Kg/m}^2/\text{s}$ . The initial conditions used are  $\theta=2^\circ$ ,  $d\theta/dt=0.57^\circ/\text{s}$ ,  $\psi=1^\circ$  and  $d\psi/dt=0^\circ$ . Figure 3 shows that the LQR control law performance is better than the LQG. The main reason is because the LQR control law considers that the sloshing variables are available to be feedback which is not true. Figure 4 shows that both the torque and the force of the LQR controller are smaller than the LQG controller.

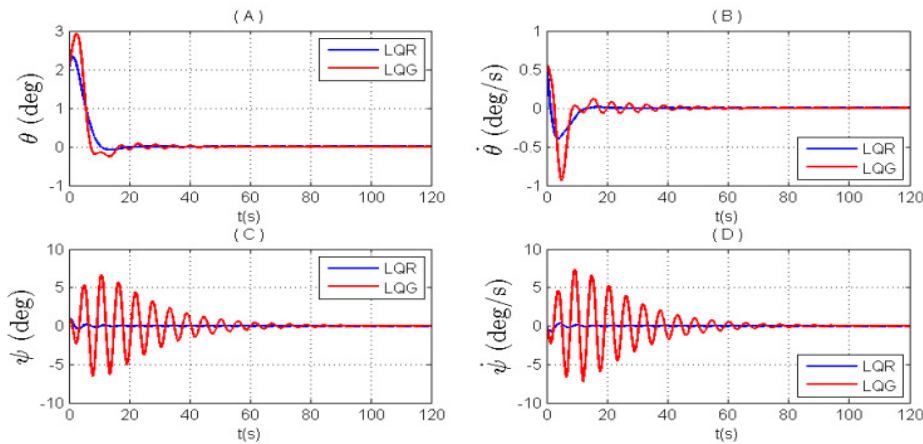


Figure 3 – The angular and sloshing control by the LQR and LQG controller.

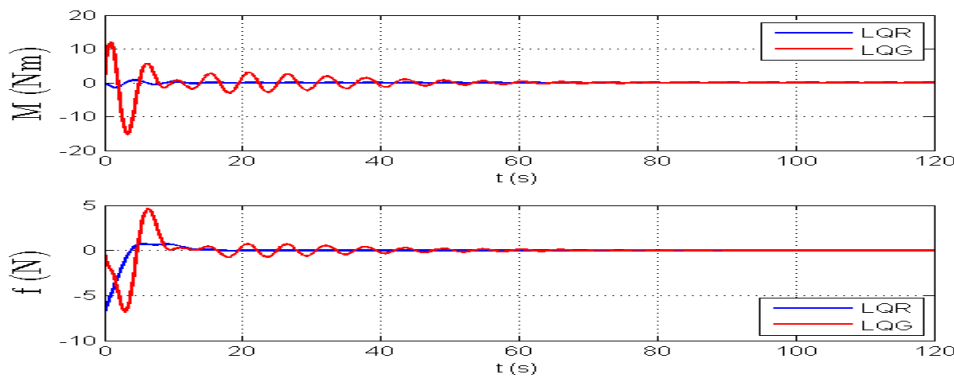


Figure 4 – The LQR and LQG controller effort to control the angular and sloshing motion.

The second simulation is also the comparison between the LQR and LQG control law, but now the satellite model has the sloshing dynamics plus the flexible dynamics of panel,



which the data values are  $m_p=10\text{kg}$ ,  $\ell = 1.5\text{m}$ ,  $k = 320\text{Kgrad}^2/\text{s}^2$ ,  $k_d = 0.48\text{Kgrad}^2/\text{s}$ . The simulations initials conditions are  $\theta=2^\circ$ ,  $d\theta/dt=0.57^\circ/\text{s}$ ,  $\psi=1^\circ$ ,  $d\psi/dt=0^\circ/\text{s}$ ,  $\delta = \dot{\delta}=0$ .

Figure 5 and 6 show that the LQR control law performance is better than the LQG only for controlling the angular motion. However, for controlling the flexible motion the LQR controller performance is damage.

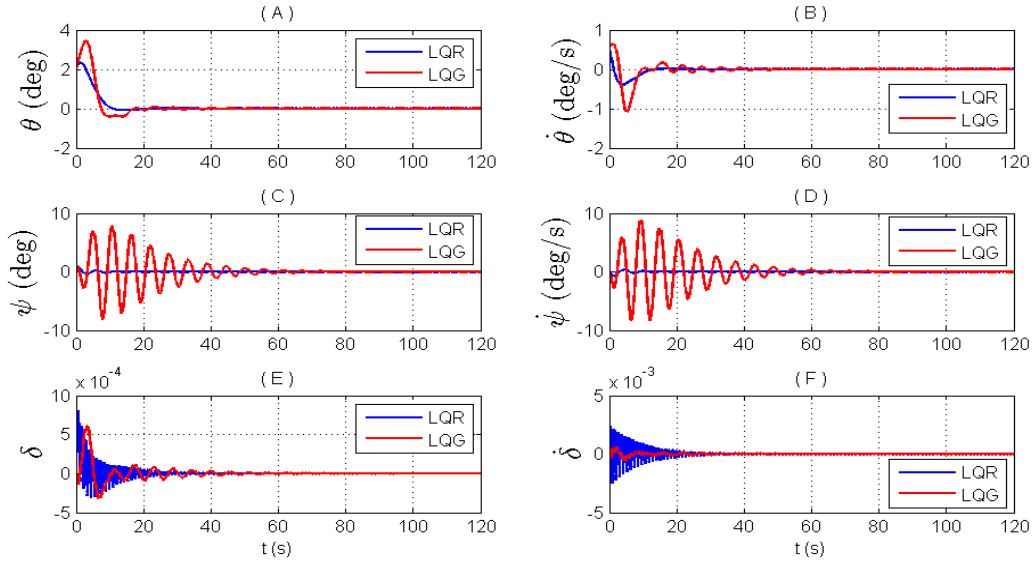


Figure 5 : Control of the angular motion, sloshing and flexibly.

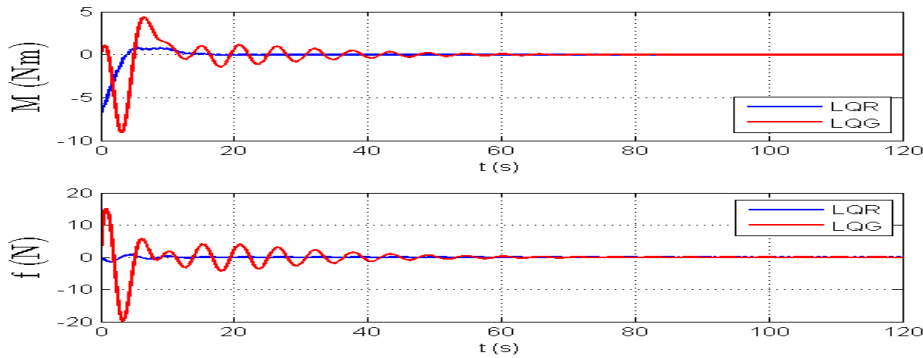


Figure 6: The LQR and LQG performance controlling angular motion, sloshing and flexibly.

## 7 CONCLUSIONS

In this paper one described the concepts of the sloshing phenomenon which is associated with the dynamics of a liquid moving into at partially fills reservoir. To derive the equation of motion of a spacecraft with liquid inside the sloshing phenomenon is represented by its mechanical analog of a pendulum type. One shows that the performance of the LQR control is better

than and LQG control and the reason that the LQG control is degraded is because the sloshing states need to be estimated by the filter, besides that there is noises representing the imperfections of the models acting over the system. These comparisons show that when all states are available to be feedback the LQR controller is better than LQG. However, in a realistic situation one has not the sloshing states to be feedback, so one must use the KF to estimate them. Besides, one observes that flexibility still has small fluctuation, which is not appropriated when one needs high precise pointing.

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