11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

A STATISTICAL ANALYSIS OF THE DYNAMIC RESPONSE OF A VIADUCT

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Keywords: dynamic analysis, railway bridges, resonance, factorial design, residuals, response function

Abstract. *In this contribution a statistical analysis of the dynamic response of a railway viaduct, modelled after an actual structure, is presented. The finite element model of the viaduct is based on the data provided by the Portuguese Railway Company REFER EPE. The train load is simplified by a set of constant moving forces and the range of velocities implemented belongs to the range of typical velocities of circulation. The viaduct is composed of eight modules, but, for the sake of simplicity, only the first viaduct module is included in the analysis.*

This paper, in particular, addresses the construction and utility of the response functions. With this in mind, the viaduct is subjected to a two-level factorial design and the response function is formed according to the standard statistical literature. Accuracy and viability is confirmed by residuals analysis with respect to the probability level separating the significant effects and interactions.

In addition, normal probability plots are shown and their usage is demonstrated on the analysis of significant effects.

This type of straightforward application of statistical analysis necessitates adequately selected key parameters and key results. Conclusions provide useful information for design guidelines and therefore lead to better planning and more realistic representation of the actual response of railway bridges.

1 Introduction

1.1 General

Railway bridges are important connecting infrastructures that due to the current trend of expanding and increasing the railway network capacity require specific design considerations supported by an adequate numerical modelling. To idealize railway bridges for numerical models, the associated components of the structure are subject to certain simplifications and the input data to a related numerical model are supplied with a certain level of uncertainty. When approximations of the input data are combined with the uncertainties of the solicitations the calculated response by deterministic approach can be greatly misled. Therefore a statistical treatment of input as well as result data should be accomplished. Statistical analysis of numerical results allows defining a set of key input data and key results, in order to study complex mechanisms interactions and understand if the involved factors play a role in the response in an interactive or simply additive way. Key factors are selected by the user, it should be the ones governing the dynamic response of the system.

Previous works addressed these issues by parametric analyses. It is clearly shown in many statistical publications [3, 5, 17] that this one-variable-at-a-time strategy fails frequently because it tacitly assumes that the maximizing value of one variable is independent of the level of the other. Simultaneous consideration of the influence of several key parameters provides a better representation of reality.

Although simplified models of railway tracks are widely used, the growth in numeric and computational efficiency made complete models involving several structural details viable and preferable. The computational speed is a very important factor and has been constantly improving over the years. Therefore certain methods that require several runs turned out to be practicable within an acceptable period of time. Statistical methods belong to these methods and thus can enhance the analysis by providing results that are more realistic, which consequently gives a better insight of the situation of interest and helps in the calibration of the models.

When classifying a bridge model with respect to the train load there are mainly three types of models: (i) the moving force model [15, 4]; (ii) the moving mass model [2, 1]; and (iii) the moving system model [19, 13] that comprehends a system of masses, springs and dampers. In the present work the moving force model can be safely used since the ratio of the load mass over the mass of the bridge does not exceed 30 %. This simplification was already used in the authors previous work [11] and it is justified in the monographs [9, 20].

Statistical analysis of dynamic response of bridges by design of experiments is under growth in the scientific community. It can be found in [18] and related works of the first author. In Karalar [12] statistical methods are applied to the analysis of isolation of bridges. Structural health monitoring (SHM) on bridges is another field of structural engineering that is currently employing statistical treatment [7, 8]. The factorial experiment stands for the statistical analysis of the variance of the results due to the changes in the key input data. The question of whether the train speed can or cannot be a valid factor in the two-level factorial analysis is addressed in detail in [10]. It is known (see e.g. Yang [20]) that structures subjected to repetitive moving loads increase their dynamic response when the speed equals the resonant one. Consequently the interval of velocities to be considered in a factorial analysis cannot involve any of such velocities because while considering only marginal values, an important response increase could be overlooked. It was concluded in [10] that it is safe to perform two-level factorial design, where one of the factors is the typical train speed.

This paper, in particular, addresses the construction and utility of the response functions.

With this in mind, the viaduct is subjected to a two-level factorial design and the response function is formed according to the standard statistical literature. Accuracy and viability is confirmed by residuals analysis with respect to the probability level separating the significant effects and interactions. In addition, normal probability plots are shown and their usage is demonstrated on the analysis of significant effects.

2 The Santana do Cartaxo viaduct

Train specification and in-situ measurements of the soil foundation properties were supplied by REFER EPE [16]. A more detailed description of the structure can be found in previous work of the authors [11].

The viaduct is composed by a set of eight module sections in the longitudinal direction. Each module is connected to the other through transition pillars which are larger and have more piles than the intermediate pillars.

(a) General view (b) Embankment

Figure 1: Santana do Cartaxo viaduct

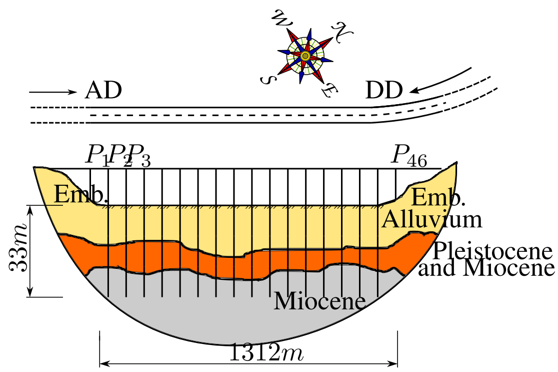


Figure 2: Viaduct top and left view

Designating by ascending departing direction (AD) the one from south to north and by (DD) the descending return one, then the first of the eight modules comprises three spans of 25, 30, and 25 m, finalizing a length of 80 m, while the other seven modules have spans of 25, 4×30 and 25 m, yielding the length of 170 m, bringing the total viaduct length to 1312 m. On the plan view (see Figure 2) the viaduct develops linearly and at the end starts a left transition curve of the final radius 1750 m.

The geological layers are visualized in Figure 2. The alluvium layer is subdivided in two categories and three subcategories A1, A2 and A3. For the sake of simplicity, in this paper a single layer with average properties is considered.

(a) CP5600 Locomotive (b) Corail carriage

Figure 3: Train photos

The traffic over the viaduct is practically equally distributed between Alfa Pendular and Intercidades trains (Figure 3).

Traveling speeds can be checked in Table 1.

	Locomotive	Length [m]	Weight [ton]	Speed [km/h]
Merchandise TAKARGO	6000	536	1156	100
Passangers Intercidades	2600	267.5	190	190 (AD); 180 (DD)
Passangers international	5600	267.5	250	190 (AD); 180 (DD)

Table 1: Circulation characteristics

The Intercidades train was selected to perform analyses in this paper.

3 Finite element model

The numerical model is developed with an ANSYS/LS-DYNA module. The parametric analyses with automatic extraction of key results are coded by APDL (ANSYS parametric design language).

For the sake of simplicity, only the first of the eight modules, the one having three spans of 25, 30 and 25 m, is modelled. This module is supported at its ends on one embankment and one transition pillar that connects it with the rest of the viaduct. In the numerical model, the other seven modules have been replaced by representative springs and dampers. It was verified numerically, based on bending natural frequencies, that valid conclusions can be obtained on such one module set up.

Rail-pads and ballast are represented by linear and rotational spring and damper elements acting in three directions. The arrangement is a three-dimensional extension of the system used in [14] and it is displayed in Figure 4. A more detailed description of the finite element model can be found in previous work of the authors [11].

The eight lower mode shapes are illustrated in Figure 5. The first three bending modes are the 5th, the 7th and the 8th mode, respectively, shown in Figures 5e, g and h.

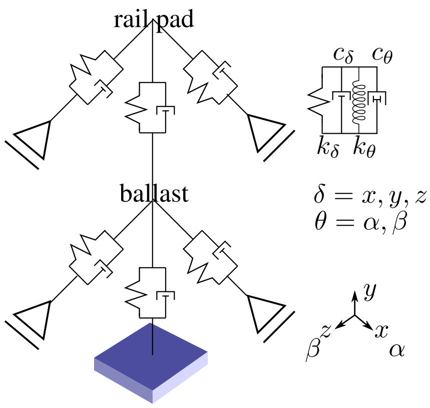
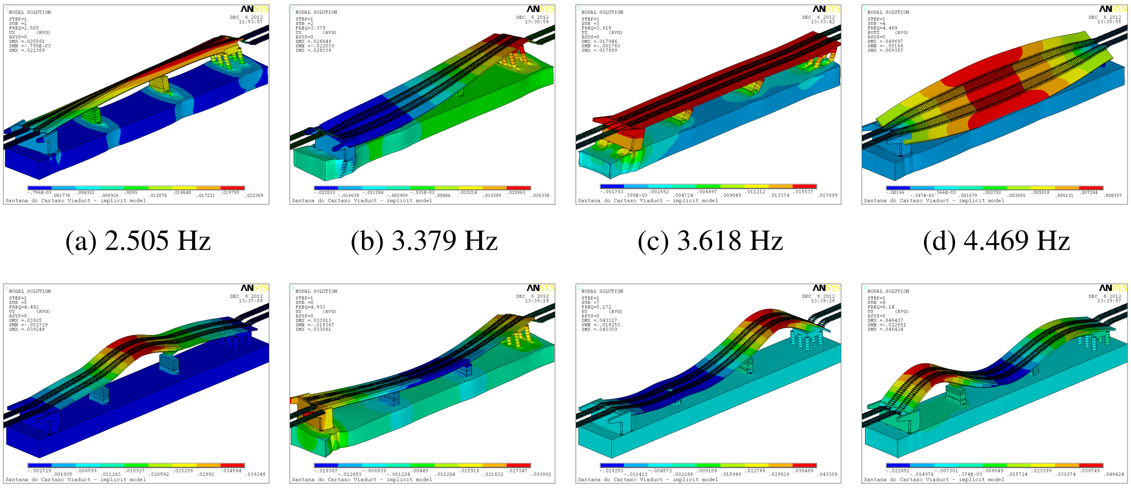


Figure 4: Spring-damper ballast system



(a) 2.505 Hz (b) 3.379 Hz (c) 3.618 Hz (d) 4.469 Hz

(e) 4.492 Hz (f) 4.833 Hz (g) 5.172 Hz (h) 6.14 Hz

Figure 5: Mode shapes

Ballast behaviour was selected as a qualitative factor for the factorial design and a nonlinear behaviour (high level) was tested against the linear one (low level). In order to make some valid comparison, a type of the nonlinear behaviour curve and a connection between the linear and nonlinear behaviours was established. According to [6], the appropriate function describing the nonlinear ballast behaviour has a cubic polynomial form. It was decided to calculate the parameters (coefficient with the linear $\tilde{K}_l$ and cubic terms $\tilde{K}_c$), that govern the nonlinear behaviour from two conditions: (i) same elastic force at a given displacement and (ii) same elastic energy accumulated at a given displacement. Such conditions are dependent and therefore the linear elastic force in the former condition was reduced to 90 %.

Solving the equations for the coefficients of the cubic spring, one gets:

$$\begin{cases} \tilde{K}_l &= \frac{0.1K_l(-20\delta_1^2+9\delta_2^2)}{-2\delta_1^2+\delta_2^2} \\ \\ \tilde{K}_c &= \frac{0.2K_l}{-2\delta_1^2+\delta_2^2} \end{cases} \tag{1}$$

where F_l and F_c represent the elastic forces and δ_1 and δ_2 are the specified displacements. δ_2 was chosen as a typical displacement of 1 mm and δ_1 as 50 % of δ_2, i.e. 0.5 mm. It was verified numerically that changing δ_2 does not affect the results significantly.

The graph in Figure 6 presents a typical force displacement relation of the linear and cubic springs related to the vertical stiffness value, i.e. when K_l is equal to 120000 kN/m.

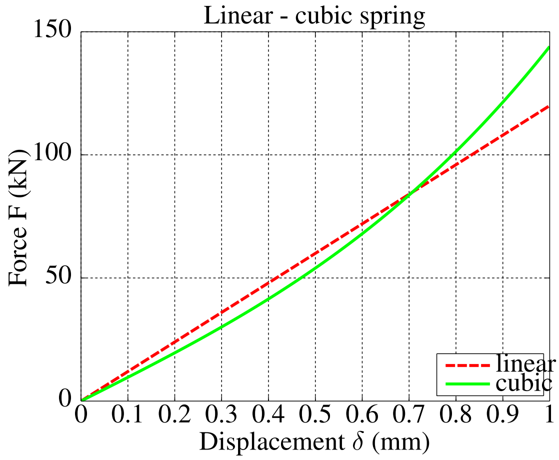


Figure 6: Cubic (solid green) and linear (dashed red) spring force-displacement graph

4 Results

Explicit analysis is performed with LS-DYNA software with a time step calculated according to the element sizes and properties as 0.017 ms. The model has 64878 elements and each analysis took around 2 h and a half for the speed of 185 km/h. Mesh size is variable over the model; it gradually increases from 10 cm in rail to 1 m in soil. It was verified numerically that soil elements are sufficiently small.

The statistics toolbox of Matlab software was used to produce normal probability plots of single effects and interactions from a reference normal distribution, response functions and residuals diagnostic checks.

Factors (variation)	Key results (vertical) Peak acceleration velocity, displacement
A. Ballast stiffness (40 %)	rail level (a)
B. Ballast mass (6 %)	sleeper level (b)
C. Ballast behaviour (L-NL)	deck level (c)
D. Loads speed (2.7 %)	free rail level (e)
E. Ballast damping (30 %)	soil level (d)
F. Rail pad damping (15 %)	

Table 2: Factors, their variation and key results

The selected factors, their variations and key results are presented in Table 2. The key results are extracted from the middle point of the middle span of the viaduct at several levels that are placed on a vertical line that passes the external rail of the track that is subjected to the load passage.

The value of the reference train velocity considered was v_0=185 km/h. A two-level full factorial analysis was accomplished. This means that $2^6 = 64$ runs were performed, leading to a total computing time of 184.5 hours.

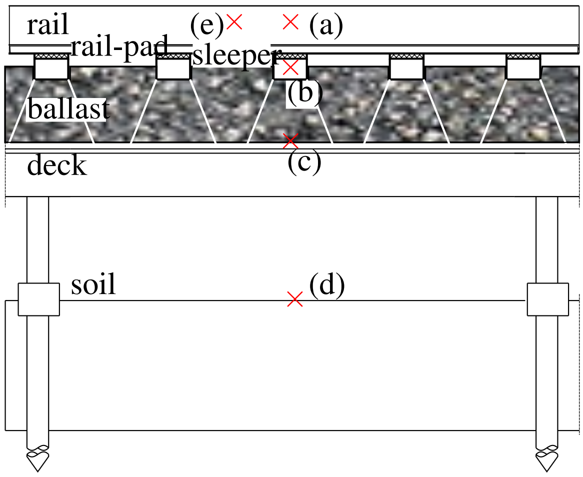


Figure 7: Points of results extraction (not scaled)

The results selected are the peak velocity at the free-rail and deck, and the peak acceleration at the sleeper. These results exhibit dominant single and/or interaction effects and therefore are suitable for drawing some interesting conclusions.

4.0.1 Normal probability plots

The normal probability plots are shown in Figures 8-9. In these graphs the effects are plotted on the horizontal axes and their probability from the corresponding normal distribution is added and scaled on the vertical axes. The dashed red line, called the error line, represents the situation when the results exactly obey the normal distribution. Thus the significant effects and combinations do not match the dashed red line and fall outside. The remaining effects can readily be explained as a random effect.

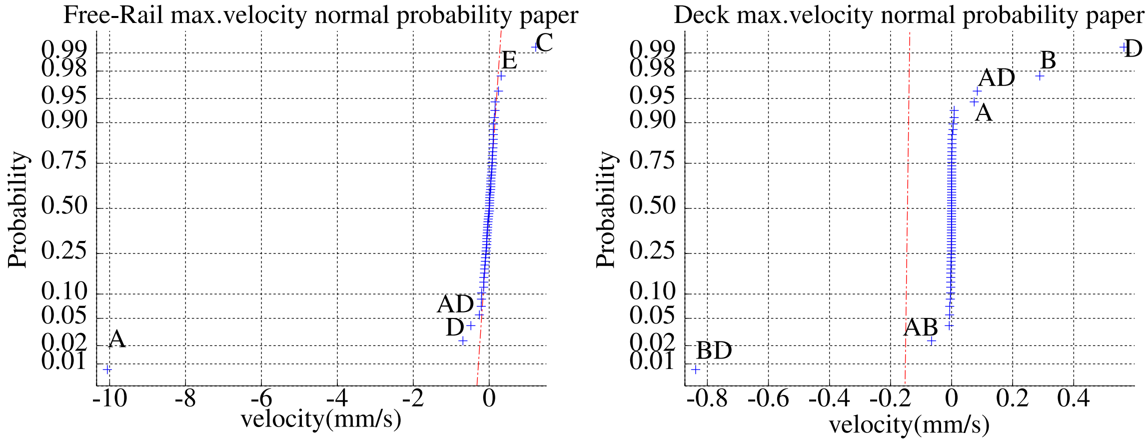
According to [3] significant effects and interactions are defined as the ones that overpass in absolute value the "simultaneous margin of error (SME)", defined as the horizontal coordinate (measured from the zero mean) of the t-student distribution that encompasses the probability of $\phi = (1 + 0.95^{(1/n)})/2$, where n is the number of effects, i.e. $2\phi - 1$ confidence interval. Using this definition the effects are represented in the Figures below. However, and for the sake of clarity, in Figure 8b only for four of the fourteen significant effects are represented. These are BD, D, B, AD, A, AB, ABD, BC, CD, C, BE, ABE, ABC and AC.

4.0.2 Response functions

The response functions of the presented results are written in Equations (2) to (4). The coefficients of each term are half of the respective effect. Only significant effects are implemented. Residuals are defined as the differences of the calculated results and estimated results by the response function.

Besides SME definition, it is possible to use "margin of error (ME)" that considers a smaller probability of $\phi = 97.5\%$. The distinction between these two is that there is at most a 5% chance that one individual inactive effect will exceed the ME, while there is at most a 5% chance that *any* inactive effect will exceed the SME.

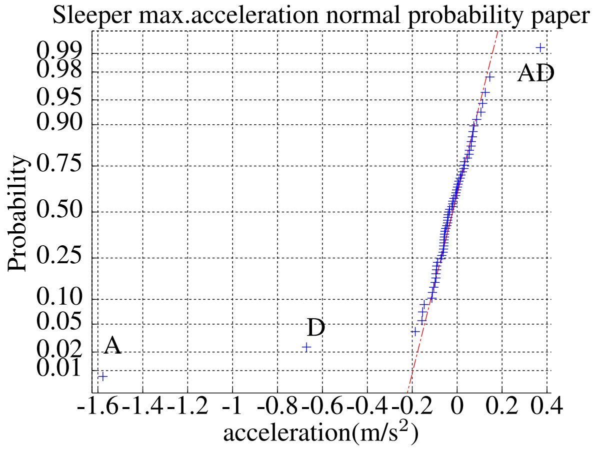
In order to compare the behaviour of the residuals of these two possible choices, a sum of residuals relative to the average, S has been calculated. The smaller this value is the more accurate the response function is. The comparison between sums of residuals for the ME and SME



(a) Free rail level; S=0.4989

(b) Deck level; S=0.0266

Figure 8: Normal probability plots peak velocity



(a) Sleeper level; S=3.5222

Figure 9: Normal probability plots peak acceleration

is presented in Table 3. It is clear that the SME approach is a rougher and more conservative estimate, while ME approach involves more effects and thus residuals are getting closer to the error line. One thus has to make a proper choice between complexity and simplicity versus accuracy.

S	ME	significant eff.	SME	significant eff.
free-rail(vel.)	0.3845	10	0.4989	5
deck(vel.)	0.0161	21	0.0266	14
sleeper(accel.)	3.0254	6	3.5222	3

Table 3: Sums of residuals for the SME and ME

The free-rail velocity response function in SME approach is

$$v^e = 39.07 - 5.030x_A + 0.61x_C - 0.34x_D - 0.24x_Ax_D + 0.16x_E \tag{2}$$

In the deck velocity response function only the first six effects are written below, for the sake of simplicity. The function is given by

$$v^c = 7.402 - 0.42x_Bx_D + 0.28x_D + 0.14x_B + 0.042x_Ax_D + 0.037x_A - 0.033x_Ax_B \quad (3)$$

Finally the sleeper acceleration response function is given by

$$a^b = 4.27 - 0.79x_A - 0.34x_D + 0.19x_Ax_D \quad (4)$$

5 Conclusions

A complete statistical analysis, based on a two-level factorial design of experiments was presented and several analysis tools were applied to a real case study. The statistical theory proved to be relevant, solid, meaningful and easy to implement.

The main conclusions are listed as: Interaction effects can be more important than single effects, and therefore key parameters cannot be analysed individually. This is supported by the peak velocity at the deck level (Figure 8b); in this case the interaction BD, i.e. the ballast mass with the loads speed is the most relevant effect.

Regarding the analysis of effects, in general, results revealed that the factors A (the ballast stiffness) and D (the loads speed), and the interaction BD (the interaction of ballast mass and loads speed) are the most important single and combined effects.

The construction and utility of the response functions was shown. Accuracy and viability was confirmed by residuals analysis. At first SME definition was used. Then the probability level separating the significant effects and interactions was decreased at ME level and shown that, as a consequence, the residuals fell closer to the error line.

In summary, the usefulness of the statistical analysis was demonstrated.

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