11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

DYNAMIC MODELING AND SIMULATION OF SPATIAL MANIPULATOR WITH FLEXIBLE LINKS AND JOINTS

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Keywords: Spatial Flexible Manipulators, Flexible Links, Flexible Joints, Dynamic Modeling.

Abstract. *Dynamic modeling of the robot manipulator is very important to design model-based control and for simulation purposes. A steady increase in the demand of high speed operations, low energy consumption and increase in payload to arm weight ratio motivates the use of light weight materials to build manipulators. With the use of light weight material, the rigid link assumption is no longer valid and, in some cases, the transmission system at manipulator joints can introduce flexibility. Ignoring the link and joint flexibility can cause poor estimation of dynamic parameters and, eventually, poor performance of the control design. To obtain an accurate dynamic model of a robot manipulator, the flexibility of the links and joints should be accounted. The inclusion of the dynamics due to flexibility makes the robot manipulator a continuous system and requires infinite degrees of freedom to estimate the dynamic parameters. It also establishes strong coupling between gross rigid body motion and elastic deformations of links and joints.*

In this paper, a general purpose algorithm is presented that allows to obtain a dynamic model of spatial flexible manipulators for model based control design and simulation purposes. Both link and joint flexibilities are considered in the dynamic modeling. The flexible links are discretized to get a finite dimensional dynamic model. The deformation of each link is assumed to be due to both bending and torsion. The deformation of the joints is assumed to be due to pure torsion. The deformation of each link is assumed to be small relative to the rigid body motion. Thus, the configuration of each link is defined as the sum of rigid and elastic coordinates using a floating reference frame. The dynamic model is first derived using the principle of virtual work along with finite element method in generalized coordinates for general purpose implementation. Then, the system of equations in generalized coordinates is converted into independent coordinate form using a recursive kinematic formulation based on the topology of a manipulator. The advantage of general purpose algorithm is it uses minimum set of equations that define the dynamics of flexible manipulator, which is required in control design to reduce computation cost. In addition, it allows the dynamic modeling of any arbitrary manipulator configuration. Numerical simulation results of an open chain RRR manipulator with flexible links and flexible joints is presented to show the effects of flexibility on robot manipulator dynamics.

1 INTRODUCTION

Industrial manipulators are designed to increase the productivity and to help humans in tedious and hazardous work environment. These manipulators are made of heavy and stiff materials to achieve high precision on endeffector motion. Heavy industrial manipulators have high mass to payload ratio, consume more power and have limited operation speed. To improve the performance of industrial manipulators, the focus on light weight manipulators has been increased in recent years [1]. Other applications of light weight manipulators can be found in space applications for communication, space exploration, solar power generation [2], fire rescue turntable ladders [3] and crane boom [4].

The side effects of light weight manipulators are their low stiffness especially of the links. In addition, also the transmission system at manipulator joints can introduce flexibility. The deformations of links and joints have a strong coupling with the gross rigid body motion. Ignoring these flexibilities in control design can cause poor performance [5]. To improve the performance of control, an accurate dynamic model that incorporates the link and joint flexibilities is mandatory. Flexible manipulators are distributed parameter systems, hence infinite degrees of freedom are required to characterize the dynamic behavior of the system. However, the exact dynamic modeling of such systems is not feasible from the control point of view. A finite dimensional dynamic model can be obtained by discretizing the continuous systems using assumed mode method (AMM) or finite element method (FEM).

In [6] dynamic modeling of flexible link manipulator using recursive lagrangian dynamics via transformation matrix is presented. It is an extension of the dynamic modeling of rigid manipulators. The kinematics of joint rotations and link deformations is defined using a 4×4 transformation matrix. The link deformations are assumed to be small and approximated using assumed mode method. In [7] a closed form dynamic model using Lagrangian approach and assumed mode method for a planar multi-link light weight robot is presented whereas [8] proposes a linearized dynamic model for a multilink planar flexible link manipulator for simulation and control design. Elastic deformations are defined using Euler-Bernoulli beam theory, and the total deformations of each link are approximated using assumed mode method. The main drawback of assumed mode method is that it is not suitable of finding modes for non-regular cross-sections and that the choice of boundary conditions for multilink manipulator is not unique. The possible boundary conditions reported in the literature are clamped, pinned-pinned and free-free boundary conditions.

In [9] numerical and experimental investigation on the dynamic modeling of planar flexible manipulators is presented. The dynamic model is developed using Lagrangian approach and finite element method. The experimental validation is carried out considering a single link flexible manipulator and comparing the results with numerical finite element model in both frequency and time domain. The FEM model shows closer agreement with experimental results. In [10] a general purpose computer program SPACAR for numerical simulations of flexible mechanism and manipulators is shown. SPACAR uses a finite element based Lagrangian formulation to define the dynamics of the system. The program incorporates a virtual power type approach, to automatically eliminate the non-working constraints forces and reactions. This approach reduces the Lagrangian formulation to a minimal set of ordinary differential equations. In [11] is presented a dynamic modeling of spatial flexible link manipulators using principle of virtual power and finite element method is presented whereas [12] proposes a redundant Lagrangian FEM formulation for the dynamic modeling of flexible links and joints. The elastic deformation on each link is assumed to be due to bending and torsion. The deformation of each link is

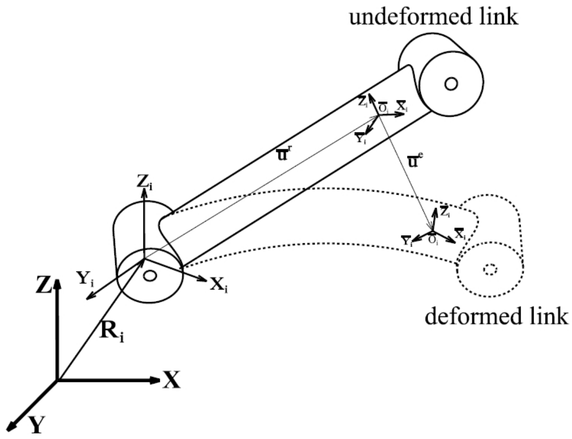

Figure 1: Arbitrary point on a flexible link.

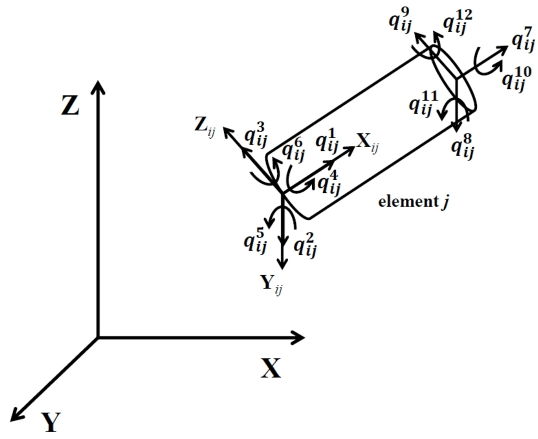

Figure 2: Elastic coordinates on finite element.

expressed in a tangential local floating frame. The constrained equations due to the connectivity of each link are added to equations of motion by using Lagrangian multiplier. The redundant dynamic model requires more computational power. In [13] develops a recursive Newton-Euler formulation for a flexible link open loop robot system. The recursive formulation converts the redundant form into minimum set of equations. However, this formulation ignores the joint flexibility. Ignoring the joint flexibility can significantly affect the system Eigen frequencies [14].

In this paper, the systematic approach for the dynamic modeling of spatial flexible manipulators considering both link and joint flexibility is presented. The link flexibility is assumed due to bending and torsion, where shear deformations are neglected. The joint flexibility is assumed due to pure torsion. The deformation of each link is assumed to be small with respect to gross rigid body motions. Hence, the configuration of each link is defined as the sum of rigid and elastic coordinates using floating reference frame. A general purpose program is developed based on the principle of virtual work and finite element method. The dynamic model is first derived using generalized coordinates for general purpose implementation. Then, a recursive kinematic formulation is presented based on the topology of the flexible manipulator which converts the system of equations in absolute coordinates into relative coordinates.

2 Kinematics of Flexible Manipulators

The kinematic equations that define an arbitrary displacement of a flexible link *i* shown in Figure 1 is derived using floating reference frame formulation. Floating reference frame formulation uses two sets of coordinates i.e., body reference and elastic coordinates. The body reference coordinates describe the position and orientation of body reference frame $X_iY_iZ_i$ with respect to global coordinate system XYZ. The elastic coordinates describe the deformations on flexible link *i* with respect to body reference frame $X_iY_iZ_i$. The elastic deformations on flexible link are approximated using finite element method to obtain finite set of elastic coordinates. The elastic coordinates of finite element shown in Figure 2 is defined using floating reference frame $X_{ij}Y_{ij}Z_{ij}$ with respect to body reference frame $X_iY_iZ_i$. The position of an arbitrary point on flexible link *i* can be defined as

$$r_i = R_i + A_i\overline{u}_i \tag{1}$$

where R_i is the position vector of body reference frame $X_iY_iZ_i$. A_i is the transformation matrix defined using euler parameters $\beta_i = [\beta_0\ \beta_1\ \beta_2\ \beta_3]$. $\overline{u}_i$ is the local position vector defined with respect to $X_iY_iZ_i$.

$$\overline{u}_i = \overline{u}_i^r + \overline{u}_i^e \tag{2}$$

where $\overline{u}_i^r$ is the undeformed position vector and $\overline{u}_i^e$ is deformation vector which is defined as

$$\overline{u}_i^e = S_i q_i^e \tag{3}$$

In which S_i is the shape function matrix and q_i^e is elastic coordinates vector. Differentiating Eq.(1) with respect to time gives the velocity of an arbitrary point on a flexible link. It is written as

$$\dot{r}_i = \dot{R}_i + A_i(\overline{\omega}_i \times \overline{u}_i) + A_i S_i \dot{q}_i^e \tag{4}$$

where $\overline{\omega}_i$ is angular velocity vector defined in body coordinate system $X_iY_iZ_i$.

3 Dynamic Modeling of Flexible Manipulators

The equations of motion of a flexible manipulator are derived using the principle of virtual work. The dynamics of flexible links is first derived using absolute coordinates for the general purpose implementation. Then the system of equations in absolute coordinates is converted into minimum set of equations or relative coordinate form using recursive kinematic equations.

3.1 Flexible link modeling

The virtual work of total forces acting on flexible link *i* is defined as

$$\delta W_i = \delta W_i^i + \delta W_i{}^s + \delta W_i{}^e \tag{5}$$

where δW_i^i, δW_i^s, and δW_i^e are respectively the virtual work of inertia forces, elastic forces, and external forces. The flexible link *i* is discretized using finite element method to get finite dimensional dynamic model. The representation of finite element *ij* on link *i* is shown in Figure 2. The virtual work of flexible link *i* can be obtained by summing up the virtual work of its elements.

The virtual work of inertia forces acting on element *ij* is written as

$$\delta W_{ij}^i = \int_{V_{ij}} \rho_{ij}\, \ddot{r}_{ij}^T\, \delta r_{ij}\, dV_{ij} \tag{6}$$

where ρ_{ij} and V_{ij} are respectively, the mass density and volume of element *ij*. $\ddot{r}_{ij}$ and δr_{ij} are respectively the acceleration vector and virtual displacements of an arbitrary point on element *ij*. The virtual displacement δr_{ij} is written as

$$\delta r_{ij} = L_{ij} \delta q_{ij} \tag{7}$$

where

$$L_{ij} = \begin{bmatrix} I & -A_i \tilde{\overline{u}}_{ij} \overline{G}_i & A_i S_{ij} \end{bmatrix}, \;\; \delta q_{ij} = \begin{bmatrix} \delta R_{ij} & \delta \beta_{ij} & \delta q_{ij}^e \end{bmatrix}^T \tag{8}$$

where q_{ij} is generalized coordinates of element *ij*. The acceleration vector $\ddot{r}_{ij}$ of an arbitrary point can be obtained by differentiating Eq. 4 with respect to time.

$$\ddot{r}_{ij} = L_{ij} \ddot{q}_{ij} + Q_{ij} \tag{9}$$

in which $\ddot{q}_{ij}$ is the generalized accelerations and Q_{ij} is the quadratic term which is written as

$$Q_{ij} = \ A_i \left(\tilde{\overline{\omega}}_i\right)^2 \overline{u}_{ij} + 2A_i \tilde{\overline{\omega}}_i S_{ij} \dot{q}^e_{ij} \tag{10}$$

Substituting acceleration vector $\ddot{r}_{ij}$ and virtual displacements δr_{ij} in Eq. 6 gives

$$\delta W^i_{ij} = \int_{V_{ij}} \rho_{ij}\ \ddot{q}^T_{ij}\ L^T_{ij}\ L_{ij} \delta q_{ij}\ dV_{ij} + \int_{V_{ij}} \rho_{ij}\ Q^T_{ij}\ L_{ij} \delta q_{ij}\ dV_{ij} \tag{11}$$

$$\delta W^i_{ij} = \left[\ddot{q}^T_{ij}\ M_{ij} - Q^{v^T}_{ij}\right] \delta q_{ij} \tag{12}$$

where M_{ij} and Q^v_{ij} are respectively the inertia matrix and quadratic velocity term.

$$M_{ij} = \int_{V_{ij}} \rho_{ij} \begin{bmatrix} I & -A_i\ \tilde{\overline{u}}_{ij}\ \overline{G}_i & A_i\ S_{ij} \\ & \overline{G}^T_i \tilde{\overline{u}}^T_{ij}\ \tilde{\overline{u}}_{ij} \overline{G}_i & \overline{G}^T_i \tilde{\overline{u}}^T_{ij} S_{ij} \\ symmetric & & S^T_{ij} S_{ij} \end{bmatrix} dV_{ij} \tag{13}$$

and

$$Q^v_{ij} = -\int_{V_{ij}} \rho_{ij} \begin{bmatrix} I \\ -\overline{G}^T_i \tilde{\overline{u}}^T_{ij} A^T_i \\ S^T_{ij} A^T_i \end{bmatrix} Q_{ij}\ dV_{ij} \tag{14}$$

The virtual work of elastic forces due to the deformation of element *ij* can be defined as

$$\delta W^s_{ij} = -\textstyle\int_{V_{ij}} \sigma^T_{ij} \delta \varepsilon_{ij} dV_{ij} \tag{15}$$

where σ_{ij} and ε_{ij} are stress and strain vectors of element *ij*.

$$\varepsilon_{ij} = D_{ij} \overline{u}^e_{ij} = D_{ij} S_{ij} q^e_{ij} \tag{16}$$

$$\sigma_{ij} = E_{ij} \varepsilon_{ij} = E_{ij} D_{ij} S_{ij} q^e_{ij} \tag{17}$$

Substituting Eq. 16 and Eq. 17 in Eq. 15 gives

$$\delta W^s_{ij} = -q^{e^T}_{ij} \left[\textstyle\int_{V_{ij}} (D_{ij} S_{ij})^T E_{ij} D_{ij} S_{ij} dV_{ij}\right] \delta q^e_{ij} = -q^{e^T}_{ij} K^e_{ij} \delta q^e_{ij} \tag{18}$$

where D_{ij} is the differential operator, S_{ij} is the element shape function matrix and E_{ij} is the elastic coefficient. The virtual work of external forces acting on element *ij* is defined as

$$\delta W^e_{ij} = -Q^{e^T}_{ij} \delta q^i \tag{19}$$

Substituting δW^i_{ij}, δW^s_{ij} and δW^e_{ij} in Eq. 5 yields

$$\delta W_{ij} = \left[\ddot{q}^T_{ij}\ M_{ij} - Q^{v^T}_{ij} - q^{e^T}_{ij} K^e_{ij} - Q^{e^T}_{ij}\right] \delta q_{ij} \tag{20}$$

From the Eq. 20 the equations of motion can be rearranged as

$$M_{ij} \ddot{q}_{ij} = Q^e_{ij} + Q^v_{ij} + Q^s_{ij} \tag{21}$$

where Q^e_{ij} are the external forces applied. Q^v_{ij} and Q^s_{ij} are respectively the quadratic velocity term and elastic forces. Eq. 21 can be extended to all finite elements in flexible link and assembled based on element connectivity to form a dynamic model of flexible link.

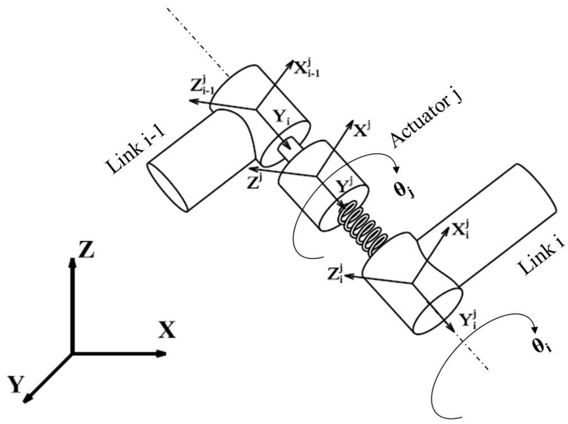

Figure 3: Flexible joint *j* assembly

3.2 Flexible joint modeling

The revolute joint *j* with actuator and transmission system is shown in Figure 3. The actuator is assumed as electric motor and torsional spring represents the flexibility induced due to transmission system. θ_j and θ_i respectively the rotations of actuator *j* and link *i*. The virtual work of torque exserted on link *i* by actuator *j* and transmission system is defined as

$$\delta W_j = (J_j\ddot{\theta}_j + C_j(\dot{\theta}_j - \dot{\theta}_i) + K_j(\theta_j - \theta_i) - T_j)\, \delta\theta_{ij} \tag{22}$$

where J_j is the inertia of the motor.K_j and C_j are the stiffness and damping coefficients of transmission system. T_j is the torque produced by motor. $\delta\theta_{ij}$ is the virtual change at joint. The equations of motions of joint assembly is written as

$$J_j\ddot{\theta}_j + C_j(\dot{\theta}_j - \dot{\theta}_i) + K_j(\theta_j - \theta_i) = T_j \tag{23}$$

3.3 Recursive Kinematic Formulation

Consider two flexible link *i-1* and *i* shown in Figure 4 which are connected by a revolute joint *j*. The joint allows relative rotation along joint axis and has one rigid body coordinates θ_i. The following kinematic relationship for revolute joint holds the relation between generalized

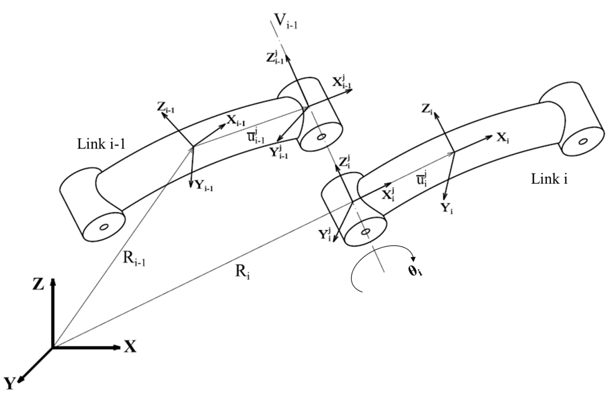

Figure 4: Representation of Relative body Coordinates

coordinates and joint coordinates

$$R_i + A_i\overline{u}_i^j - R_{i-1} - A_{i-1}\overline{u}_{i-1}^j = 0 \tag{24}$$

$$\omega_i = \omega_{i-1} + \omega_{i-1}^j - \omega_i^j + \omega_{i,i-1} \tag{25}$$

where $\overline{u}_i^j$ and $\overline{u}_{i-1}^j$ are local position vectors of joint defined on link *i* and *i-1* respectively. ω_i^j and ω_{i-1}^j are respectively the local angular velocity vectors of joint due to elastic deformations on link *i* and *i-1*. These vector quantities are defined as

$$\omega_{i-1}^j = A_{i-1}S_{i-1}^{jr}\dot{q}_{i-1}^e \tag{26}$$

$$\omega_i^j = A_i S_i^{jr}\dot{q}_i^e \tag{27}$$

In which S_i^{jr} and S_{i-1}^{jr} are respectively the constant shape function matrix of joint rotations due to elastic deformations on link *i* and *i-1*. $\omega_{i,i-1}$ is relative angular velocity vector of link *i* with respect to link *i-1* is expressed as

$$\omega_{i,i-1} = \nu_{i-1}\dot{\theta}_i \tag{28}$$

where ν_{i-1} is rotation axis defined with respect to link *i-1* in global coordinate system XYZ.

$$\nu_{i-1} = A_{i-1}\overline{\nu}_{i-1} \tag{29}$$

$\overline{\nu}_{i-1}$ is constant vector defined with respect to link *i-1* in body coordinate system $X_{i-1}Y_{i-1}Z_{i-1}$. Differentiating Eq. 24 twice with respect to time and Eq. 25 once with respect to time gives

$$\ddot{R}_i - A_i\tilde{\overline{u}}_i^j\overline{G}_i\ddot{\beta}_i + A_i S_i^{jt}\ddot{q}_i^e = \ddot{R}_{i-1} - A_{i-1}\tilde{\overline{u}}_{i-1}^j\overline{G}_{i-1}\ddot{\beta}_{i-1} + A_{i-1}S_{i-1}^{jt}\ddot{q}_{i-1}^e + \gamma_R \tag{30}$$

$$\dot{\omega}_i = \dot{\omega}_{i-1} + A_{i-1}S_{i-1}^{jr}\ddot{q}_{i-1}^e - A_i S_i^{jr}\ddot{q}_i^e + A_{i-1}\overline{\nu}_{i-1}\ddot{\theta}_i + \gamma_\beta \tag{31}$$

where S_i^{jt} and S_{i-1}^{jt} are respectively the shape functions of joint translations defined on link *i* and *i-1*. γ_R and γ_β are written as

$$\gamma_R = -A_i\left(\tilde{\overline{\omega}}_i\right)^2\overline{u}_i^j + A_{i-1}\left(\tilde{\overline{\omega}}_{i-1}\right)^2\overline{u}_{i-1}^j - 2A_i\tilde{\overline{\omega}}_i S_i^{jt}\dot{q}_i^e + 2A_{i-1}\tilde{\overline{\omega}}_{i-1}S_{i-1}^{jt}\dot{q}_{i-1}^e \tag{32}$$

$$\gamma_\beta = A_{i-1}\tilde{\overline{\omega}}_{i-1}\overline{\nu}_{i-1}\dot{\theta}_i + A_{i-1}\tilde{\overline{\omega}}_{i-1}S_{i-1}^{jr}\dot{q}_{i-1}^e - A_i\tilde{\overline{\omega}}_i S_i^{jr}\dot{q}_i^e \tag{33}$$

The equation Eq. 30 and Eq. 31 can be written in a compact form as

$$D_i\ddot{q}_i = D_{i-1}\ddot{q}_{i-1} + H_i\ddot{P}_i + \gamma_i \tag{34}$$

where

$$D_i = \begin{bmatrix} I & -A_i\,\tilde{\overline{u}}_i^j\,\overline{G}_i & A_i S_i^{jt} \\ 0 & A_i\overline{G}_i & A_i S_i^{jr} \\ 0 & 0 & I \end{bmatrix} \tag{35}$$

$$D_{i-1} = \begin{bmatrix} I & -A_{i-1}\tilde{\overline{u}}^{j}_{i-1}\overline{G}_{i-1} & A_{i-1}S^{jt}_{i-1} \\ 0 & A_{i-1}\overline{G}_{i-1} & A_{i-1}S^{jr}_{i-1} \\ 0 & 0 & 0 \end{bmatrix} \tag{36}$$

$$H_i = \begin{bmatrix} A_{i-1}\overline{\nu}_{i-1} & 0 \\ 0 & I \end{bmatrix} \tag{37}$$

$$\ddot{P}_i = \begin{bmatrix} \ddot{\theta}_i & \ddot{q}^e_i \end{bmatrix}^T \tag{38}$$

$$\gamma^i = \begin{bmatrix} \gamma_R & \gamma_\beta \end{bmatrix}^T \tag{39}$$

The generalization of recursive formulation for a n link manipulator is expressed as

$$\begin{bmatrix} D_1 & 0 & \cdots & 0 \\ -D_1 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & -D_{n-1} & D_n \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix} = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & H_n \end{bmatrix} \begin{bmatrix} \ddot{P}_1 \\ \ddot{P}_2 \\ \vdots \\ \ddot{P}_n \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix} \tag{40}$$

The generalized accelerations $\ddot{q}_i$ in absolute coordinates can be expressed in terms of relative coordinates as

$$\ddot{q}_i = B_i\ddot{P}_i + \gamma_i \tag{41}$$

where

$$B_i = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ -D_1 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & -D_{n-1} & D_n \end{bmatrix}^{-1} \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & H_n \end{bmatrix} \tag{42}$$

And

$$\gamma_i = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ -D_1 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & -D_{n-1} & D_n \end{bmatrix}^{-1} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix} \tag{43}$$

Substituting the generalized acceleration $\ddot{q}_i$ in Eq. 21 and premultiplying with B_i^T gives the dynamic model of flexible links in relative coordinates form.

$$B_i^T M_i B_i \ddot{P}_i = B_i^T (Q^e_i + Q^v_i + Q^s_i - M_i\gamma_i) \tag{44}$$

which can written as

$$\bar{M}_i\ddot{P}_i = \bar{Q}_i \tag{45}$$

where

$$\bar{M}_i = B_i^T M_i B_i, \quad \bar{Q}_i = B_i^T (Q^e_i + Q^v_i + Q^s_i - M_i\gamma_i) \tag{46}$$

The Eq. 45 along with Eq. 23 provides a coupled and nonlinear dynamic model of flexible link and flexible joint which can be used for simulation and model based control design purpose.

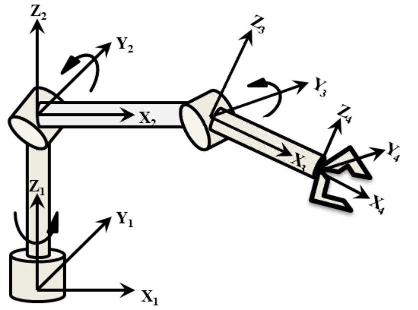

Figure 5: Spatial RRR flexible manipulator.

4 Numerical Simulation

A spatial RRR manipulator shown in Figure 5, is considered to demonstrate the effect of link and joint flexibility on manipulator dynamics. Each flexible link is discretized using two finite element beams with six degrees of freedom on each node and one degree of freedom for rigid body rotations i.e. θ_i where $i = 1, 2, 3$. Manipulator joint has one rigid body rotation i.e. θ_j where $j = 1, 2, 3$. The torsional stiffness K_j for $j = 1, 2, 3$ at manipulator joints is defined as 5000 Nm/rad. The damping effects on links and joints are ignored. The physical parameters of a RRR spatial manipulator is presented in Table 1. Uniform cross-section and material properties are assumed on each link. The numerical simulation for different cases considering rigid links and rigid joints, flexible links and rigid joints, flexible links and flexible joints is performed to study the effect of flexibility on manipulator dynamics. A constant torque of 400 Nm, is applied at manipulator joints for each case to compare manipulator motion. The deformations of manipulator endeffector $X_4Y_4Z_4$ along the X,Y and Z direction is shown in Figure 6, Figure 7 and Figure 8 respectively. It shows the deformations of flexible manipulator endeffector with respect to the rigid manipulator endeffector motion. The deformation of flexible manipulator joint θ_i with respect to rigid manipulator joint motion is shown in Figure 9, Figure 10 and Figure 11 respectively. The numerical simulation results show significant effect of link and joint flexibility on the overall manipulator motion. In addition, the joint flexibility in manipulator can significantly alter the motion of the manipulator.

Parameter	Link 1	Link 2	Link 3
Link Length (m)	1	4.0	3.5
C/s Area (m^2)	0.028	0.0020	0.0008
Moment of Inertia (m^4)	8.33×10^{-7}	6.24×10^{-7}	5.37×10^{-7}
Polar moment of Inertia (m^4)	1.66×10^{-6}	1.24×10^{-6}	1.07×10^{-6}
Tensile Modulus (MPa)	206000		
Shear Modulus (MPa)	79300		
Density (Kg/m^3)	8253		

Table 1: The Physical Parameters of a RRR flexible manipulator.

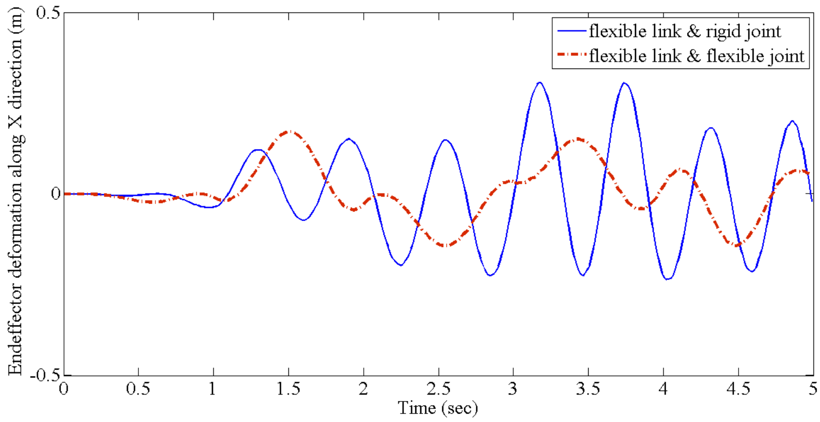

Figure 6: Deformation of flexible manipulator endeffector with respect to rigid manipulator endeffector motion in X-direction.

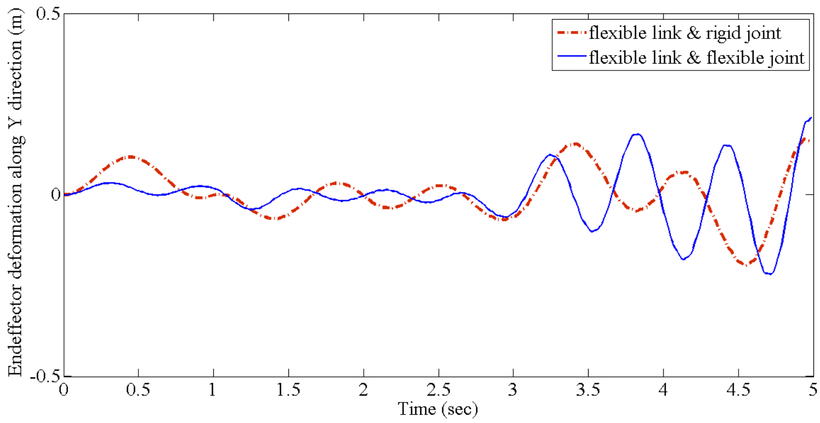

Figure 7: Deformation of flexible manipulator endeffector with respect to rigid manipulator endeffector motion in Y-direction.

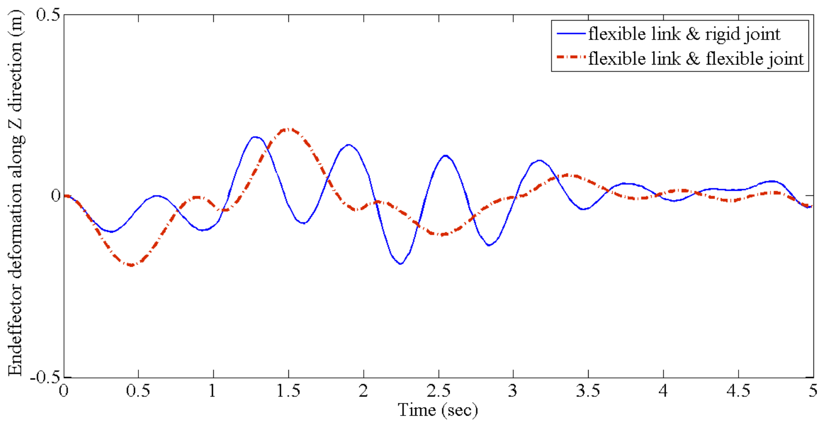

Figure 8: Deformation of flexible manipulator endeffector with respect to rigid manipulator endeffector motion in Z-direction.

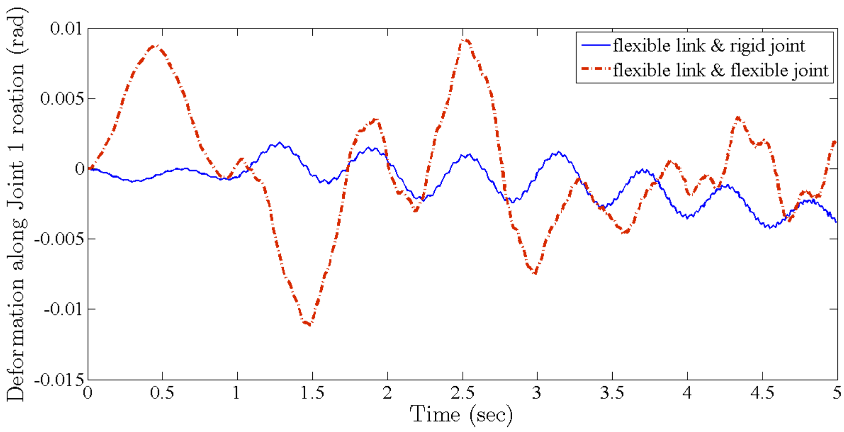

Figure 9: Deformations of flexible manipulator joint θ_1 with respect to rigid manipulator joint motion.

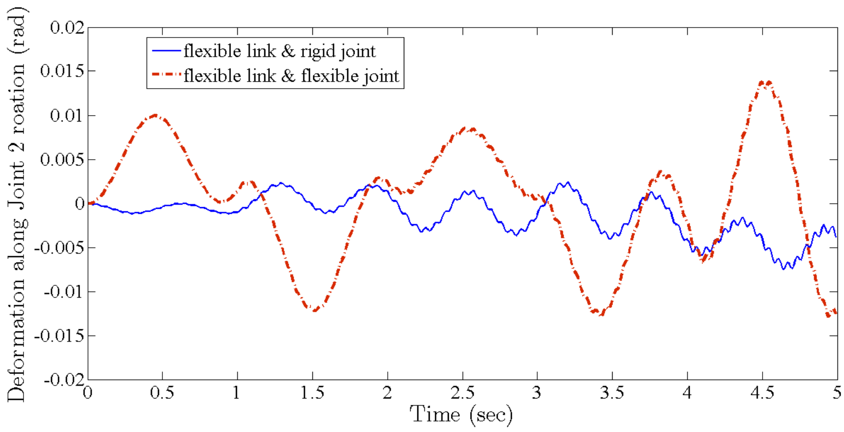

Figure 10: Deformations of flexible manipulator joint θ_2 with respect to rigid manipulator joint motion.

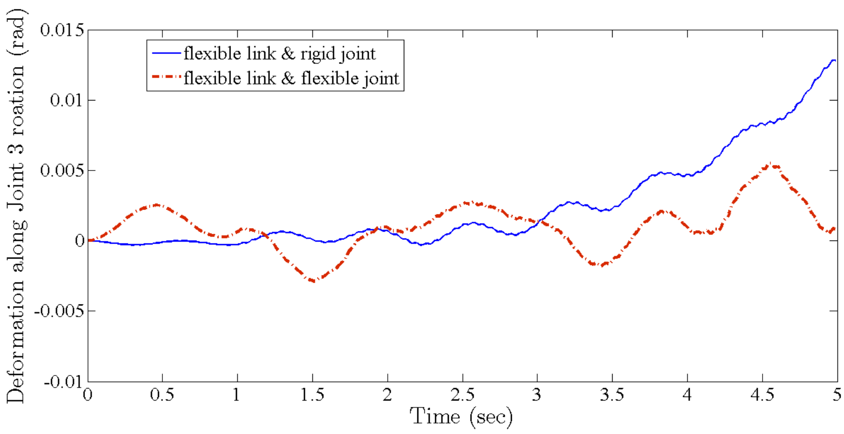

Figure 11: Deformations of flexible manipulator joint θ_3 with respect to rigid manipulator joint motion.

5 Conclusions

The light weight manipulators have many advantages over rigid manipulators. However, the low stiffness of links and in some cases the transmission systems at manipulator joints can introduce flexibility in the light weight manipulators. The effect of link and joint flexibility on manipulator dynamics is studied using a spatial RRR manipulator. The numerical simulation of different cases considering rigid links and rigid joints, flexible links and rigid joints, flexible links and flexible joints is performed. The simulation results show significant effect of flexibility on the overall manipulator motion. An accurate dynamic model that incorporates both link and joint flexibility is mandatory for model based control design. A general purpose algorithm that allows to obtain an accurate dynamic model of spatial manipulators that includes both link and joint flexibility is developed. The algorithm is developed using the principle of virtual work along with finite element method, and recursive kinematic formulation. The advantage of general purpose algorithm is that it uses minimum set of equations that define the dynamics of flexible manipulator, which is necessary for control design to reduce computational cost. In addition, it also allows the dynamic modeling of any arbitrary manipulator configuration with revolute joints.

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