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ABOUT VIBRATIONS AND STABILITY OF BORING RODS OF SHALLOW DRILLING IN VIEW OF GEOMETRICAL NONLINEARITY

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Abstract. *The paper is devoted to studying of nonlinear vibrating processes and problems of their stability in elastic rod systems and to their practical application. Nonlinear vibrations of boring rods of shallow drilling, applied in the petroleum industry, are investigated. Nonlinearity of the geometrical nature is considered. Here nonlinear vibrations of compressed - braided drill rods are investigated. Unlike the majority of well-known linear models of movement of drill rods, restrictions on sizes of elastic deformations of a rod are removed, and we suppose them to be finite. For construction of a model the V.V. Novozhilov theory of geometrically nonlinear mediums is used. The boring rod is modeled as a simply supported girder with non-approaching ends. Numerical modeling of vibratory process of the boring rod is carried out. Stability of movement of the boring rod with detection of frequencies of resonance on the basic mode and on multiple frequencies is investigated. The G. Floquet theory is used. As the equations appear to be connected through nonlinear members, excitation of one form of vibrations can lead to transfer of part of its energy to other originally unexcited forms and cause vibrations in these forms. Therefore here the behavior and stability of the not only excited form of vibrations of the boring rod are considered, but also of other forms conditioned by geometrical nonlinearity of models. Influence of the initial curvature of the boring rod on its oscillations is considered. Dynamics of the boring rod which has lost static stability is investigated.*

1 INTRODUCTION

The work is dedicated to applied problems of dynamics of nonlinear deformable media. Movement of chisel bars of shallow drilling (up to 500 м), applied in oil and gas extraction industry (Figure 1) is considered.

The branch of oil-and-gas equipment is known to be dynamically developing. The speed of oiler drilling and crude output depends on quality and perfection of the drilling unit. The operating mode of the boring machine in full measure depends on stability and strength of drill rods.

Besides, loss of stability of the rectilinear shaped bar causes curvature of the borehole, which results in unfitness. Numerous factors are the reason for that - dynamic cross-section impact; variable axial forces and twisting moments; large inertial forces arising at drilling; the initial curvature of the bar; concentrators of pressure, etc.

The analysis of works on dynamics of chisel bars shows, that research is conducted by authors in three mainstreams:

· Perfection of dynamic models of chisel bars, which take into account most generally every possible of factors, which reflect real conditions of operation of designs.

· Perfection of analytical and numerical methods of calculation, which allow to receive the solution of complex and nonlinear problems of the dynamic analysis of chisel bars with necessary in practice accuracy.

· Creation of new experimental-and-theoretical methods, which allow to carry out research and calculation of designs such as chisel bars in view of casual character of physical-and-technical and operational conditions of their work.

The present paper is dedicated to modeling of vibrations of chisel bars of shallow drilling. In view of variety of complicating of rotation of the chisel column factors the nominal movement of the chisel column is not obviously possible. In the chisel bar complex dynamic processes arise, modeling and analysis of which represents difficulties. Therefore, it is more expedient to consider the most typical modes of weighting and deformation of the chisel columns (their longitudinal and torsional [1-3, etc.], flexural [4-7, etc.], flexural-and-torsional and other vibrations), as components of complex movement.

The majority of models of movement of chisel bars are linear in view of restrictions of quantities of their deformations, or the nonlinear model linearizes, as in work [7]. The present restriction can affect the results of research. It is known, that under the action of variable external forces and moments, large inertial forces, etc. deformations of chisel bars can be final, causing nonlinearity of the dynamic model of geometrical nature.

In the present paper movement and stability of vibrations of chisel bars in the assumption of finiteness of deformations of the chisel bar are modeled.

2 MODEL OF MOVEMENT OF THE CHISEL BAR IN VIEW OF INITIAL CURVATURE

One of the reasons for curvature of borehole is static (according to Euler) or dynamic curvature of the rectilinear shaped chisel bar [8-9, etc.] due to its initial curvature. It causes vibrations of the chisel bar and destruction of the borehole.

Here vibrations of the chisel bar with initial curvature are modeled. The bar is considered as a freely-supported beam with not approaching ends and lost static stability, as in work [10]. The equations of cross-section vibrations are set by the equation of the kind:

$$EI\frac{\partial^4 w}{\partial x^4} - \frac{Eh}{l}\left\{V_0 + \frac{1}{2}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx\right\}\frac{\partial^2 w}{\partial x^2} + \rho h\frac{\partial^2 w}{\partial t^2} = F(x,t), \quad (1)$$

where V_0 - initial displacement of the chisel bar from unstrained position of its longitudinal axis, caused by the initial longitudinal exertion; w - deflection of the chisel bar; E- module of elasticity; h- thickness of the beam; I - moment of inertia of cross-section; ρ - density of the material of the bar; l - length of the bar $F(x,t)$ - cross-section dynamic loading on the bar. Nonlinearity of model (Eq.1) is caused by influence of deflection on longitudinal exertion, which arises for the lack of displacement of the support of the chisel bar.

Quasianalytic research of dynamics of the chisel bar has been carried out. Bubnov-Galerkin's method is used. The solution of the model is defined as decomposition under the basic forms of vibrations:

$$w(x,t)=\sum_{n=1}^{m} A_n(t)\Phi_n(x), \qquad (2)$$

where $\Phi_n(x)=\sin n\pi x$ - the main forms of vibrations of the bar, which satisfy its geometrical boundary conditions.

As the equations appear connected through nonlinear members, excitation of one form of vibrations can bring to downloading of part of its energy in other originally not excited forms and cause vibrations in these forms.

Thus, here not only the behavior of the excited form of vibrations, but also the behavior of other forms is considered.

In dimensionless quantities: $\xi_n=\frac{A_n}{l}$, $\vec{x}=\frac{x}{l}$, $\tau=\left(\frac{E}{\rho}\right)^{\frac{1}{2}}\frac{t}{l}$, $\alpha=\frac{h}{l}$, $\lambda=\frac{V_0}{V_{0k}}$, Eq. (1) results in the following :

$$\xi_n(\tau)+p_n\xi_n(t)+q_n\left\{\sum_{j=1}^{m} j^2\xi_j^2(\tau)\right\}\xi_n(\tau)=Q_n, \qquad n=\overline{1,m.}, \qquad (3)$$

Here $p_n=\frac{l^2 I}{h}(\pi n)^4+(\pi n)^2 V_0 l$, $q_m=\frac{\pi^4 n^4}{4}l^3$, $Q_n=\frac{2}{Eh}\int_0^1 F(\bar{x},\tau)\sin n\pi\bar{x}d\bar{x}$.

Considering a special case of harmonious perturbation

$Q_n=Q_i=B_i\cos\omega t$ for $n=i$; $Q_n=0$ for $n\neq i$,

the numerical analysis of nonlinear vibrations of the chisel bar is carried out. The amplitude of perturbing force is set as $B_i=5\times i\times 10^{-6}$. Frequency of vibrations is $\omega=\sqrt{P_n}$. Initial curvature of the bar is considered equal $V_0=a\sin\pi x;$, where x - coordinate of section, a =0,003. Entry conditions are set:

$$\xi\big|_{\tau=0}=0, \quad \left.\frac{d\xi}{d\tau}\right|_{\tau=0}=0,5, \qquad (4)$$

As a result of the research the following is established: the initial curvature of the chisel bar renders an essential impact on the amplitude of vibrations (Figure 2); duralumin chisel bars have the amplitude of vibrations smaller than steel ones (Figure 3); the amplitude of vibrations of the duralumin chisel bar is smaller than the amplitude of vibrations of the steel bar (Figure 4). As a test a linearized model is considered (Figure 5). ($E_{dur}=0{,}7\cdot10^5$ MPa, $\rho_{dur}=2{,}7\cdot10^3$ $кг/м^3$, $E_{st}=2{,}1\cdot10^5$ MPa, $\rho_{st}=7{,}8\cdot10^3$ $кг/м^3$, $D=0{,}200$ $м$, $d=0{,}140 м$).

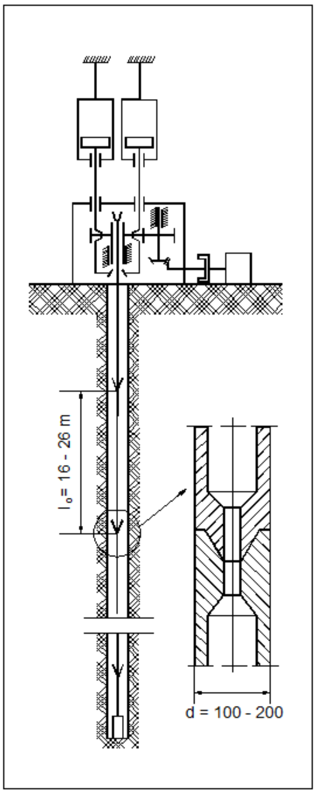


Figure 1: The kinematic circuit of the boring machine.

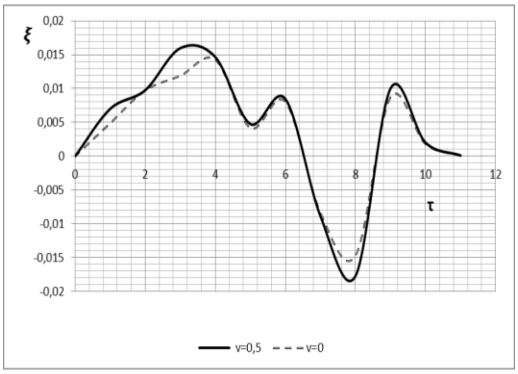


Figure 3: Vibrations of the steel rod with Initial curvature and without it.

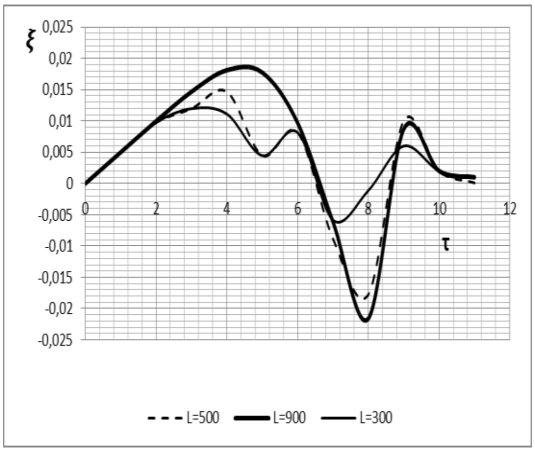


Figure 4: Influence of length of the steel rod on its amplitude.

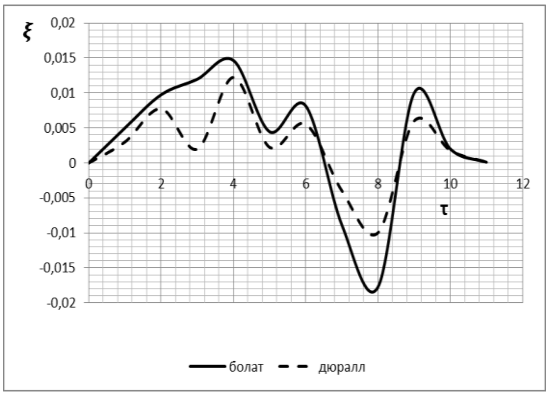


Figure 2: Vibrations of the ___steel and _ _ _duralumin rods (L=500 m).

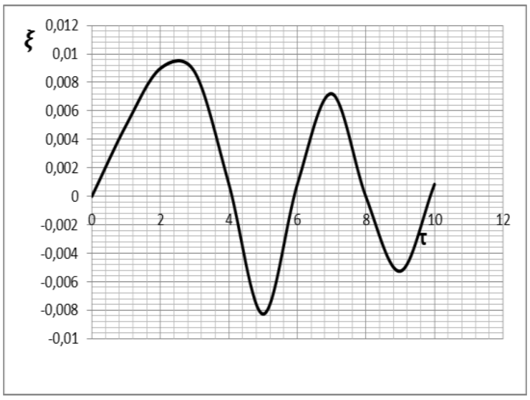


Figure 5: Linear vibrations of the drill rod

3 MODELLING OF LONGITUDINAL - CROSS-SECTION VIBRATIONS OF THE CHISEL COLUMN

A case of a flat bend of the chisel bar is considered under the action of variable axial compressing loading $P(t) = P_0 + P_t \Phi(t)$. Makivor-Bernard's model of movement of the core in plane x - y (Figure 6) [11] is applied.

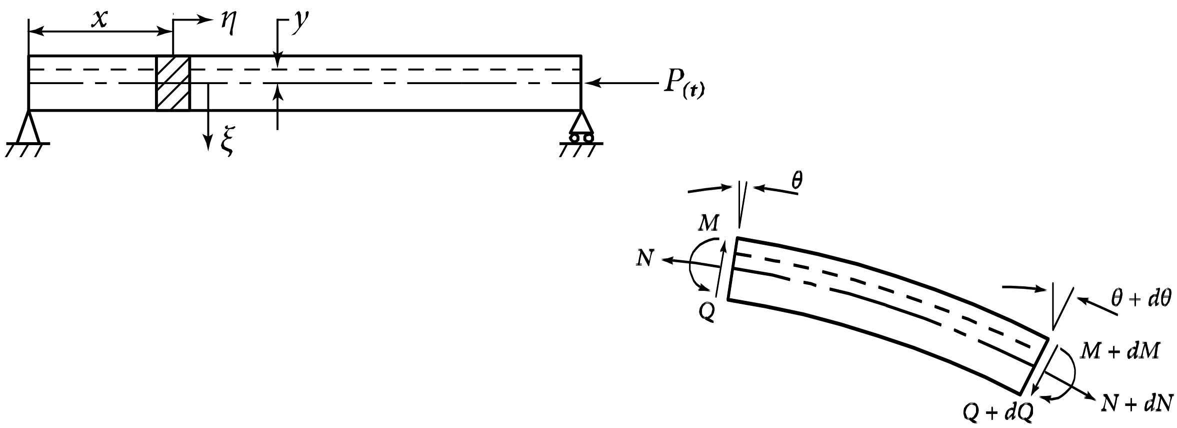


Figure 6: Geometry of the core and its element at the moment of time t.

The authors of paper [11] neglect rotary inertia. They are limited to the analysis of small deformations, but take into account the effect of rotation on axial deformation. That is pre-conditions for the elementary nonlinear theory, which takes into account mutual influence of cross-section and longitudinal movements. In this case it is possible to replace $\cos\theta$ with one, and quantities θ and $\sin\theta$ - with quantity $\frac{\partial \xi}{\partial x}$ in the equations of movement. Then curvature and axial deformation are written as:

$$k = \frac{\partial^2 \xi}{\partial x^2}, \quad \varepsilon_\alpha = \frac{\partial \eta}{\partial x} + \frac{1}{2}\left(\frac{\partial \xi}{\partial x}\right)^2, \tag{5}$$

accordingly. Dissipative properties of the material are modeled, proceeding from the assumption that the material behaves like Calvin's body, submitting to the equation for single-axis stressed state [11]:

$$\sigma = E\varepsilon + \mu \frac{\partial \varepsilon}{\partial t}. \tag{6}$$

According to Eqs. (5) and (6), it is possible to set axial loading and bending moment as:

$$N = EA\left(\varepsilon_\alpha + \frac{\mu}{E}\frac{\partial \varepsilon_\alpha}{\partial t}\right), \qquad M = EI\left(k + \frac{\mu}{E}\frac{\partial k}{\partial t}\right). \tag{7}$$

The specified model is considered with reference to the vertical chisel core with addition of the linear body weight. In this case movement of the element of the vertical core is described by the following equations:

$$\begin{aligned} &\rho\frac{\partial^2 \eta}{\partial t^2} = -N\frac{\partial \theta}{\partial x}\sin\theta + \frac{\partial N}{\partial x}\cos\theta - Q\frac{\partial \theta}{\partial x}\cos\theta - \frac{\partial Q}{\partial x}\sin\theta - \frac{\rho g\cos\theta}{L}, \\ &\rho\frac{\partial^2 \xi}{\partial t^2} = N\frac{\partial \theta}{\partial x}\cos\theta + \frac{\partial N}{\partial x}\sin\theta - Q\frac{\partial \theta}{\partial x}\sin\theta + \frac{\partial Q}{\partial x}\cos\theta - \frac{\rho g\cos\theta}{L}, \\ &\frac{\partial M}{\partial x} = Q = 0, \end{aligned} \tag{8}$$

Introducing dimensionless quantities:

$$u = \frac{\eta}{L},\ v = \frac{\xi}{L},\ s = \frac{x}{L},\ \tau = \frac{aL}{t},\ \alpha^2 = \frac{r^2}{L^2},\ \beta^2 = \frac{A\mu}{\rho La},\ \gamma = \frac{\rho gL}{EA},$$

where $a^2 = EA/\rho,\ r^2 = 1/A$, Eq. (8) are led to a kind:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial \tau^2} - \frac{\partial}{\partial s}\left[\frac{\partial u}{\partial s} + \frac{1}{2}\left(\frac{\partial v}{\partial s}\right)^2 + \beta\left(\frac{\partial^2 u}{\partial \tau \partial s} + \frac{\partial v}{\partial s}\frac{\partial^2 v}{\partial \tau \partial s}\right)\right] ts\gamma = 0,\\
&\frac{\partial^2 v}{\partial \tau^2} + \alpha^2\left(\frac{\partial^4 v}{\partial s^4} + \beta^2 \frac{\partial^5 v}{\partial \tau \partial s^4}\right) - \frac{\partial}{\partial s}\left[\left(\frac{\partial u}{\partial s} + \beta^2 \frac{\partial^2 u}{\partial \tau \partial s}\right)\frac{\partial v}{\partial s}\right] tv\gamma = 0.
\end{aligned} \tag{9}$$

For the model of elastic deformation of the chisel bar (Figure 6) the corresponding boundary conditions are set:

$$\begin{aligned}
&u(1,\tau) = 0;\ \ v(0,\tau) = 0;\ \ v(1,\tau) = 0,\\
&\bar{M}(0,\tau) = 0;\ \ \bar{M}(1,\tau) = 0;\ \ \bar{N}(0,\tau) = -\frac{-P(\tau)}{EA},
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
&\bar{M} = \frac{ML}{EI} = \frac{\partial^2 v}{\partial s^2} + \beta^2 \frac{\partial^3 v}{\partial \tau \partial s^2};\\
&\bar{N} = \frac{N}{EA} = \frac{\partial u}{\partial s} + \frac{1}{2}\left(\frac{\partial v}{\partial s}\right)^2 + \beta\left(\frac{\partial^2 u}{\partial \tau \partial s} + \frac{\partial v}{\partial \tau}\frac{\partial^2 v}{\partial \tau \partial s}\right).
\end{aligned} \tag{11}$$

Production of entry conditions will close the model of longitudinal - cross-section vibrations of the chisel bar.

As well as in [11], Bubnov-Galerkin's method is applied. Eq. (9) result in an infinite system of ordinary differential equations:

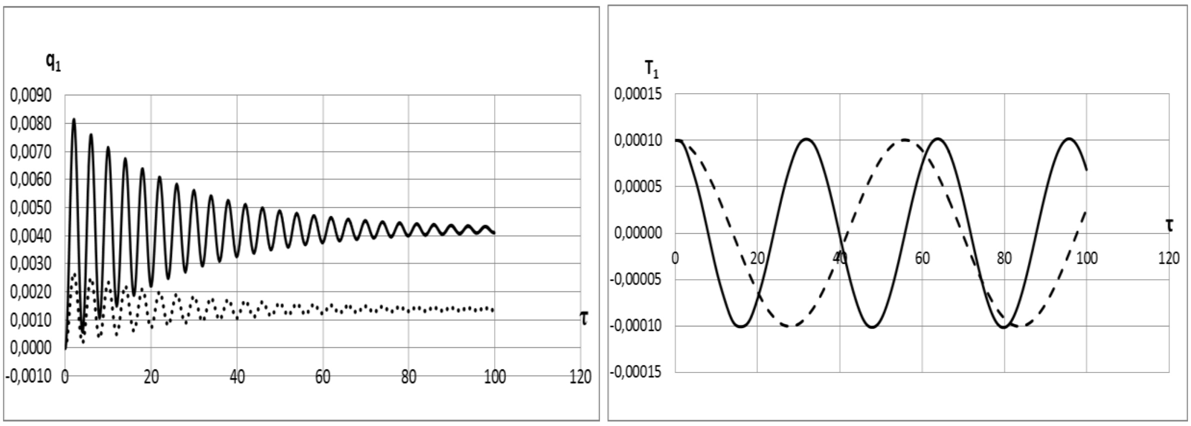
$$\begin{aligned}
&\int_0^1 \left\{\ddot{u}\delta u + \left[u' + \frac{1}{2}v'^2 + \beta\left(\dot{u}' + v'\dot{v}'\right)\right]\delta u' + tjs\delta u\right\} ds - \frac{P(\tau)}{EA}\delta u(0,\tau) = 0,\\
&\int_0^1 \left\{\ddot{v}\delta v + a^2(v'' + \beta^2\dot{v}'')\delta v'' + \left[(u' + \beta^2\dot{u}')\dot{v}'\right]\delta v' + tjsv'\delta u\right\} ds = 0,
\end{aligned} \tag{12}$$

where satisfying to geometrical conditions u and v are set as follows:

$$\begin{aligned}
&u = \sum q_j \cos\left(\frac{j\pi s}{2}\right),\ \ j = 1,3,5...,\\
&v = \sum T_j \cos\left(m\pi s\right),\ \ m = 1,2,3...,
\end{aligned} \tag{13}$$

Dots designate differentiation on τ , and strokes - on s. In Figure 7 numerical calculations of longitudinal (a) and cross-section (b) vibrations of the chisel bar on the first form of the bend of the bar axis are submitted. Calculations are carried out for: $D = 0{,}14m$, $d = 0{,}12m$,

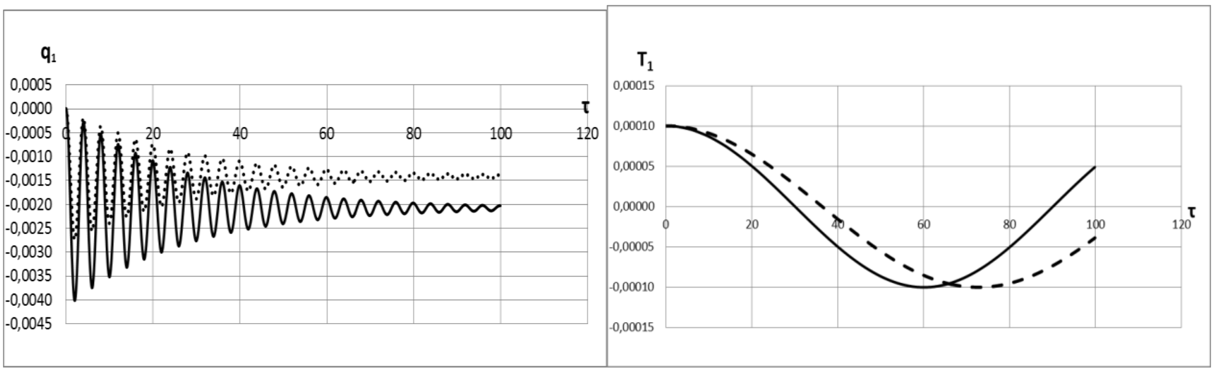
$P(\tau) = 2,2 \cdot 10^6 N, \ \ \mu = 6,07083 \cdot 10^8, \ \ L = 500m$.



a) b)

Figure 7: Impact of the material of the chisel bar on longitudinal (a) and cross-section components (б) of its deformations (_ _ _ Steel**,** ____Duralumin).

It is established, that change of geometrical and physical parameters of the researched system, and also end loadings **P** (Figure 8) renders a greater impact on longitudinal vibrations than on cross-section ones.



a) b)

Figure 8: Impact of end loading of the chisel bar on longitudinal (a) and cross-section component (б) of its deformations (_ _ _ P= const, , ___ P(τ)).

Thus, the quantity of variable end loading P (τ) is taken smaller than constant loading $P = 2,2 \cdot 10^6 N$.

4 MODELING OF VIBRATIONS AND STABILITY OF CHISEL BAR IN VIEW OF FINITE DEFORMATIONS

Nonlinear vibrations of rotating compressed - twisted chisel bars are investigated. The diagram of elastic deformation of the bar is submitted in Figure 9 [12]. Unlike the majority of known linear models of movement of chisel bars, restrictions on quantities of elastic deformations of the bar are removed. We believe them to be finite. For construction of the model V.V. Novozhilov [13] theory of geometrically nonlinear media is used. As well as in work [14], the chisel bar is modeled as a freely supported beam with non-approaching ends. According to the diagram of deformation the nonlinear dynamic model of rotation of the chisel bar is constructed:

$$
\begin{aligned}
&EJ_V \frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 V}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial V}{\partial x}\right)^2\right)\right]+\frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial U}{\partial \mathrm{x}}\right]+\frac{\partial}{\partial x}\left[\mathrm{N}(\mathrm{x},\mathrm{t})\frac{\partial \mathrm{V}}{\partial \mathrm{x}}\right]+K_1 V=-\frac{\gamma F}{g}\frac{\partial^2 V}{\partial t^2},\\
&EJ_U \frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 U}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial U}{\partial x}\right)^2\right)\right]+\frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial V}{\partial \mathrm{x}}\right]+\frac{\partial}{\partial x}\left[\mathrm{N}(\mathrm{x},\mathrm{t})\frac{\partial U}{\partial \mathrm{x}}\right]+K_1 U=-\frac{\gamma F}{g}\frac{\partial^2 U}{\partial t^2},
\end{aligned}
\tag{14}
$$

where N (t), M (t) - variable external influences: axial force and the twisting moment, accordingly; ω - angular velocity of the bar rotation; $K_1=\gamma F\omega^2/g$, U and V are motions on elastic line of a column in planes XOY and XOZ, accordingly. The boundary conditions in case of hinge leaning columns are set as equality to zero of motions and the bending moment on the ends:

$$V=EJ_v\frac{\partial^2 V}{\partial x^2}=0,\quad U=EJ_u\frac{\partial^2 U}{\partial x^2}=0,\qquad (x=0,\ x=l) \tag{15}$$

Numerical experiment is carried out. The steel chisel bar under the action of variable axial forces $N(t)=N_0+N_t\cdot\sin(\omega\cdot t)$ and moments $M(t)=M_0+M_t\cdot\sin(\omega\cdot t)$ is considered. Model (14) - (15) is solved with the method of division of Bubnov-Galerkin variables. Using a package of symbolical mathematics Wolfram Mathematica (WM), a comparative analysis of linear and nonlinear models at $M_0=7\,\kappa Nm$; $M_t=3\,\kappa Nm$; $N_0=60\,\kappa N$; $N_t=2140\,\kappa N$; D=0,2 m; d=0,12 m ; $\omega=2\,c^{-1}$ is carried out. It is established, that the linear model of the steel bar overestimate results (Figure 10).

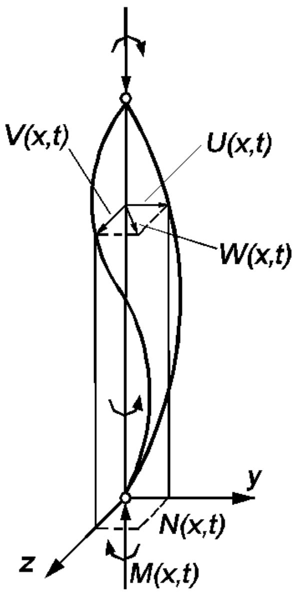


Figure 9: The form of the curved axis of a rod.

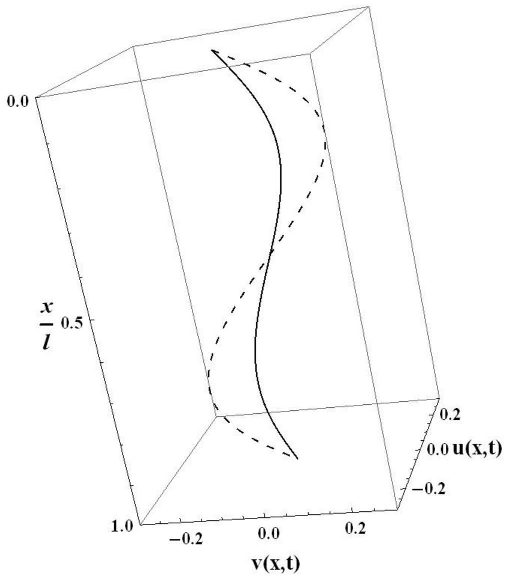


Figure 10: Comparative analysis of deformations of the bar _ _ _linear model, ___ nonlinear model).

Stability of movement of the chisel bar is researched. As well as in work [15], steady movement of the chisel bar is understood as its movement in absence of resonance. It is known, that in case of static or dynamic loss of stability of cores the cross-section shifts of their axis represent greater interest than longitudinal ones. In this connection a special case of model Eq. (14) - a case of a flat curving of the rod rotating at speed ω under the action of longitudinal force $N(t) = N_0 + N_t \Phi(t)$ is considered. Setting the solution as $U(x,t) = \sum_{k=1}^{\infty} f_k(t) \sin \frac{k\pi x}{l}$, with Bubnov method the multivariate model results in the system of parametrical equations with one degree of freedom:

$$\ddot{f} + C_k^2 (1 - 2\nu_k \cos \Omega t) f + \alpha f^3 = 0, \tag{16}$$

where

$$C_k = \frac{k^2 \pi^2}{l^2} \sqrt{\frac{EI}{m} \left(1 - \frac{N_0}{N_k}\right)} = \omega_0, \;\; N_k = \frac{k^2 \pi^2 EI}{l^2}, \;\; \nu_k = \frac{N_t}{2(N_k - N_0)}, \;\; \alpha = \frac{3Ek^4 \pi^4}{8\rho l^4} \tag{17}$$

Equations of rod movement are made up in view of proper forms of vibrations. As the equations appear to be connected through nonlinear members, excitation of one form of vibrations can lead to transfer of part of its energy to other originally unexcited forms and cause vibrations in these forms. Therefore, here the behavior and stability of not only excited form of vibrations of the boring rod are considered, but also of other forms conditioned by geometrical nonlinearity of models. The Floquet theory for delimitation of areas of instability of vibrations on resonant frequencies [16] is used.

For this purpose resonance on the basic frequency is considered:

$$f_0 = r_1 \cos(\Omega t - \varphi_1) \tag{18}$$

Small increment $f = f_0 + \delta f$ is set, equation of the perturbed state of Hill type is received:

$$\frac{d^2 \delta f}{dt^2} + \delta f \left[C_k^2 + 1{,}5\alpha r_1^2 - 2C_k^2 \nu \cos \Omega t + 1{,}5\alpha r_1^2 \cos 2\varphi_1 \cos 2\Omega t + 1{,}5\alpha r_1^2 \sin 2\varphi_1 \sin 2\Omega t \right] = 0$$

According to Floquet theory characteristic determinant is constructed, which specifies boundaries of zones of instability of the basic resonance. As a result of the numerical analysis of zones of instability of fluctuations of the chisel bar the following is established: zones of instability are extended to the right in comparison with a linear case; they increase with growth of the bar length. Besides, in case of duralumin bars they are wider than for steel bars.

5 CONCLUSIONS

It is established, that nonlinearity of dynamic models introduces essential amendments to the results of the dynamic analysis of chisel bars.

- Initial curvature of chisel bars causes growth of amplitude of their fluctuations.
- At a rating of shifts of linear models overestimate of results is observed in comparison with solutions of nonlinear models which take into account finite deformations.
- Geometrical nonlinearity of models causes displacement of zones of instability into the zone of large frequencies. That indicates that it is necessary to expect the basic resonance and the resonance on the maximum frequencies later than under forecasts of the analysis of linear models.

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