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FAULT DETECTION AND ISOLATION FOR A NONLINEAR RAILWAY VEHICLE SUSPENSION SYSTEM

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Abstract. *Reliability and dependability in complex mechanical systems can be improved by fault detection and isolation methods (FDI). These techniques are key elements for maintenance on demand, which could decrease service cost and time significantly. This paper addresses FDI for a railway vehicle: The mechanical model is described as a multibody system, which is excited randomly due to the wheel-rail interaction. Various parameters, like masses, spring- and damper-characteristics, influence the nonlinear dynamics of the vehicle. Often, the exact values of the parameters are unknown and might even change over time. Some of these changes are considered critical with respect to the operation of the system and they require immediate maintenance. The aim of this work is to detect faults in the nonlinear suspension system of the vehicle. An optimal and robust filter is used in order to estimate the states. The main focus lies on the parameters describing the dynamics of the wheelset. The paper describes several steps in the improvement of a FDI process. Linear and nonlinear residual generators are applied for the detection and isolation of faults in the system. A full train model with multiple nonlinear components serves as an example for the described techniques. Numerical results for different test cases are presented. The analysis shows that for the given system it is possible not only to detect a failure of the suspension system from the system's dynamic response, but also to distinguish clearly between different possible sources for the changes in the dynamical behavior.*

1 INTRODUCTION

Increasing system reliability and dependability while decreasing maintenance cost is a major topic in the railway industry. On-line fault detection and isolation offers advantages when early detection of faults and wear is crucial. In railway suspension systems the most critical parts are the secondary lateral and anti-yaw damper. The first one is important for ride quality and the second one for running stability. Faults in the process often cause changes in the model parameters this could have a direct influence on the system stability.

In this study, nonlinear suspension characteristics are used. The full scale railway vehicle model includes the car body, the two bogies, two motors and four wheelsets. The FDI process is divided into two steps. The first step is the observation of the system with an optimal and robust Filter and the second step is the detection and isolation of faults by a residual generator.

2 Vehicle dynamics

The railway vehicle model, shown in Figure 1 and Figure 2, consists of a car body, two bogies, four wheelsets and two motors. The blue circles in Figure 2 indicate the position, where the suspension elements are fixed to the bodies. The red lines between the blue circles denote suspension elements.

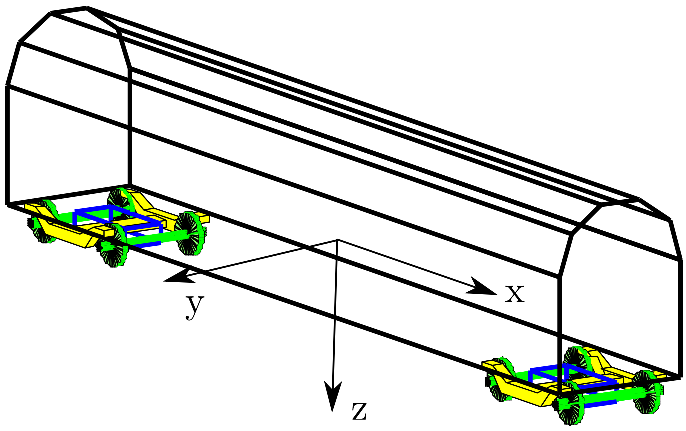


Figure 1: Matlab train model

The wheelsets are connected to bogies by the so-called primary suspensions. The bogies are connected to the car body with secondary suspensions. The motor is connected with suspensions to the bogie.

Car body, bogie and motor dynamics can be characterized by six degrees of freedom each. Wheelset dynamics are considered as two dimensional and characterized by lateral y^w and yaw motion ψ^w. By assuming constant cruising speed, the rotational motion of the wheelset around the Y-axis is neglected. The roll angel and the movement in the z-direction of the wheelset is given by the track irregularities. The dynamics of one wheelset without the primary suspension is described by [1]:

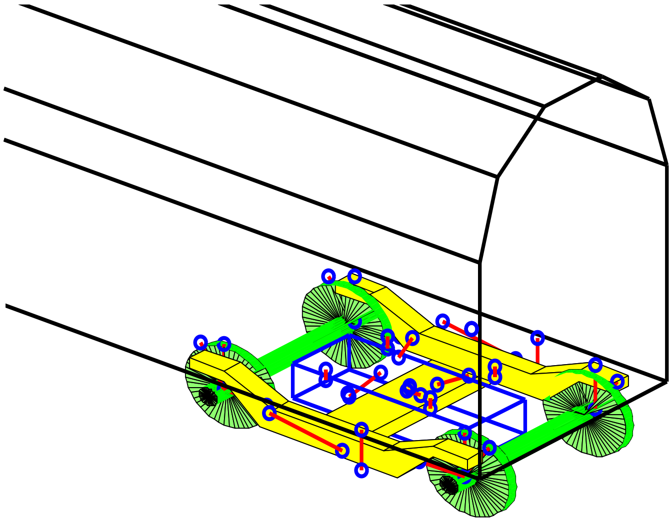


Figure 2: Matlab train model

$$\begin{bmatrix} \dot{y}^w \\ \dot{\psi}_i^w \\ \ddot{y}^w m \\ \ddot{\psi}_i^w I \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_g & +2f_{22} & -\frac{2f_{22}}{V}(1+\frac{\lambda r_0}{a}) & -\frac{2f_{23}}{V} \\ -\frac{2af_{11}\lambda}{r_0} & -2f_{23} & \frac{2f_{23}}{V}(1+\frac{\lambda r_0}{a}) & -2(\frac{a^2 f_{11}}{V}+\frac{f_{33}}{V}) \end{bmatrix} \begin{bmatrix} y^w \\ \psi_i^w \\ \dot{y}^w \\ \dot{\psi}_i^w \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_{g1} & k_{cl} & \frac{2f_{22}}{V}(1+\frac{\lambda r_0}{a}) & 0 \\ \frac{2af_{11}\lambda}{r_0} & 0 & -\frac{2f_{23}}{V}(1+\frac{\lambda r_0}{a}) & 0 \end{bmatrix} \begin{bmatrix} y^r \\ \phi^{cl} \\ \dot{y}^r \\ \dot{\phi}^{cl} \end{bmatrix}$$

where y^r is the lateral track alignment, ϕ^{cl} the cross level track alignment, V the vehicle velocity, a track gauge and λ the conicity. The equation of motion is derived by the procedure of Newton-Euler. The global position vector of an arbitrary point on the rigid body i can be written as

$$\mathbf{r}^i = \mathbf{R}^i + \mathbf{A}^i_{rot}\bar{\mathbf{u}}^i \tag{1}$$

where $\mathbf{R}^i$ is the global position vector of the origin of the body coordinate system defined as

$$\mathbf{R}^i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

and $\mathbf{A}^i_{rot}$ is a 3×3 rotation matrix that defines the orientation of the axes of the body coordinate system with respect to the global coordinate system. The vector $\bar{\mathbf{u}}^i$ is the position vector of the arbitrary point on the body with respect to the origin of the body coordinate system and is defined as

$$\bar{\mathbf{u}}^i = \begin{bmatrix} \bar{u}^i_x \\ \bar{u}^i_y \\ \bar{u}^i_z \end{bmatrix} \tag{2}$$

When rotating about the z-x-y-axes, in this order, the rotation matrix results to

$$\mathbf{A}_{rot} = \begin{bmatrix} \cos\psi\cos\phi - \sin\psi\sin\phi\sin\theta & -\sin\psi\cos\phi & \cos\psi\sin\theta + \sin\psi\sin\phi\cos\theta \\ \sin\psi\cos\phi - \cos\psi\sin\phi\sin\theta & \cos\psi\cos\phi & \sin\psi\sin\theta + \cos\psi\sin\phi\cos\theta \\ -\cos\phi\sin\theta & \sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

and for the linearized case to

$$\mathbf{A}_{rot} \approx \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix}$$

Here, ϕ describes the roll motion, θ the pitch motion and ψ the yaw motion.

Two bodies are connected with a suspension at the points denoted by $\mathbf{r}^j$ and $\mathbf{r}^i$. Defining

$$\mathbf{l} = \mathbf{r}^j - \mathbf{r}^i, \tag{3}$$

the spring force f_c and the damper force f_d can be expressed as functions of $\mathbf{l}$ and its derivative:

$$f_c = f_c(||\mathbf{l}||_2 - l_0), \;\; f_d = f_d(\frac{\mathbf{l}^T\dot{\mathbf{l}}}{||\mathbf{l}||_2}). \tag{4}$$

where l_0 is the unstrained spring length. The force vectors acting on the points $\mathbf{r}^j$ and $\mathbf{r}^i$ are

$$\mathbf{F}_c = \pm f_c \frac{\mathbf{l}}{||\mathbf{l}||_2}, \;\; \mathbf{F}_d = \pm f_d \frac{\mathbf{l}}{||\mathbf{l}||_2} \tag{5}$$

The Newton-Euler equations are used to derive the equation of motion. For this case, the origin of the body coordinate system is attached to the center of mass of the body. The Newton-Euler equations of motion are given in matrix form as follows [4]

$$\begin{bmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{J} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{R}} \\ \dot{\bar{\boldsymbol{\omega}}} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \bar{\mathbf{M}} - \bar{\boldsymbol{\omega}} \times (J\bar{\boldsymbol{\omega}}) \end{bmatrix},$$

where m is the mass of the rigid body, $\mathbf{I}$ is the 3×3 identity matrix, $\mathbf{J}$ is the moment of inertia tensor, $\mathbf{F}$ is the resultant of the external forces and $\bar{\mathbf{M}}$ is the resultant of the external moments defined in the body coordinate system. The vector $\bar{\boldsymbol{\omega}}$ results from

$$\bar{\boldsymbol{\omega}} = \bar{\mathbf{G}}\dot{\boldsymbol{\alpha}}$$

where

$$\bar{\boldsymbol{G}} = \begin{bmatrix} -\cos\phi\sin\theta & \cos\theta & 0 \\ \sin\phi & 0 & 1 \\ \cos\phi\cos\theta & \sin\theta & 0 \end{bmatrix}, \;\; \boldsymbol{\alpha} = \begin{bmatrix} \psi \\ \phi \\ \theta \end{bmatrix}$$

For the reconstruction of the system dynamics, the vehicle is prepared with 14 acceleration sensors, each wheelset has two sensors measuring the acceleration in z-direction and in y-direction. Each bogie has one sensor measuring the acceleration of all three directions. It is assumed that the measurement noise of all sensors is white with a standard deviation of $\sigma_s = 10^{-4} m/s^2$. Then, the equations describing the vehicle system dynamics and measurement output are written in the form

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathbf{A}_s\mathbf{x}_s + \mathbf{B}_s\mathbf{x}_r \\ \mathbf{y}_s &= \mathbf{C}_s\mathbf{x}_s + \mathbf{D}_s\mathbf{v} \end{aligned}$$

where $\mathbf{A}_s$ is the state matrix, $\mathbf{B}_s$ is the process noise matrix resulting from the track, $\mathbf{C}_s$ is the output matrix and $\mathbf{D}_s$ is the measurement noise matrix.

2.1 Random track irregularities

For the generation of random track irregularities, power spectral densities obtained from measurement data are commonly used. The PSDs for horizontal, lateral and cross level track irregularities are defined in ERRI B176 [3]. For high track irregularities the polynomials are

$$
\begin{aligned}
S_h(\Omega) &= \frac{b_{h0}}{a_{h0} + a_{h2}\Omega^2 + \Omega^4} \\
&= \frac{b_{h0}}{0.00028855 + 0.6803895\Omega^2 + \Omega^4} \quad , \\
S_v(\Omega) &= \frac{b_{v0}}{a_{v0} + a_{v2}\Omega^2 + \Omega^4} \\
&= \frac{b_{v0}}{0.00028855 + 0.6803895\Omega^2 + \Omega^4} \quad , \\
S_{cl}(\Omega) &= \frac{b_{cl2}\Omega^2}{a_{cl0} + a_{cl2}\Omega^2 + a_{cl4}\Omega^4 + \Omega^6} \\
&= \frac{b_{cl2}\Omega^2}{5.535659 \cdot 10^5 + 0.1308172\Omega^2 + 0.8722335\Omega^4 + \Omega^6} \quad .
\end{aligned}
$$

For a vehicle travelling at velocity V this gives

$$
\omega = V\Omega, \ \Omega = \frac{2\pi}{L} \quad ,
$$

where ω is the angular frequency and L is the spatial wave length of the rails. With this assumption it follows

$$
\begin{aligned}
S_h(s) &= \frac{b_{h0}}{0.00028855 + 0.6803895(s/V)^2 + (s/V)^4} \\
&= \frac{\sqrt{b_{h0}}V^2}{s^2 + 0.8452s + 0.01698676V^2} \frac{\sqrt{b_{h0}}V^2}{s^2 + 0.8452s + 0.01698676V^2} \quad , \\
S_v(s) &= \frac{b_{v0}}{0.00028855 + 0.6803895(s/V)^2 + (s/V)^4} \\
&= \frac{\sqrt{b_{v0}}V^2}{s^2 + 0.8452Vs + 0.01698676V} \frac{\sqrt{b_{v0}}V^2}{s^2 + 0.8452Vs + 0.01698676V} \quad , \\
S_{cl}((s) &= \frac{b_{cl2}(s/V)^2}{5.535659 \cdot 10^5 + 0.1308172(s/V)^2 + 0.8722335(s/V)^4 + (s/V)^6} \\
&= \left(\frac{\sqrt{b_{cl2}}sV^2}{0.00744V^3 + 0.387184V^2 s + 1.2832Vs^2 + s^3} \right)^2 \quad .
\end{aligned}
\tag{6}
$$

In order to consider the stochastic characteristics of the track irregularities in the estimation process, the train model is extended with form filters. Resulting from (6) the form filters for the horizontal, lateral and cross level track irregularities are

$$
\begin{aligned}
\dot{\mathbf{x}}_{ffi} &= \mathbf{A}_{ffi} x_{ffi} + \mathbf{B}_{ffi} \\
\mathbf{y}_{ffi} &= \mathbf{C}_{ffi} \mathbf{x}_{ffi} \quad ,
\end{aligned}
$$

with the matrices

$$\mathbf{A}_{ffh} = \begin{bmatrix} 0 & 1 \\ -0.01698676V^2 & -0.8452V \end{bmatrix}, \; \mathbf{B}_{ffh} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; \mathbf{C}_{ffh} = \begin{bmatrix} \sqrt{b_0}V^2 & 0 \\ 0 & \sqrt{b_0}V^2 \end{bmatrix} \quad ,$$

$$\mathbf{A}_{ffv} = \begin{bmatrix} 0 & 1 \\ -0.01698676V^2 & -0.8452V \end{bmatrix}, \; \mathbf{B}_{ffv} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; \mathbf{C}_{ffv} = \begin{bmatrix} \sqrt{b_0}V^2 & 0 \\ 0 & \sqrt{b_0}V^2 \end{bmatrix} \quad ,$$

$$\mathbf{A}_{ffcl} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.00744V^3 & -0.387184V^2 & -1.2832V \end{bmatrix}, \; \mathbf{B}_{ffcl} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$\mathbf{C}_{ffcl} = \begin{bmatrix} 0 & \sqrt{b_0}V^2 & 0 \\ 0 & 0 & \sqrt{b_0}V^2 \end{bmatrix} \quad .$$

Combining all three irregularities in one set of equations the resulting matrices are

$$\mathbf{A}_{ff} = \begin{bmatrix} \mathbf{A}_{ffh} & 0 & 0 \\ 0 & \mathbf{A}_{ffv} & 0 \\ 0 & 0 & \mathbf{A}_{ffcl} \end{bmatrix}, \; \mathbf{B}_{ff} = \begin{bmatrix} \mathbf{B}_{ffh} & 0 & 0 \\ 0 & \mathbf{B}_{ffv} & 0 \\ 0 & 0 & \mathbf{B}_{ffcl} \end{bmatrix},$$

$$\mathbf{C}_{ff} = \begin{bmatrix} \mathbf{C}_{ffh} & 0 & 0 \\ 0 & \mathbf{C}_{ffv} & 0 \\ 0 & 0 & \mathbf{C}_{ffcl} \end{bmatrix}$$

Even thought the four wheelsets are running over the same railway track, any correlation is neglected and the excitation of the wheelsets is assumed to be independent. By the assumptions given above, the matrices for the form filter including all four wheelsets are given by:

$$\mathbf{A}_{FF} = \begin{bmatrix} \mathbf{A}_{ff} & 0 & 0 & 0 \\ 0 & \mathbf{A}_{ff} & 0 & 0 \\ 0 & 0 & \mathbf{A}_{ff} & 0 \\ 0 & 0 & 0 & \mathbf{A}_{ff} \end{bmatrix}, \; \mathbf{B}_{FF} = \begin{bmatrix} \mathbf{B}_{ff} & 0 & 0 & 0 \\ 0 & \mathbf{B}_{ff} & 0 & 0 \\ 0 & 0 & \mathbf{B}_{ff} & 0 \\ 0 & 0 & 0 & \mathbf{B}_{ff} \end{bmatrix}$$

$$\mathbf{C}_{FF} = \begin{bmatrix} \mathbf{C}_{FF} & 0 & 0 & 0 \\ \mathbf{C}_{FF}\mathbf{A}_{FF} & 0 & 0 & 0 \\ 0 & \mathbf{C}_{FF} & 0 & 0 \\ 0 & \mathbf{C}_{FF}\mathbf{A}_{FF} & 0 & 0 \\ 0 & 0 & \mathbf{C}_{FF} & 0 \\ 0 & 0 & \mathbf{C}_{FF}\mathbf{A}_{FF} & 0 \\ 0 & 0 & 0 & \mathbf{C}_{FF} \\ 0 & 0 & 0 & \mathbf{C}_{FF}\mathbf{A}_{FF} \end{bmatrix}$$

The system containing the train model and the form filter for the track irregularities is written as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{w} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{v} \quad ,\end{aligned} \tag{7}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_s & \mathbf{B}_s\mathbf{C}_{FF} \\ 0 & \mathbf{A}_{FF} \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{B}_{FF} \end{bmatrix}, \; \mathbf{C} = \begin{bmatrix} \mathbf{C}_s & 0 \end{bmatrix}, \; \mathbf{x} = \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix}.$$

3 State estimation

In the system (7), the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are unknown, but they belong to a given convex bounded domain $\mathcal{D}_c$. The aim is to design an optimal state observer of the form

$$\begin{aligned}\dot{\hat{\mathbf{x}}} &= \mathbf{A}_f\hat{\mathbf{x}} + \mathbf{B}_f\mathbf{y} \\ \hat{\mathbf{y}} &= \mathbf{C}\hat{\mathbf{x}}\end{aligned} \tag{8}$$

which minimizes

$$\sup_{\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}\in\mathcal{D}_c} ||\mathbf{e}(t)||_2$$

where $\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}$, and $||\cdot||_2$ denotes the H_2 system norm defined later.

For the observer design process, the system equation (7) and the observer equation (8) are combined to one state space model

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\tilde{\mathbf{w}} \\ \mathbf{e} &= \tilde{\mathbf{C}}\tilde{\mathbf{x}}\end{aligned}$$

with the matrices

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{B}_f\mathbf{C} & \mathbf{A}_f \end{bmatrix}, \; \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{B}_f\mathbf{D} \end{bmatrix}, \; \tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{I} & -\mathbf{I} \end{bmatrix}, \; \tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix}$$

and the transfer function G(s). The H_2 norm is now defined by

$$||G(s)||_2 = \sqrt{\text{trace} \int_0^\infty (\mathbf{g}^T(t)\mathbf{g}(t))dt} \tag{9}$$

where

$$\mathbf{g}(t) = \tilde{\mathbf{C}}e^{\tilde{\mathbf{A}}t}\tilde{\mathbf{B}}$$

is the impulse response matrix.

Substituting this expression into (9) yields

$$||G(s)||_2 = \sqrt{\text{trace} \int_0^\infty \tilde{\mathbf{B}}^T\mathbf{e}^{\tilde{\mathbf{A}}^T t}\tilde{\mathbf{C}}^T\tilde{\mathbf{C}}\mathbf{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{B}}dt}. \tag{10}$$

Using the fact that trace $\mathbf{MN}$ = trace $\mathbf{NM}$ for two matrices $\mathbf{M}$ and $\mathbf{N}$ with compatible dimension, (10) can be rewritten as

$$||G(s)||_2 = \sqrt{\text{trace} \int_0^\infty \tilde{\mathbf{C}}\mathbf{e}^{\tilde{\mathbf{A}}^T t}\tilde{\mathbf{B}}\tilde{\mathbf{B}}^T\mathbf{e}^{\tilde{\mathbf{A}}t}\tilde{\mathbf{C}}^T dt}. \quad (11)$$

Defining

$$\mathbf{P} = \int_0^\infty \mathbf{e}^{\tilde{\mathbf{A}}^T t}\tilde{\mathbf{B}}\tilde{\mathbf{B}}^T\mathbf{e}^{\tilde{\mathbf{A}}t} dt,$$

equation (11) leads to

$$||G(s)||_2 = \sqrt{\text{trace } \tilde{\mathbf{C}}\mathbf{P}\tilde{\mathbf{C}}^T}$$

The matrix $\mathbf{P}$ is the solution of the Lyapunov equation

$$\tilde{\mathbf{A}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}^T + \tilde{\mathbf{B}}\tilde{\mathbf{B}}^T = \mathbf{0}.$$

Therefore, the H_2 norm can be computed by solving a single Lyapunov equation. It could be shown that the H_2 norm $||G(s)||_2$ is smaller than a given value ν^2 if and only if there exists a positive definite matrix $\mathbf{P}$ that satisfies

$$\tilde{\mathbf{A}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}^T + \tilde{\mathbf{B}}\tilde{\mathbf{B}}^T < \mathbf{0} \quad (12)$$

and

$$\text{trace } \tilde{\mathbf{C}}\mathbf{P}\tilde{\mathbf{C}}^T < \nu^2. \quad (13)$$

Using the fact that

$$\begin{bmatrix} \mathbf{M} & \mathbf{L} \\ \mathbf{L}^T & \mathbf{N} \end{bmatrix} > \mathbf{0}$$

where $\mathbf{M} = \mathbf{M}^T$ and $\mathbf{N} = \mathbf{N}^T$, is equivalent to

$$\mathbf{N} > \mathbf{0} \text{ and } \mathbf{M} - \mathbf{L}\mathbf{N}^{-1}\mathbf{L}^T > \mathbf{0}$$

equation (12) and (13) can be rewritten to

$$\begin{bmatrix} \tilde{\mathbf{A}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}^T & \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}}^T & -\mathbf{I} \end{bmatrix} < \mathbf{0}, \quad (14)$$

$$\begin{bmatrix} \mathbf{P} & \mathbf{P}\tilde{\mathbf{C}}^T \\ \tilde{\mathbf{C}}\mathbf{P} & \mathbf{W} \end{bmatrix} > \mathbf{0}, \quad (15)$$

$$\text{trace } \mathbf{W} < \nu^2. \quad (16)$$

The matrix $\mathbf{P}$ and the filter matrices $\mathbf{A}_f$ and $\mathbf{B}_f$ are variables and therefore the inequalities are nonlinear because the term $\mathbf{P}\tilde{\mathbf{A}}$ contains products of variables. To get linear inequalities matrix $\mathbf{P}$ and its inverse is partitioned as [4]

$$\mathbf{P} = \begin{bmatrix} \mathbf{X} & \mathbf{U} \\ \mathbf{U}^T & \hat{\mathbf{X}} \end{bmatrix}, \ \mathbf{P}^{-1} = \begin{bmatrix} \mathbf{Y} & \mathbf{V} \\ \mathbf{V}^T & \hat{\mathbf{Y}} \end{bmatrix}$$

where $\mathbf{X}, \mathbf{Y} \in R^{n\times n}$ and $\hat{\mathbf{X}}, \hat{\mathbf{Y}} \in R^{n_f \times n_f}$ are all symmetric and positive definite matrices. In addition, the non-singular matrix

$$\mathbf{T} = \begin{bmatrix} \mathbf{X}^{-1} & \mathbf{Y} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

is introduced. Multiplying (15) to the left by $\mathbf{T}^T$ and to the right by $\mathbf{T}$, yields [4]

$$\begin{bmatrix} \mathbf{Z} & \mathbf{Z} & \mathbf{L}^T - \mathbf{G}^T \\ \mathbf{Z} & \mathbf{Y} & \mathbf{L}^T \\ \mathbf{L} - \mathbf{G} & \mathbf{L} & \mathbf{W} \end{bmatrix} > \mathbf{0} \tag{17}$$

where $\mathbf{Z} = \mathbf{X}^{-1}$ and $\mathbf{G} = \mathbf{C}_f \mathbf{U}^T \mathbf{Z}$. Doing the same with (14) gives

$$\begin{bmatrix} \mathbf{ZA} + \mathbf{A}^T\mathbf{Z} & \mathbf{ZA} + \mathbf{A}^T\mathbf{Y} + \mathbf{C}^T\mathbf{F}^T + \mathbf{Q}^T & \mathbf{ZB} \\ \mathbf{A}^T\mathbf{Z} + \mathbf{YA} + \mathbf{FC} + \mathbf{Q} & \mathbf{YA} + \mathbf{FC} + \mathbf{A}^T\mathbf{Y} + \mathbf{C}^T\mathbf{F}^T & \mathbf{YB} + \mathbf{FD} \\ \mathbf{B}^T\mathbf{Z} & \mathbf{B}^T\mathbf{Y} + \mathbf{D}^T\mathbf{F}^T & -\mathbf{I} \end{bmatrix} < \mathbf{0}$$

The matrix variables appearing only in the linear matrix inequality (LMI) (17) can be eliminated. Permuting the second and third columns and rows and using Schur complements LMI (17) can be rewritten to [4]

$$\begin{bmatrix} \mathbf{Z} - \mathbf{Z}\mathbf{Y}^{-1}\mathbf{Z} & \mathbf{L}^T - \mathbf{G}^T - \mathbf{Z}\mathbf{Y}^{-1}\mathbf{L}^T \\ \mathbf{L} - \mathbf{G} - \mathbf{L}\mathbf{Y}^{-1}\mathbf{Z} & \mathbf{W} - \mathbf{L}\mathbf{Y}^{-1}\mathbf{L}^T \end{bmatrix} > \mathbf{0} \tag{18}$$

The inequality

$$\begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 - \mathbf{N} \\ \mathbf{M}_2^T - \mathbf{N}^T & \mathbf{M}_3 \end{bmatrix} > \mathbf{0}$$

has a solution with respect to $\mathbf{N}$ if and only if $\mathbf{M}_1 > 0$ and $\mathbf{M}_3 > 0$. Moreover, if the later conditions are fulfilled, then a possible solution is $\mathbf{N} = \mathbf{M}_2$.

Applying this, the LMI (18) can be replaced by

$$\begin{bmatrix} \mathbf{Y} & \mathbf{L}^T \\ \mathbf{L} & \mathbf{W} \end{bmatrix} > \mathbf{0}, \ \begin{bmatrix} \mathbf{Z} & \mathbf{Z} \\ \mathbf{Z} & \mathbf{Y} \end{bmatrix} > \mathbf{0}.$$

In the end, the equations (14) - (16) are formulated as the minimization problem [4]

$$\min \operatorname{trace}(\mathbf{W}) \tag{19}$$

$$\begin{bmatrix} \mathbf{Y} & \mathbf{L}^T \\ \mathbf{L} & \mathbf{W} \end{bmatrix} < \mathbf{0} \tag{20}$$

$$\begin{bmatrix} \mathbf{Z} & \mathbf{Z} \\ \mathbf{Z} & \mathbf{Y} \end{bmatrix} > \mathbf{0} \tag{21}$$

$$\begin{bmatrix} \mathbf{ZA} + \mathbf{A}^T\mathbf{Z} & \mathbf{ZA} + \mathbf{A}^T\mathbf{Y} + \mathbf{C}^T\mathbf{F}^T + \mathbf{Q}^T & \mathbf{ZB} \\ \mathbf{A}^T\mathbf{Z} + \mathbf{YA} + \mathbf{FC} + \mathbf{Q} & \mathbf{YA} + \mathbf{FC} + \mathbf{A}^T Y + \mathbf{C}^T\mathbf{F}^T & \mathbf{YB} + \mathbf{FD} \\ \mathbf{B}^T\mathbf{Z} & \mathbf{B}^T\mathbf{Y} + \mathbf{D}^T\mathbf{F}^T & -\mathbf{I} \end{bmatrix} < \mathbf{0}, \tag{22}$$

where $\mathbf{W} = \mathbf{W}^T$, $\mathbf{Z} = \mathbf{Z}^T$, $\mathbf{Y} = \mathbf{Y}^T$, $\mathbf{Q}$ and $\mathbf{F}$ are the optimal solution to the convex programming problem. The minimum error variance filter is defined by matrices

$$\mathbf{A}_f = -\mathbf{Y}^{-1}\mathbf{Q}(\mathbf{I} - \mathbf{Y}^{-1}\mathbf{Z})^{-1}, \quad \mathbf{B}_f = -\mathbf{Y}^{-1}\mathbf{F}.$$

As mentioned in the beginning of this section, it is assumed that matrices A,B,C and D are unknown, but belong to a given convex bounded domain D_C. It is proven in [4], that for the case, when only some elements are unknown, the uncertain matrices can be written as an unknown convex combination of N extreme matrices

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix}, \begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix}, ..., \begin{bmatrix} \mathbf{A}_N & \mathbf{B}_N \\ \mathbf{C}_N & \mathbf{D}_N \end{bmatrix},$$

which are assumed to be known. To consider the extreme matrices in the filter design equation (14) is replaced by

$$\begin{bmatrix} \mathbf{Z}\mathbf{A}_i + \mathbf{A}_i^T\mathbf{Z} & \mathbf{Z}\mathbf{A}_i + \mathbf{A}_i^T\mathbf{Y} + \mathbf{C}_i^T\mathbf{F}^T + \mathbf{Q}^T & \mathbf{Z}\mathbf{B} \\ \mathbf{A}_i^T\mathbf{Z} + \mathbf{Y}\mathbf{A}_i + \mathbf{F}\mathbf{C}_i + \mathbf{Q} & \mathbf{Y}\mathbf{A}_i + \mathbf{F}\mathbf{C}_i + \mathbf{A}_i^T\mathbf{Y} + \mathbf{C}_i^T\mathbf{F}^T & \mathbf{Y}\mathbf{B}_i + \mathbf{F}\mathbf{D}_i \\ \mathbf{B}_i^T\mathbf{Z} & \mathbf{B}_i^T\mathbf{Y} + \mathbf{D}_i^T\mathbf{F}^T & -\mathbf{I} \end{bmatrix} < \mathbf{0}$$

for $i = 1, 2, ..., N$.

4 Fault Detection and Isolation

The aim of this work is to detect and isolate faults in the suspension. This is done by comparing the real process outputs with their estimates. The procedure of creating the estimation of the process outputs and building the difference between the process output and their estimates is called residual generation [5]. Like many real engineering systems the train model is governed by continuous-time dynamics whereas the measurements are obtained at discrete instants of time. Thus the system as well as the observer have to be discretized, which is denoted by the subscript d in A_{fd} and B_{fd}. In order to isolate the faults clearly, the procedure of the residual generation is improved in three steps. In the first step the residual $\mathbf{x}_k^r$ is simply generated by comparison of the filter estimation and the estimation by using only the system matrix

$$\mathbf{x}_k^r = (\mathbf{A}_{fd}\hat{\mathbf{x}}_k + \mathbf{B}_{fd}\mathbf{y}) - \mathbf{A}\hat{\mathbf{x}}_k \quad .$$

In order to distinguish the effects of n different faults, the residual vector is multiplied by specific vectors $\mathbf{f}_i$ for $i = 1 : n$. Each vector is characteristic for a suspension faults. The fault vector $\mathbf{f}_i$ corresponds to an additional actuator in the suspension. The fault vectors are collected in a fault matrix:

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 & \mathbf{f}_2 & ... & \mathbf{f}_n \end{bmatrix} \quad .$$

With

$$\boldsymbol{\epsilon}_k^1 = \mathbf{F}\mathbf{x}_k^r$$

a vector ϵ_k^1 is defined which contains n values, each of them is a measure of the particular fault.

The second improvement is to scale $\boldsymbol{\epsilon}^1$ with the state acting on the particular suspension. Having a faulty damper, the difference between the measurements and their estimates depends on the velocity acting on this damper. Having a faulty spring, the difference between the measurements and their estimates depends on the relative position on this spring. If there is no velocity at the faulty damper or the spring is relaxed no error could be detected. This issue has been used and $\boldsymbol{\epsilon}$ are weighted at each time step with the actual estimated state $g_1(\hat{x}), ..., g_n(\hat{x})$,

$$\begin{aligned}
\epsilon_k^2(1) &= \epsilon_k^1(1) \cdot \|g_1(\hat{\mathbf{x}})\| \quad , \\
\epsilon_k^2(2) &= \epsilon_k^1(2) \cdot \|g_2(\hat{\mathbf{x}})\| \quad , \\
&\vdots \\
\epsilon_k^2(n) &= \epsilon_k^1(n) \cdot \|g_n(\hat{\mathbf{x}})\| \quad .
\end{aligned}$$

Given that some faults are more sensible than others in the fault detection, the third step is to multiply specific constant values a_1 to a_n.

$$\begin{aligned}
\epsilon_k^2(1) &= \epsilon_k^1(1) \cdot \|g_1(\hat{\mathbf{x}})\| \cdot a_1 \\
\epsilon_k^2(2) &= \epsilon_k^1(2) \cdot \|g_2(\hat{\mathbf{x}})\| \cdot a_2 \\
&\vdots \\
\epsilon_k^2(n) &= \epsilon_k^1(n) \cdot \|g_n(\hat{\mathbf{x}})\| \cdot a_n \\
&.
\end{aligned}$$

The values a_1-a_n are selected such that in the fault free case all values ϵ_k^2 have the same magnitude.

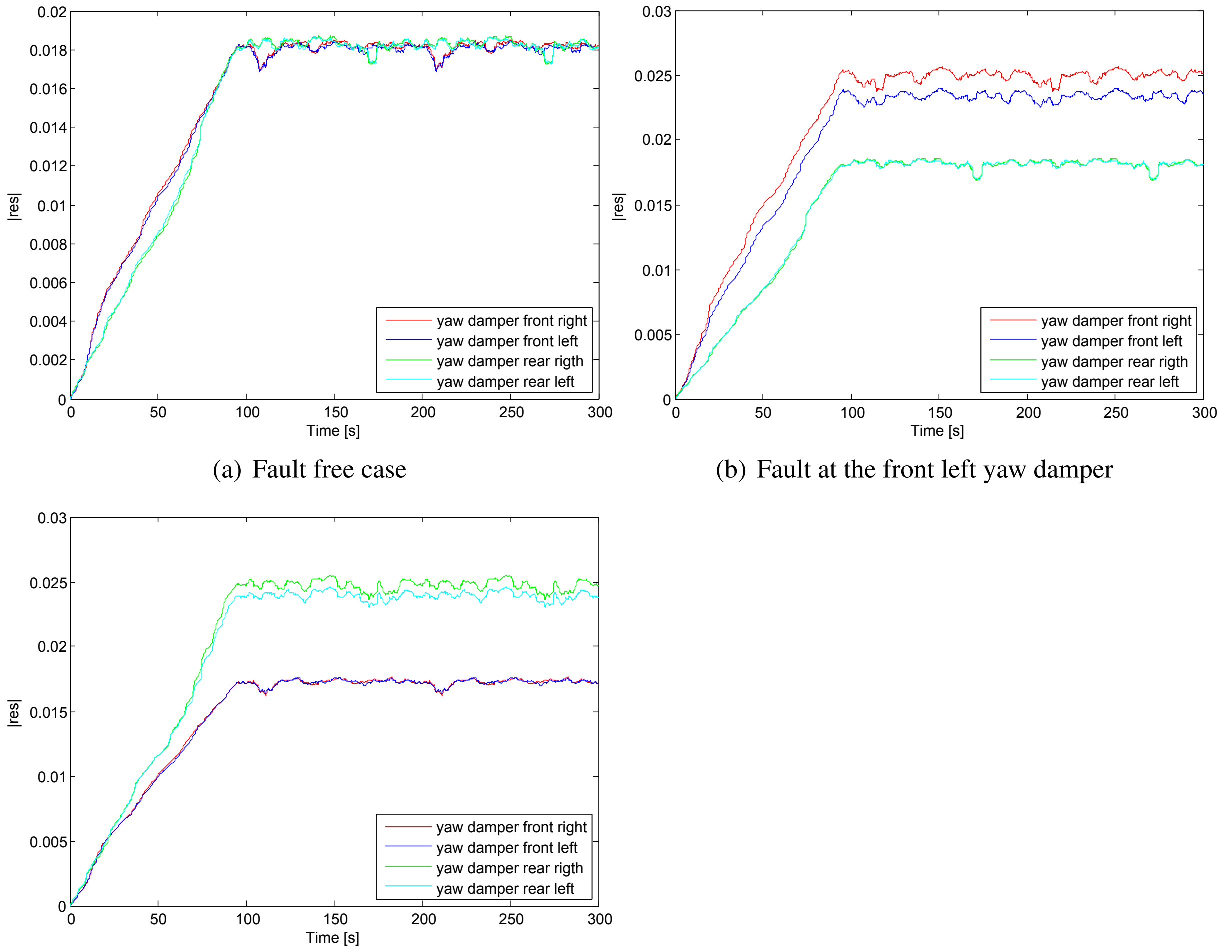
The generated values ϵ_k^2 are oscillating around zero, in this form the fault detection is difficult. To evaluate the results the absolute values of the generated scalar residuals are calculated and these values are filtered with a moving average filter.

$$\begin{aligned}
\zeta_k(1) &= \frac{\sum_{m=0}^{M} \|\epsilon_{k-n}^2(1)\|}{M} \quad , \\
\zeta_k(2) &= \frac{\sum_{m=0}^{M} \|\epsilon_{k-n}^2(2)\|}{M} \quad , \\
&\vdots \\
\zeta_k(n) &= \frac{\sum_{m=0}^{M} \|\epsilon_{k-n}^2(n)\|}{M} \quad .
\end{aligned}$$

The values ζ_k now serve as indicators in the fault detection process. In steady state operation they should all have the same constant value. A fault in a specific component corresponds to an offset of the corresponding indicator value ζ_k as compared to the other indicator values.

5 Results

The fault detection and isolation design procedure explained in the last chapter is used to detect faults in a full scale train model. The Velaro Rus is used as an example. For primary and



(a) Fault free case

(b) Fault at the front left yaw damper

(c) Fault at the rear rigth yaw damper

Figure 3: Fault detection and isolation

secondary suspensions nonlinear characteristics are applied for the simulation process as well as for the observer design. The nonlinear characteristics of the anti yaw damper has a major influence on the system behavior especially for the stability, hence this dampers are used to show the results. Three different test cases were considered, firstly a fault free train, secondly a fault at the front left anti yaw damper and thirdly a fault at the rear right anti yaw damper. The failure is modeled by multiplying the suspension forces times 0.5. Figure 3 shows the results of the three cases, ζ is plotted over time. In all three cases ζ started at zero and the moving average time is 100 seconds. For a fault and disturbance free system ζ should be zero, as the system is corrupted by measurement and process noise, this is not the case. Figure 3 (a) shows the fault free case it can be seen that all four FDI signals have the same magnitude. Figure 3 (b) shows the results for a failure at the front right and Figure 3 (c) the results for a failure at the left rear damper. The figures show that it is possible to detect the occurrence of a fault. Also, it can be determined whether the fault occurs at the front or the rear bogie. Since the influence of the left and right anti yaw damper on the system dynamics is highly correlated, it is not possible to distinguish between left or right.

6 Conclusion

A Hybrid Extendet Kalman Filter combined with a nonlinear residual generator was used to detect and isolate faults in a nonlinear suspension system of a railway vehicle. With this method faults in the anti yaw damper can be detect. Further it could be distinguish between faults at the front and rear bogie. Because the influence of the left and right anti yaw damper to the system dynamics is highly correlated, it is not possible to distinguish between this two faults.

Further work will determine whether the application could be applied to different track conditions.

REFERENCES

[1] K. Zaazaa and A. Shabana, *Study of the lateral stability of railroad vehicle systems using the elastic contact approach*, The Arabian Journal for Science and Engineering, 29 (1C) (2004).

[2] A. Shabana, K. Zaazaa and H. Sugiyama *Railroad Vehicle Dynamics: A Computational Approach*, Crc Pr Inc, (2007).

[3] *Simpack*, Simpack Documentation (11) (2012), pp. 1618.

[4] Li, P. and Goodall, R. M., *Optimal linear filtering under parameter uncertainty* , IEEE Transactions on signal processing, 47 (1) (1999), pp. 168–175.

[5] S.X. Ding, *Model-based Fault Diagnosis Techniques*, Springer (2008).