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MATERIALS SELECTION AND TOPOLOGY OPTIMIZATION DESIGN ON TRUSS STRUCTURE VIBRATION REDUCTION

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Abstract. *The mismatch design of stiffness, mass and damping characteristics plays important roles in vibration reduction on structures. The optimal mismatch design of structures involves simultaneous structural materials selection, the determination of component dimensions, geometry and topology, even damping materials distributions in vibration energy transmitting path. Hybrid structures will be obtained after the optimal mismatch design, such as steel-composite hybrid structure. The employment of different topology and materials design variables, like elastic modulus of materials, materials topology distribution, density and thickness of components, may lead to different structural optimization models with a mixture of continuous and discrete variables. In this paper, using the laminate component method (LCM) and solid isotropic microstructure with penalty method (SIMP), the mathematical model for the concurrent optimization of materials selection, size and topology optimizations of truss structure is established. A unified structural analysis model is used during the optimization procedure. The vibration level difference (VLD), acceleration, stress and displacement constraints are considered in optimization model. The vibration reduction design examples for 3 bar truss and 10 bar truss are implemented.*

1 INTRODUCTION

Composite materials begin to play important roles in vibration and noise reduction in ship structures. The composite axle, composite mounting, composite propeller as well as composite ship hull have been put into use successively with effective reductions on weight, vibration and noise (Mouritz 2001). If all hull structures are manufactured by composite materials, the cost will be more expensive than that of metal materials. The practical solution is the use of hybrid structures in ships. That means, the majority of the hull structures are manufactured by metal materials which can decrease the costs, while some important components and cabins are fabricated by composites materials which can decrease structural weight and vibration level (Barsoum 2003, Cao 2007). There are a number of fundamental challenges with steel-composite hybrid structures involving design and fabrication. In this paper, we focus our study on optimal design methods of steel-composite hybrid truss structure considering vibration reduction.

The design of hybrid metal-composite structure involves simultaneous materials selection, component dimensions, geometry and topology determination. For example, material types (steel, aluminum, glass fiber reinforced composites, etc.), topology (layout), shape and sizes of each component. Models and methods on optimization design of composite structure and topology have been widely investigated. Sigmund (1997) takes topology optimization procedure as a tool for smart materials design and discusses two applications in composite structures. Sourav Rakshit (2008) explored simultaneous geometry design and material selection for statically determinate trusses by posing it as a continuous optimization problem. The available materials for selection were put in a database with design index along with the corresponding best geometry. S. Šilih (2010) presented shape and discrete sizing optimization of timber trusses with the consideration of joint flexibility. Jose Pedro Blasques (2012) presents a novel framework for simultaneous optimization of topology and laminate properties in structural design of laminated composite beam cross sections.

Deqing Yang(2012) proposed a laminate component method (LCM) on modeling of hybrid structure during materials selection optimal design. By introducing the concept of laminate component, hybrid structure design problems are formulated as a size or topology optimization problem with all materials considered for selection within one structural analysis model. However, investigations on simultaneous materials selection, topology and dimensions optimization design for hybrid truss structures are very scarce. One challenge in this optimization problem is the need for switching the structural analysis models during optimization iterations for different settings of material selection and topology distributions. The other challenge is that the definitions of material selection design variables and topology design variables in one optimization model, these design variables all belong to discrete variables.In this paper, by applying SIMP method and LCM method, the mathematical models for concurrent optimization of materials types, size and topology of hybrid structure with respect to vibration constraints are proposed. Typical metal-composite hybrid truss structures optimization examples are performed to demonstrate the effectiveness of the proposed models.

2 CONCURRENT OPTIMIZATION MODELS FOR METAL-COMPOSITE HYBRID TRUSS STRUCTURES

2.1 SIMP model for structural topology optimization

SIMP method is a widely used standard topology optimization approach (Bendsøe MP

1986 & 2003, Zhou M and Rozvany G 1992). A topology optimization problem based on SIMP where the objective is to minimize compliance can be written as

$$\begin{aligned} &\underset{\mathbf{X}}{Min} \quad C(\mathbf{X}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N}(x_e)^p \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \\ &s.t. \quad \frac{V(\mathbf{X})}{V_0} = f \\ &\mathbf{K}\mathbf{U} = \mathbf{F} \\ &0 < x_{\min} \le x_e \le 1 \end{aligned} \tag{1}$$

where $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}_e$ and $\mathbf{k}_e$ are the element displacement vector and stiffness matrix, respectively, $\mathbf{X} = [x_1, x_2, \cdots, x_e, \cdots x_N]^T$ is the vector of design variables, $x_{\min}$ is a vector of minimum relative densities (non-zero to avoid singularity), N is the number of elements used to discretize the design domain, p is the penalization power (typically p=3), $V(\mathbf{X})$ and V_0 is the material volume and design domain volume, respectively and f is the prescribed volume fraction.

In this paper, SIMP method is used to define the topological design variables of elements in each laminate component or components without materials selection design. For the elements needed materials selection design, each element consists of multiple different material plies according to laminate component method, and x_e is the topological value of the eth laminate element. x_e has not connection with materials selection, and it only decides the existence of the element. For the elements without materials selection design requirements in components, each element consists of isotropic material, x_e is the existence of the isotropic material bar element.

2.2 Laminate component method for materials selection optimization

In this paper, laminate component method for materials selection optimization will be applied. The laminate component method has three main steps (Deqing Yang, 2012). First, the design components in a hybrid structure are replaced with laminated composite components that different materials can be chosen for each ply. Second, a finite element model is constructed for the hybrid structure. Finally, a material selection optimization model is established by defining the design variables of materials topology distribution and the thickness of each ply at component level and element level, as well as design objective and constraints that capture the relationship between two levels. Details can be seen in reference[8].

2.3 Concurrent optimization formulations for materials selection, size and topology design of truss structures

By applying LCM method to deal with materials selection optimization, and applying SIMP method to define topology distribution in the components in hybrid truss structure, the concurrent optimization model for materials selection, size and topology design can be established as Eqs.2. The weight of truss structure is considered as the objective function with acceleration vibration level difference (AVLD), stress and displacement on designated points and elements as constraints. The meanings of design variables and constraints are including in the Appendix I. $n \geq m$ means some steel components no materials selection optimization design requirements.

$$\left.\begin{array}{ll}
Find & \mathbf{T}_s = [t_{1s}, t_{2s}, \cdots, t_{ms}, t_{m+1s} \cdots, t_{ns}]^T \\
 & \mathbf{T}_c = [t_{1c}, t_{2c}, \cdots, t_{ic}, \cdots, t_{mc}]^T \\
 & \tilde{\mathbf{X}}_t = [\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_i, \cdots, \mathbf{X}_n]^T \\
Min & Weight = \sum_{i=1}^{m}\sum_{t=1}^{T_{iT}} (t_{ic} \cdot \rho_c + t_{is} \cdot \rho_s) \cdot se_{it} \cdot x_{it} + \sum_{i=1}^{m}\sum_{t=T_{iT}+1}^{T_i} (t_{ic} \cdot \rho_c + t_{is} \cdot \rho_s) \cdot se_{it} + \\
 & \sum_{i=m+1}^{n}\sum_{t=1}^{T_{iT}} t_{is} \cdot \rho_s \cdot se_{it} \cdot x_{it} + \sum_{i=m+1}^{n}\sum_{t=T_{iT}+1}^{T_i} t_{is} \cdot \rho_s \cdot se_{it} \\
s.t. & \sigma_e^L \leq \sigma_{ek}[\mathbf{T}_s, \mathbf{T}_c, \tilde{\mathbf{X}}_t] \leq \sigma_e^U \\
 & \delta_{pk}[\mathbf{T}_s, \mathbf{T}_c, \tilde{\mathbf{X}}_t] \leq \bar{\delta}_p \\
 & a_{Aj}^{\max} \leq a_A^U \\
 & a_{Bj}^{\max} \leq a_B^U \\
 & 10^{L_r/20} - \varepsilon \leq a_{Aj}^{\max} / a_{Bj}^{\max} \leq 10^{L_r/20} + \varepsilon \\
 & f^L \leq j \leq f^U \\
 & t_{ic} = sca \cdot t_{is} \\
 & t_{is}^L \leq t_{is} \leq t_{is}^U \\
 & \mathbf{X}_i = [x_{i1}, x_{i2}, \cdots, x_{it}, \cdots, x_{iT_{iT}}]^T \\
 & 0 < x_{\min} \leq x_{it} \leq 1 \\
 & i = 1, 2, \cdots, n;\ e = 1, 2, \cdots, E;\ k = 1, 2, \cdots, K; \\
 & p = 1, 2, \cdots, P;\ t = 1, 2, \cdots, T_{iT}
\end{array}\right\} \tag{2}$$

3 MATERIALS SELECTION, SIZE AND TOPOLOGY DESIGN EXAMPLES FOR HYBRID TRUSS STRUCTURE

3.1 10-bar truss

Figure 1 shows the dimensions and connection of each bar of 10-bar truss (Jin PAN, Deyu WANG, 2006). Parameters for the materials selection design are listed in table 1. For the initial design, bars are made of aluminum equilateral L-bar, heights and thicknesses for the bars are listed in Table 2.

aluminum	Young’s modulus	68.5 GPa
	Poisson’s ratio	0.3
	density	2768kg/m^3
fiber reinforced composite	Young’s modulus	E_{11}=27285.7MPa
		E_{22}=26142.8MPa
	shear modulus	G_{12}= 9242.9MPa
		G_{23}= 9242.9MPa
		G_{13}=9242.9MPa
	Poisson’s ratio	0.14
	damping coefficient	η_{11}=0.0729
		η_{22}=0.0715
		$\eta_{12}=\eta_{23}=\eta_{13}$=0.1068
	density	1600kg/m^3
	ply angles	$[90^0/0^0]_s$

Table 1: Materials parameters for selection

Boundary conditions and loading conditions are shown in Figure 2. There are two concentrated mass at point B and C, each one is 454kg. A 100N vertical force acts on point C in Z direction with frequency range 1-200Hz. The modal damping loss factor for structural dynamic frequency response is 2%. Constraints for optimization design are: (1) AVDL at point A and C less than 5dB. (2) Allowable displacement at point C is ±0.0508m. (3) Maximum acceleration amplitude for the referenced points is 0.1g.

Bars L_1、L_3、L_4 and X_5 are concurrent size and material selection optimized. The size, material selection and topology optimization are concurrent implemented for other bars.

Bar No.	Web and Face height (mm)	Web and Face thickness (mm) (mm)	Al. layer thickness (mm)	Comp. layer thickness (mm)
L_1	100	65.03	65	0.03
L_2	100	45.03	45	0.03
L_3	100	70.03	70	0.03
L_4	100	68.03	68	0.03
X_1	100	25.03	25	0.03
X_2	100	35.03	35	0.03
X_3	100	80.03	80	0.03
X_4	100	50.03	50	0.03
V_1	100	48.03	48	0.03
V_2	100	52.03	52	0.03

Table 2: Initial design for the 10-bar truss

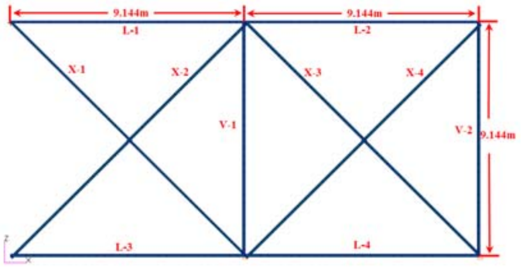

Figure 1: Dimensions of 10-bar truss

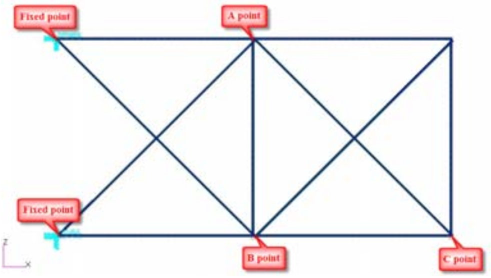

Figure 2: Boundary and loading conditions

Optimization results are shown in Table 3. Figure 3 shows topological densities of bars after optimization. Since density values for bar L_2, X_1, X_4, V_1 and V_2 are less than 0.01, all of these bars are deleted in final structural design shown in figure 5. Figure 4 shows iterative history of objective function and the weight of truss decreased from 4308kg to 2166kg.

Bar No.	Initial thickness of aluminum plate (mm)	Initial thickness of composite laminate (mm)	Optimal thickness of aluminum plate (mm)	Optimal thickness of composite laminate (mm)	Optimal topology density
L_1	100	0.03	59.38	0.03	——
L_2	100	0.03	0.005	107.58	0.0086
L_3	100	0.03	63.95	0.03	——
L_4	100	0.03	0.005	155.34	——
X_1	100	0.03	0.005	57.126	0.0035
X_2	100	0.03	0.005	79.98	0.0586
X_3	100	0.03	0.005	182.76	——
X_4	100	0.03	47.80	0.03	0.0085
V_1	100	0.03	43.85	0.03	0.0089
V_2	100	0.03	49.71	0.03	0.0082
Objective function	Initial weight （kg）	4308	Optimal result (kg)	2166	——

Table 3: Design variables and objective value

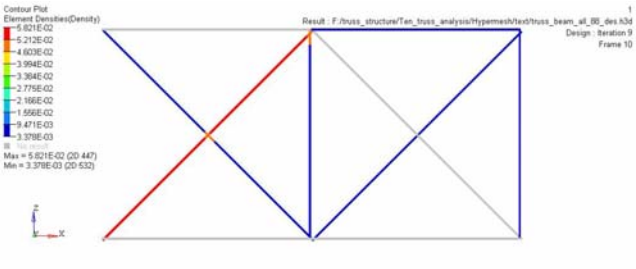

Figure 3: Topological densities of bars

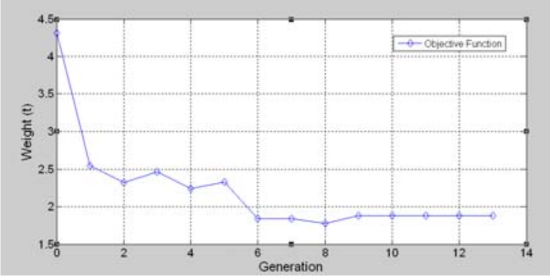

Figure 4: Iteration history of objective function

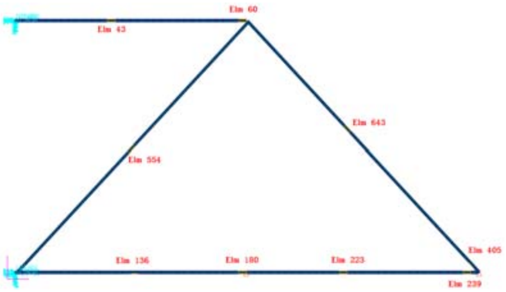

Figure 5: Final topology of 10-bar truss

3.2 3-bar truss

Figure 6 shows the dimensions and connection of each bar of 3-bar truss. Parameters for the materials selection design are steel and fiber reinforced composite listed in table 1. Thickness of steel_plate is 15mm, Girder is made of 100x50x10 steel L-bar. Physical parameters of steel are: elastic modulus E=206GPa, Poisson ratio 0.3, density 7850kg/m^3. For the initial design, bars are made of square steel, areas and thicknesses for the bars are listed in table 2. Boundary conditions and loading conditions are shown in Figure 7~9. There is a 454kg concentrated mass at point A. A 100N vertical force acts on point A in Z direction with frequency range 1-100Hz. The modal damping loss factor for structural dynamic frequency response is 2%. Constraints for optimization design are: (1) AVDL at point A, B and E less than 5dB. (2) Allowable displacement at point C is 1mm. (3) Maximum acceleration amplitude for the referenced points is 0.5g.

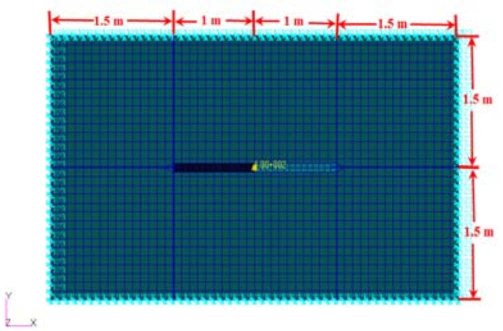

Figure 6: Dimensions of 10-bar truss

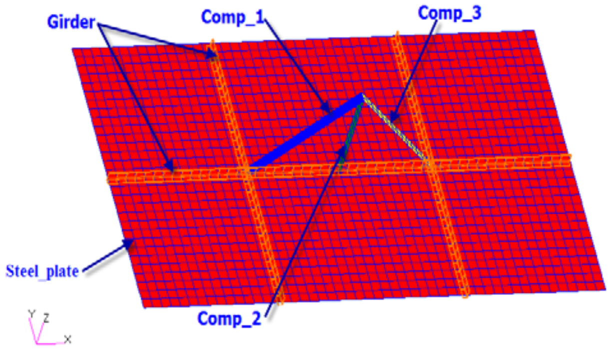

Figure 7: Boundary and loading conditions

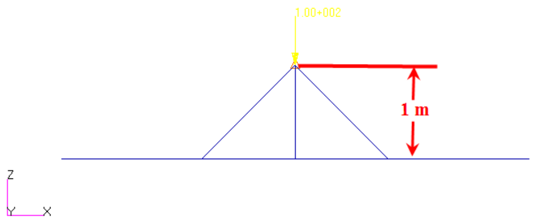

Figure 8: Dimensions of 10-bar truss

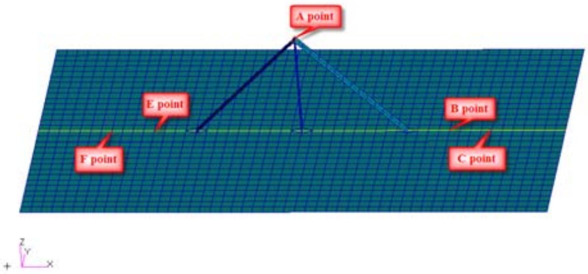

Figure 9: Boundary and loading conditions

Optimization results are shown in table 4. Figure 10 shows topological densities of bars after optimization. Since density value for bar Comp_3 is less than 0.062, this bar is deleted in final structural design shown in Figure 12. Figure 11 shows iterative history of objective function and the weight of truss decreased from 60.5kg to 33.7kg.

Bar No.	Initial thickness of steel plate (mm)	Initial thickness of composite laminate (mm)	Optimal thickness of steel plate (mm)	Optimal thickness of composite laminate (mm)	Optimal topology density
Comp_1	20	0.03	0.005	46.632	0.435
Comp_2	20	0.03	0.005	46.632	0.385
Comp_3	20	0.03	19.1	0.03	0.062
Objective value	Initial （kg）	60.5	optimum (kg)	33.7	——

Table 4: Design variables and objective value

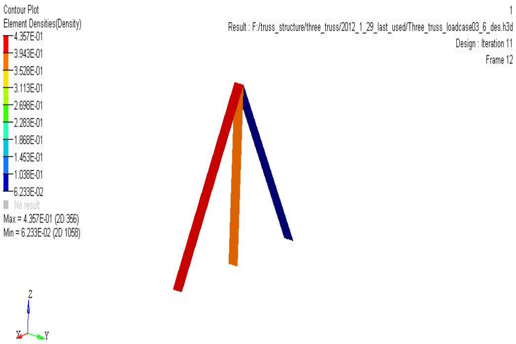

Figure 10: Topological densities of bars

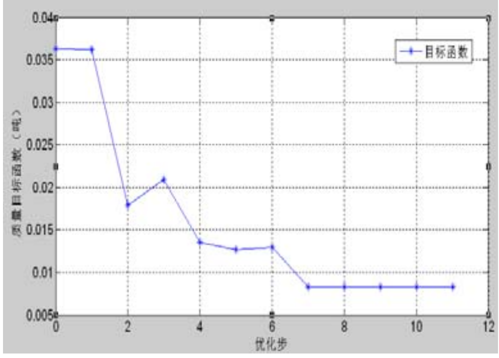

Figure 11: Iteration history of objective function

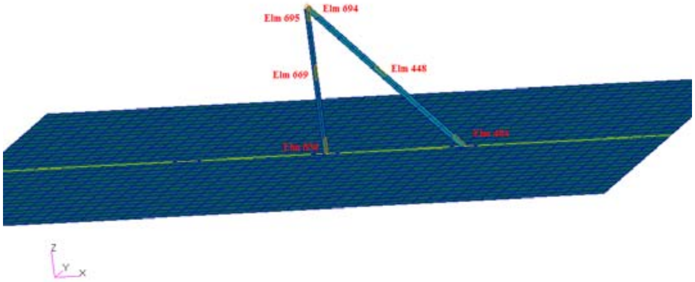

Figure 12: Final topology of 10-bar truss

4 CONCLUSIONS

- According to laminate component method (LCM) and solid isotropic microstructure with penalty (SIMP), concurrent optimization model (materials selection, size and topology optimization) for hybrid metal-composite truss structures is established. The design variables for topology and materials selection can be combined in a unified structural finite element analysis model, in particular, combined in each element in the design domain of structure.

- Typical steel-composite hybrid 10-bar plane truss and 3-bar truss are optimized for vibration reduction by the proposed models. The optimization results demonstrated the validity of the proposed approach.
- The optimization design of 10-bar truss shows that concurrent optimization can obtain the better results with structural weight from 4308kg to 2166kg. For 3-bar space truss, the structural weight decreases from 60.5kg to 33.7kg after the concurrent optimization.

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APPENDIX I: Nomenclature

AVLD	acceleration vibration level difference , $\mathrm{AVLD} = 20\lg \frac{a_\mathrm{A}}{a_\mathrm{B}}$ (dB)
a_A and a_B	accelerations of points A and B
T_s	thickness design variables for the components made by steel
T_C	thickness design variables for the components made by composite materials
n	Total number of the steel components
m	Total number of the composite components
$\tilde{\mathbf{x}}_t$	Overall topological design variables for truss defined by SIMP method
$\mathbf{X}_i$	topology design variables in the ith component
T_i	total number of finite elements in the ith component
T_{iT}	total number of topology design variables in the ith component
se_{it}	area of the tth element in the ith component
ρ_s	density of the steel component
ρ_c	density of the steel component and composite component
σ_{ek}	stress of the eth element under the kth loading case
σ_e^L	the lower limit of element stress
σ_e^U	the upper limit of element stress
δ_{pk}	the displacement at the designated point p under the k load case
$\bar{\delta}_p$	the upper limit of displacement at the designated point p
k	the loading case number
$a_{Aj}^{\max}$	the maximum acceleration amplitude at the designated point A in the frequency j Hz
a_A^U	upper limit of the maximum acceleration amplitude at point A
L_r	lower limit of AVLD
f^U and f^L	the upper and the lower bounds of frequency domain
ε	the relax parameter of the acceleration vibration level difference constraints
sca	the value of the equivalent scale of the thickness for steel and composite material
$x_{\min}$	the minimum relative density (non-zero to avoid singularity)
t_{is}^L t_{is}^U	the upper and the lower bounds of the thickness design variables for steel components