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A COMPARATIVE STUDY OF THE CORRELATION-FUNCTION-BASED STRUCTURAL DAMAGE DETECTION METHODS UNDER SINUSOIDAL EXCITATION

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Abstract *Similar to the already proposed CorV/IPV method use the correlation function for damage detection, this paper proposes three new damage detection methods (AMV method, CMV method and CZV method) also based on that, which can all be called as correlation-function-based structural damage detection methods. Firstly the theory of the correlation function of vibration response under sinusoidal excitation is studied, based on which the three new damage indexes are proposed. Difference of the damage index is formed as the damage location index to locate the damage. Numerical simulation of the stiffness reduction of an 8-story frame structure is provide to compare the detect ability of the proposed three new methods and CorV/IPV method. The damage detection results show that the proposed three new methods can not only effectively locate the damage even when noise exists but also require no reference point compared to CorV/IPV method, among which AMV method is the best. As only the structural vibration responses of measurement points before and after damage are needed for damage detection, these methods can be applied to structural health monitoring.*

1 INTRODUCTION

During functional age many large structures may experience some local damage, which reduces structural reliability and durability, this may cause catastrophic, economic and human life loss. How to detect the damage as soon as it appears attracts significant attention of engineers and researchers in recent years. The process of utilizing a damage detection strategy for engineering structures can be defined as Structural Health Monitoring (SHM) [1].

Damage detection by correlation function has been widely discussed by researchers and utilized in practical applications. Yang and Yu proposed a damage detection method based on Cross Correlation Function Amplitude Vector (CorV) [2]. The CorV of a structure is related to the frequency response function (FRF) under a steady random excitation with a specific frequency spectrum and the normalized CorV has a specific shape, which is useful for structural damage detection. Based on the concept of CorV and NExT [3, 4], Yang and Wang [5-7] proposed a damage detection method using the inner product vector (IPV) of structural responses. The IPV is proved to be a weighted summation of the mode shapes and can be directly calculated from the time domain vibration responses subject to white noise excitation. The effectiveness of the IPV method was demonstrated by delamination damage detection for a composite laminate beam and various damage detection experiments. Both of the CorV and IPV method have a high degree of accuracy in locating the damage.

In this paper three new correlation-function-based damage detection methods are proposed. Firstly, the theory of cross correlation function of vibration response under sinusoidal excitation is studied and based on that the damage indexes are proposed. Then stiffness reduction detection of an 8-story frame structure is used to compare these proposed methods and CorV/IPV method. Finally, some conclusions are summarized.

2 THEORY OF CROSS CORRELATION FUNCTION AND DAMAGE INDEX

2.1 Cross correlation function

Assuming the standard matrix equations of motion is given by

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}$ is a vector of forcing functions and $\mathbf{x}$ is the vector of displacements. Real normal modes and proportional damping $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ are assumed.

Suppose x_{ik} and y_{jk} are responses at point i and j due to input f_k at point k, x_{ik} is from structural health state s1 and y_{jk} is from structural health state s2

$$x_{ik}(t)=\sum_{r=1}^{n}\psi_{ir}\psi_{kr}\cdot\int_{-\infty}^{t} f_k(\tau)g^{s1,r}(t-\tau)d\tau \tag{2}$$

$$y_{jk}(t)=\sum_{s=1}^{n}\phi_{js}\phi_{ks}\cdot\int_{-\infty}^{t} f_k(\tau)g^{s2,s}(t-\tau)d\tau \tag{3}$$

where ψ_{ir}, ψ_{kr} is the ith and kth component of mode shape $\mathbf{\Psi}_r$ of structural health state s1 and ϕ_{js}, ϕ_{ks} is the jth and kth component of mode shape $\mathbf{\Phi}_s$ of structural health state s2, respectively. The function

$$g^i(t)=\begin{cases} 0 & t<0 \\ \dfrac{1}{m^i\omega_d^i}e^{-\zeta^i\omega_n^i t}\sin(\omega_d^i t) & t\ge 0 \end{cases} \tag{4}$$

where m^i is the i th modal mass, ω_n^i is the i th modal frequency, $\zeta^i = \frac{1}{2}(\frac{\alpha}{\omega_n^i} + \beta\omega_n^i)$ is the i th modal damping ratio, and $\omega_d^i = \omega_n^i\sqrt{1-(\zeta^i)^2}$ is the i th damped modal frequency.

The cross correlation function $R_{ij}(T)$ of x_{ik} and y_{jk} can be defined [8]

$$R_{ij}(T) = E\left[x_{ik}(t+T)y_{jk}(t)\right] \tag{5}$$

where E is the expectation operator.

Substitute Eqs. (2) and (3) into Eq. (5) results in

$$R_{ij}(T) = \sum_{r=1}^{n}\sum_{s=1}^{n}\psi_{ir}\psi_{kr}\phi_{js}\phi_{ks}\cdot\int_{-\infty}^{t}\int_{-\infty}^{t+T} g^{s1,r}(t+T-\sigma)g^{s2,s}(t-\tau)E\left[f_k(\sigma)f_k(\tau)\right]d\sigma d\tau \tag{6}$$

Suppose the input f_k is sinusoidal excitation, the auto correlation function of the input can be written as

$$R_{f_k}(\tau-\sigma) = E\left[f_k(\tau)f_k(\sigma)\right] = \frac{X_k^2}{2}\cos[2\pi f_{0,k}(\tau-\sigma)] \tag{7}$$

where X_k and $f_{0,k}$ are amplitude and frequency of the sinusoidal excitation, respectively.

Using the definition of $g(t)$ from Eq. (4) and substituting Eq. (7) into the Eq. (6), the cross correlation function $R_{ij}(T)$ can be written as

$$R_{ij}(T) = \boldsymbol{\beta}_i^T\boldsymbol{\Lambda}_k(T)\boldsymbol{\gamma}_j \tag{8}$$

where $\boldsymbol{\beta}_i = [\psi_{i1},\psi_{i2},...,\psi_{in}]^T$, $\boldsymbol{\gamma}_j = [\phi_{j1},\phi_{j2},...,\phi_{jn}]^T$ and $[\boldsymbol{\Lambda}_k(T)]_{rs} = \xi_k^{rs}(T) = f(X_k, f_{0,k}, \omega_n^{s1}, \omega_n^{s2}, \psi_{kr}, \phi_{ks}, T)$ is only related to the excitation and the modal parameters of the structure.

2.2 Damage index

While the damage index of CorV method is formed by the maximum values of the cross correlation function of responses, the damage index of IPV method is formed by the point zero value of the cross correlation function of responses, CorV method and IPV method can be both regarded as the damage detection methods based on the correlation function. But from the definition of CorV and IPV, they both need to set a reference point, which indicates special reliability of that point. In this section, three new damage indexes are proposed without the use of a reference point.

2.2.1. AMV method

If two signals are the same when calculating the cross correlation function, the cross correlation function can be also called as auto correlation function. Set the maximum value of the auto correlation function of the responses from different measurement points as a vector

$$\mathbf{R}_{AMV} = [\max(R_{11}(T)), \max(R_{22}(T)),...,\max(R_{nn}(T))]^T \tag{9}$$

where $\max(R_{ii}(T))$ $i = 1,2,...,n$ is the maximum value of the auto correlation function of the response from measurement point i, which is the same as the point zero value of the auto correlation function of the response from measurement point i.

In the definition of the cross correlation function Eq. (8), there is a multiplying constant referring to the sinusoidal excitation. In order to eliminate the influence of the excitation, the vector should be normalized. $\mathbf{R}_{AMV}$ is normalized by its root mean square value:

$$\overline{\mathbf{R}}_{AMV}=\frac{\mathbf{R}_{AMV}}{\sqrt{\frac{1}{n}\mathbf{R}_{AMV}^{T}\mathbf{R}_{AMV}}} \tag{10}$$

where n is the element number in the vector $\mathbf{R}_{AMV}$.

In the following sections the normalized AMV is still denoted by the symbol $\mathbf{R}_{AMV}$ for the sake of convenience. Then the relative change of $\mathbf{R}_{AMV}$ at measurement point i is defined as:

$$D_{AMV,i}=\frac{R_{AMV,i}^{s2}-R_{AMV,i}^{s1}}{R_{AMV,i}^{s1}} \tag{11}$$

where s2 and s1 indicate different structural health states, for example, s2 is the damaged structural health state and s1 is the intact structural health state. Thus, the damage index of the auto correlation function without reference point at maximum point value vector (AMV) method can be defined as $\mathbf{D}_{AMV}=[D_{AMV,1},D_{AMV,2},...D_{AMV,n}]^{T}$.

2.2.2. CMV method and CZV method

Set the maximum value of the cross correlation function of the response from the same measurement point between structural health state s1 and structural health state s2 as a vector

$$\mathbf{R}_{CMV}^{s1s2}=[\max(R_{11}^{s1s2}(T)),\max(R_{22}^{s1s2}(T)),...,\max(R_{nn}^{s1s2}(T))]^{T} \tag{12}$$

where $\max(R_{ii}^{s1s2}(T))$, $i=1,2,...,n$ is the maximum value of the cross correlation function of the response from measurement point i between health state s1 and health state s2.

Different from the auto correlation function, the maximum value and the point zero value of the cross correlation function is normally not the same. So similarly, set the point zero value of the cross correlation function from the same measurement point between structural health state s1 and structural health state s2 as a vector

$$\mathbf{R}_{CZV}^{s1s2}=[R_{11}^{s1s2}(0),R_{22}^{s1s2}(0),...,R_{nn}^{s1s2}(0)]^{T} \tag{13}$$

where $R_{ii}^{s1s2}(0)$, $i=1,2,...,n$ is the point zero value of the cross correlation function of the response from measurement point i between structural health state s1 and structural health state s2.

Like $\mathbf{R}_{AMV}$, $\mathbf{R}_{CMV}$ and $\mathbf{R}_{CZV}$ are normalized by their root mean square value. After they are normalized, the relative change between $\mathbf{R}_{CMV}$ and $\mathbf{R}_{AMV}$, $\mathbf{R}_{CZV}$ and $\mathbf{R}_{AMV}$ can be written

$$D_{CMV,i}=\frac{R_{CMV,i}^{s1s2}-R_{AMV,i}^{s1}}{R_{AMV,i}^{s1}} \tag{14}$$

$$D_{CZV,i}=\frac{R_{CZV,i}^{s1s2}-R_{AMV,i}^{s1}}{R_{AMV,i}^{s1}} \tag{15}$$

Thus, the damage index of the cross correlation function without reference point at maximum point value vector (CMV) method can be defined as $\mathbf{D}_{CMV}=[D_{CMV,1},D_{CMV,2},...D_{CMV,n}]^{T}$, similarly, the damage index of the cross correlation function without reference point at point zero value vector (CZV) method can be defined as $\mathbf{D}_{CZV}=[D_{CZV,1},D_{CZV,2},...D_{CZV,n}]^{T}$.

3 NUMERICAL SIMULATION

An 8-story shear frame structure, as shown in Figure 1, is adopted as the numerical simulation model in this paper. Assume that the mass of every story is centralized on its floor, the stiffness of each floor is supplied by the braces, and the stiffness in y direction is much larger than that in x direction. So only the movement in x direction is considered.

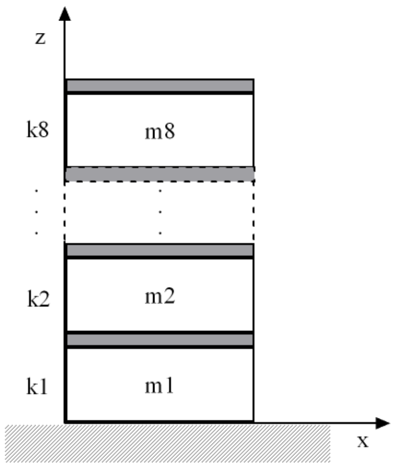


Figure 1. 8-Story Frame Structure

The mass m_i of each floor is 1 kg and the stiffness coefficient k_i of each floor is 25,000 N/m ($i = 1,2,...,8$). Based on these assumptions, the mechanical model of the frame structure can be expressed as an 8-dof discrete system. The mass matrix and stiffness matrix of the system can be written as

$$\mathbf{M} = diag[m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8] \tag{16}$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -k_7 & k_7 + k_8 & -k_8 \\ & & & -k_8 & k_8 \end{bmatrix} \tag{17}$$

The damage of the frame structure is simulated by reducing the stiffness coefficient of one floor. In the following, the stiffness coefficient of floor 3, 5, 7 is reduced by 5% as damage, respectively. The different structural health states can be seen in Table 1. The first natural frequency of the frame structure is 4.64Hz. When the damage occurs in the structure, the relative change of the first natural frequency is also shown in Table 1. As can be seen from Table 1, the first natural frequency of the structure changes only a little compares to the intact structure when damage exists. So the damage that occurs in the structure is very small.

Structural health state	I10	D13	D15	D17
Damage location	/	3	5	7
Damage extents/%	/	5	5	5
First frequency change/%	/	-0.49	-0.28	-0.08

Table 1. Different structural health state of the frame structure

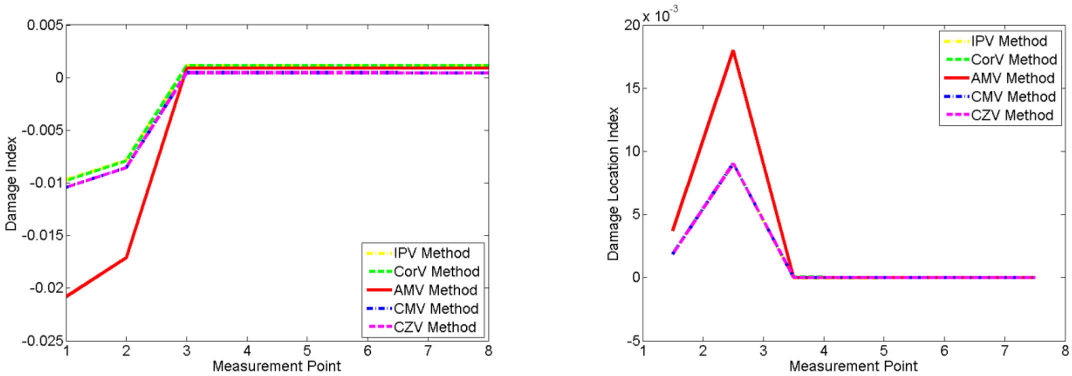
A 5 Hz sinusoidal excitation is used as excitation applied on the first floor. The excitation has the duration of 16s with sample frequency of 1024 Hz. As the acceleration response is much easier to get, we use acceleration to compute the damage index of the proposed damage detection methods. The Wilson-θ method is utilized to get the acceleration of different structural health states.

Damage index of the proposed AMV method, CMV method and CZV method respectively can be calculated. Set the structural health state I10 as s1 and the structural health state D13, D15 and D17 respectively as s2 in Eqs. (11), (14) and (15). The results are shown in Figure 2(a), Figure 3(a) and Figure 4(a). From these figures, the change of the damage index with measurement point can be called ‘step change’ [6]. So in order to make the damage more obvious, define the damage location index of these methods

$$D'_{,i+0.5} = D_{,i+1} - D_{,i} \tag{18}$$

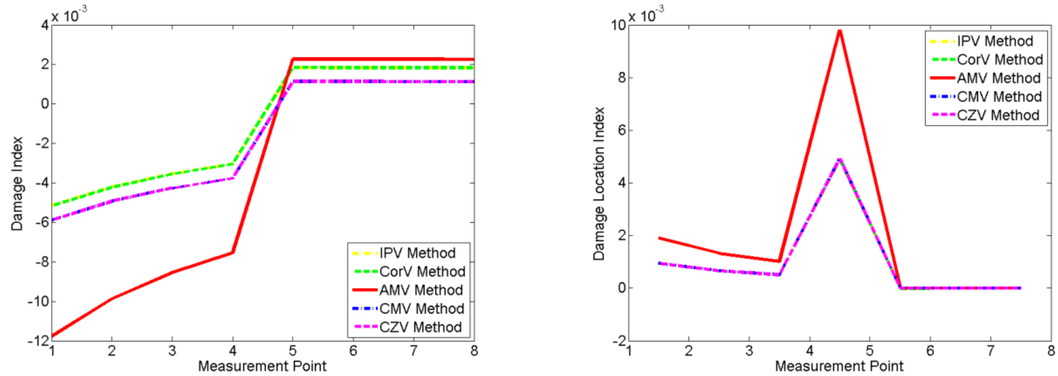
where the abrupt change in damage index D corresponds to the local maximum of D'. The abrupt change can be considered as a result of the damage in the structure. Then, if the local maximum of D' occurs in $i+0.5$, the damage is located between measurement point i and $i+1$.

The damage location index is shown in Figure 2(b), Figure 3(b) and Figure 4(b). In order to compare the detection ability of the correlation-function-based damage detection methods, the damage detection results of CorV method and IPV method are also plotted in these figures. Set the reference point of the CorV method and IPV method as measurement point 1.



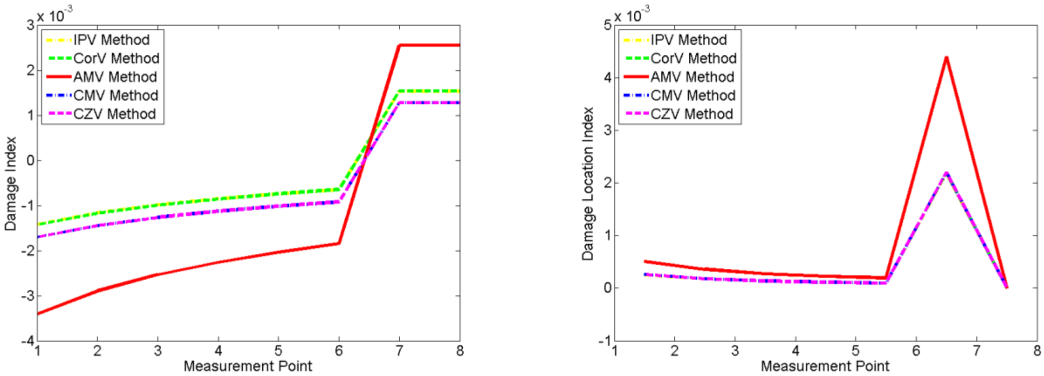
(a) Damage index (b) Damage location index

Figure 2. Damage detection result of structural state D13



(a) Damage index (b) Damage location index

Figure 3. Damage detection result of structural state D15



(a) Damage index (b) Damage location index

Figure 4. Damage detection result of structural state D17

In Figure 2(a), the damage indexes of AMV method, CMV method and CZV method change sharply from measurement point 2 to 3, correspondingly in Figure 2(b) the damage location indexes of these methods have a peak value at measurement point 2.5, which indicates that the damage is located between measurement point 2 and 3. In Figure 3(a), the damage indexes of AMV method, CMV method and CZV method change rapidly from measurement point 4 to 5, correspondingly in Figure 3(b) the damage location indexes of these methods have a peak value at measurement point 4.5, which indicates that the damage location is between measurement point 4 and 5. In Figure 4(a), the damage indexes of AMV method, CMV method and CZV method have a rapid change from measurement point 6 to 7, correspondingly in Figure 4(b) the damage location index have the peak value at measurement point 6.5, which indicates that the damage is located between measurement point 6 and 7. From these three damage detection results, the peak value of the damage location index of AMV method, CMV method and CZV method is just the location of the damage simulated. So the damage location index of these methods can all locate the damage effectively.

In Figure 2-Figure 4, the damage location index of the AMV method at the peak value is much larger than the other four methods, while at the other measurement locations, their values are nearly the same, which makes the damage location more obvious to identify. Furthermore, the AMV method uses no reference point when calculating the damage location index in compare to the IPV and CorV method. Thus we can infer that the AMV method is better than the other four correlation-function-based damage detection methods.

Figure 5-Figure 7 displays the damage location index change with the stiffness reduction of the three structural health states. The damage is simulated by reducing the stiffness of the corresponding floor from 0%-50% respectively. As CorV method and IPV method both need the reference point and have similar detection ability shown in Figure 2-Figure 4, only the damage detection results of IPV method, AMV method, CMV method and CZV method are plotted. As seen from these figures, the damage location indexes of these four methods have the same phenomenon. The damage location index of the proposed three damage detection methods at the damage location changes with the stiffness reduction: if the stiffness reduction becomes larger, the damage location index becomes larger. On the other hand, the other points nearly do not show any change when the damage becomes severer. Therefore, the damage location index of these four methods all have the ability to show how severe the damage is. If the damage location index is monitored, we can find the damage location, and we can know how severe the damage is. This is very important and suitable for the online structural health monitoring.

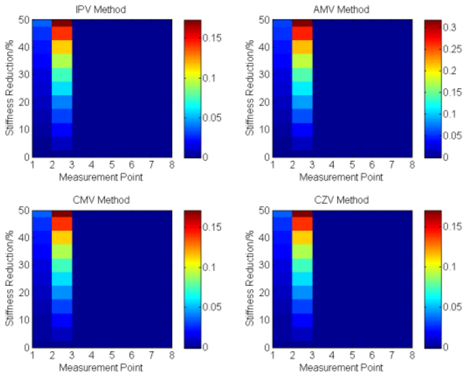


Figure 5. DLI of damage at location 3

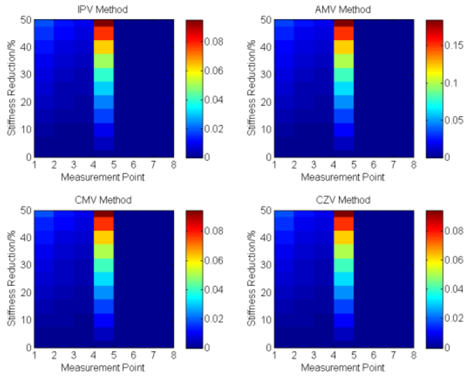


Figure 6. DLI of damage at Location 5

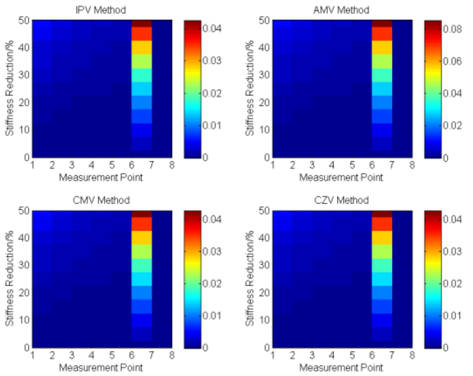


Figure 7. DLI of damage Location 7

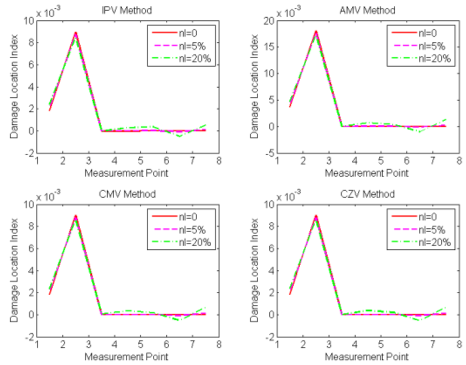


Figure 8. Structural health state D13

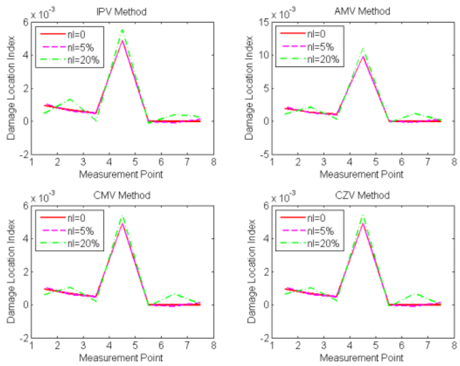


Figure 9. Structural health state D15

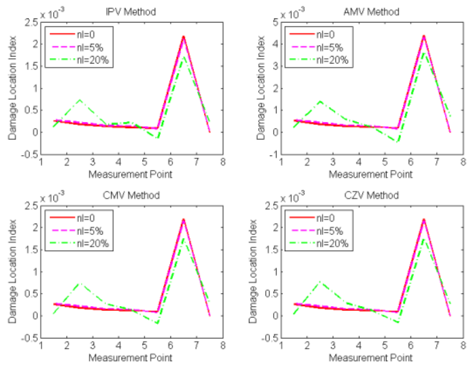


Figure 10. Structural health state D17

Noise is common in real life, which affects the response of the structure. So in this section noise is added to the response to verify the robustness of the proposed methods. Suppose that the measurement noise is Gaussian noise with standard deviation σ which depends on the noise level, and the measurement noise is independent of the value of the vibration response without measurement noise. The noise level is defined as

$$nl = \frac{rms(\delta)}{rms(x)} \times 100\% \tag{19}$$

where $rms(\cdot)$ indicates the root mean square value of the signal, x is the signal without measurement noise and δ is the added noise of x.

Figure 8-Figure 10 display the damage detection results when different noise level is added to the response signal. As can be seen in Figure 8-Figure 10, when noise level is low (5%), the damage location indexes of the AMV method, CMV method and CZV method change little from the original damage location index without noise. When the noise level increases to 20%, the changes are bigger, but even that, the damage location indexes of the proposed three methods can still locate the damage. Compared to the results obtained by IPV method, the proposed three methods have the same anti-noise ability.

4 CONCLUSIONS

In this paper the cross correlation function of vibration signals under sinusoidal excitation is investigated. Damage indexes of AMV method, CMV method and CZV method are proposed based on that. Stiffness reduction detection of an 8-story frame structure shows that the proposed methods can locate the damage effectively even when the damage is very small. Besides, they all have very good anti-noise ability. Among the three methods proposed in this paper, CorV method and IPV method, AMV method can locate the damage more efficiently. Moreover, it also has the advantage of no use of the reference point.

Although the damage location index increases when the damage becomes severer, this is just the case for same damage location, thus it can imply the damage extent of that location. While the damage location indexes of the proposed methods have an unknown relationship with the damage extent in different locations, these correlation-function-based methods cannot be utilized to evaluate the damage extent quantitatively. This is still future work be done.

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