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STRUCTURAL DAMPING IDENTIFICATION USING FINITE ELEMENT MODEL UPDATING

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Abstract. *Most of Finite Element model updating techniques do not employ damping matrices and hence, cannot be used for accurate prediction of complex frequency response functions (FRFs). Some research efforts have been made to update viscous damping matrix along with mass and stiffness matrices. In this paper, new structural damping identification method using finite element model updating technique is proposed and tested with the objective that the damped finite element updated model is able to predict the measured FRFs accurately. The proposed structure damping identification method requires prior knowledge of accurate mass and stiffness matrices. Thus, the proposed structure damping identification method is a two-step procedure. In the first step, mass and stiffness matrices are updated using FRF data and in the second step, structural damping matrix is identified using updated mass and stiffness matrices, which are obtained in the previous step. The effectiveness of the proposed procedure is demonstrated by two numerical simulated examples. Firstly, a study is performed using a lumped mass and spring system. The lumped mass and spring system study is followed by case involving numerical simulation of fixed-fixed beam. The results have shown that the proposed method able to identify the damping matrices accurately.*

1 INTRODUCTION

The modeling of damping is a very complex and still considered somewhat an unknown or grey area. The effects of damping are clear, but the characterization of damping is a puzzle waiting to be solved. A major reason for this is that, in contrast with inertia and stiffness forces, it is not in general clear which state variables are relevant to determine the damping forces. A commonly used model originated by Rayleigh [1] assumes that instantaneous generalized velocities are the only variables. The Taylor expansion then leads to a model, which encapsulates damping behavior in a dissipation matrix, directly analogous to the mass and stiffness matrices. However, it is important to avoid the misconception that, this is the only model of vibration damping. It is possible for the damping forces to depend upon values of other quantities. Any model, which guarantees that the energy dissipation rate is non-negative, can be a potential candidate to represent the damping of a given structure. The appropriate choice of damping model depends of course on the detailed mechanisms of damping. Unfortunately these mechanisms are more varied and less well-understood than the physical mechanisms governing the stiffness and inertia. In broad terms, damping mechanisms can be divided into three classes

1. Energy dissipated throughout the bulk material making up the structure which is also called as material damping.
2. Dissipation of energy associated with junctions or interfaces between parts of the structure, generally called as boundary damping.
3. Dissipation of energy associated with a fluid in contact with the structure which is also called as viscous damping.

Material damping can arise from variety of micro structural mechanisms (Bert, [2]) but for small strains it is often adequate to represent it through an equivalent linear, visco-elastic continuum model of the material. Damping can then be taken into account via the viscoelastic correspondence principle, which leads to the concept of complex moduli. Boundary damping is less easy to model than material or viscous damping but it is of crucial importance in most of engineering structures. When damping is measured on a built structure, it is commonly found out to be at least an order of magnitude higher than the intrinsic material damping of the main components of the structure. This difference is attributed to effects such as frictional micro- slipping at joints and the air pumping in riveted seams. In such a system the energy loss mechanism would no doubt be significantly non-linear if examined in detail. But it can be considered linear provided it is small. This issue is discussed in detail by Heckl [3] assuming small damping. He found that linear theory produce acceptable response predictions for panels whose damping mechanism arose from a bolted joint on beam. Oliveto and Greco [4] conducted a study on how the modal damping ratios change with different boundary conditions and they found that Rayleigh-type damping is actually independent of the boundary conditions and modal damping ratios can be easily converted from one boundary condition to another. When a structure exhibits a damped dynamic behavior that doesn’t conform to the classical and well known viscous or hysteric damping models, such problems are addressed by means of fractional derivatives leading to a model in terms of general damping parameters. Maia et al. [5] discussed the use of fractional damping concept for the modeling the dynamic behavior of the linear systems and showed how this concept allows for clearer interpretation and explanation of the behavior displayed by common viscous and hysteric damping models. Agrawal and Yuan [6] modeled the damping forces proportional to the fractional derivative of displacements and the fractional differential equations governing the dynamics of a system. Adhikari and Woodhouse [7] developed four indices to quantify non-viscous damping in discrete linear system. Two of these indices are based on non-viscous damping while third one is based on the residue matrices of the system transfer function and the fourth is based on measured complex modes of the system. Damping identification has important applications in many engineering fields such as modal analysis, condition monitoring and structural dynamic modifications. Chen et al. [8] presented a method for getting the spatial model from complex frequency response function. Unfortunately, it is unrealistic to assume that all pertinent information is given to solve for damping matrix. Actually, data from testing is neither complete nor error free. Minas and Inman [9] proposed a method which assumes that analytical mass and stiffness matrices are determined a priori from a finite element model. Eigenvalues and eigenvectors are obtained experimentally, and are allowed to be incomplete, as would be expected from modal testing. The mass and stiffness matrices are reduced to the size of the modal data available. The identified damping matrix is assumed to be real, symmetric and positive definite. The structure must exhibit complex modes for this procedure and the solution is limited to real symmetric positive definite damping matrices. Beliveau [10] uses natural frequencies, damping ratios, mode shapes and phase angles to identify parameters of viscous damping matrix. The identification is performed iteratively. The mass and stiffness matrices are reduced to the size of the modal data available. This method involves solving an n^{th} order system of linear equations for each eigenvector, making it fairly inefficient. Lancaster [11] proposed a method of identifying the mass, stiffness and damping matrices of a system directly given only the eigenvalues and eigenvectors. The input data must be normalized in a very specific way for the method to work. The mass and damping matrices to be used to normalize the eigenvectors, which are subsequently used to calculate the damping matrix. This method is only for calculating the viscous damping and Lancaster concludes by stating “the theory is there, should the experimental techniques ever become available. It is still not possible to measure the normalized eigenvectors”. The shortfall of this method comes in normalizing the eigenvectors, which requires knowledge of the very same damping matrix which we wish to, find in the end. Pilkey [12, 13] proposed two methods for computing the damping matrix using modal data. The first method is an iterative method which requires prior knowledge of the mass matrix and eigenvalues and eigenvectors. The second method requires more information but less computationally intensive. This method requires prior knowledge of the mass and stiffness matrices and eigendata. Both the methods developed from the Lancaster [11] concept. Oho et al. [14] proposed a method of identifying experimental set of spatial matrices valid only for the hysterical damping for the entire frequency range of interest using FRFs. Using this method, it is possible to set the number of degrees of freedom much larger than the number of resonant frequencies located inside the frequency range of interest and spatial matrices identified are able to represent the dynamic characteristics of the structure under arbitrary boundary conditions even though the conditions differ from those in place at the time of the identification. The limitation of this method is that it is unable to predict correctly in the modal domain. Lee and Kim [15] proposed an algorithm for the identification of the damping matrices which identifies the viscous and structural damping matrices of the equation of motion of a dynamic system using frequency response matrix. The accuracy of the identified damping matrices depends almost entirely on the accuracy of the measured FRFs, especially their phase angles. Adhikari and Woodhouse [16] identified the damping of the system as viscous damping. However, viscous damping is by no means the only damping model. Adhikari and Woodhouse [17] identified non-viscous damping model using an exponentially decaying relaxation function. Phani and Woodhouse [18] proposed that complex modes arising out of non-proportional dissipative matrix hold the key to successful modeling and identification of correct physical damping mechanisms in the vibrating systems but these identified complex modes are very sensitive to experimental errors and errors arising out from fitting algorithms. Some research efforts have also been made to update the damping matrices. Imregun et al. [19] extended the response function method (RFM) given by Lin and Ewins [20] to update proportional viscous and structural damping matrices by updating the coefficients of viscous and structural damping matrices. Arora et al. [21] identified the structural damping matrix using complex frequency response functions (FRFs) of the structure. In this method, the updating parameters are assumed complex and the imaginary part of the complex updating parameter represents structural damping in the system. Arora et al. [22] proposed a viscous damping identification method in which viscous damping is identified explicitly. This procedure is a two steps procedure. In the first step, mass and stiffness matrices are updated and in the second step, viscous damping is identified using updated mass and stiffness matrices obtained in the previous step. Pradhan and Modak [23] used the normalized FRFs for updating damping matrices. The normalized FRFs are estimated from the complex FRFs.

The proposed structural damping identification method requires prior knowledge of accurate mass and stiffness matrices. This procedure is a two-steps procedure. In the first step, mass and stiffness matrices are updated and in the second step, structural damping is identified using updated mass and stiffness matrices obtained in the previous step. The proposed method doesn't require initial damping estimates. The identified structural damping matrix [D] is both symmetric and positive definite. The effectiveness of the proposed structure damping identification method is demonstrated by two numerical simulated examples. Firstly, a study is performed using a lumped mass system. The lumped mass system study is followed by case involving numerical simulation of fixed-fixed beam. The effect of coordinate incompleteness and robustness of method under presence of noise is investigated. The performance of the proposed structural damping identification method is also investigated for cases of light, medium and heavily damped structures. The results have shown that the proposed method able to identify the damping matrices accurately in all the cases of noisy, complete and incomplete data and with all levels of damping.

2 DIRECT STRUCTURAL DAMPING IDENTIFICATION METHOD

In the following section, the theory and procedure of proposed direct structural damping identification method is developed. The proposed method structural damping identification method requires prior knowledge of accurate mass and stiffness matrices. The proposed method doesn't require initial damping estimates. The method uses complex frequency response function (FRFs) and the accurate mass and stiffness matrices. In this method, damping in the structure is modeled as structural damping. The identified structural damping matrix is both symmetric and positive definite. The governing equations of motion of a multi degree of freedom (DOF) structure with structural damping can be written in matrix form as:

$$[M]\ddot{x}(t)^{C}+([K]+j[D])x(t)^{C}=f(t) \tag{1}$$

where [M],[K] and [D] are n×n DOF mass, stiffness and structural damping matrices respectively. All these are real matrices. $j=\sqrt{-1}$. x(t) and f(t) are n×1 vectors of time-varying displacements and forces and 'N' is the total number of degrees of freedom in the finite element model. The superscript C indicates complex value corresponding to damped system. For harmonic excitation, $f(t) = F(\omega)e^{j\omega t}$ and $x(t) = X(\omega)e^{j\omega t}$. The Eq. (1) becomes:

$$([K] - \omega^2[M])X(\omega)^C + j[D]X(\omega)^C = F(\omega) \tag{2}$$

Real or normal dynamic stiffness matrix (DSM) is given by:

$$\left[Z(\omega)^N\right] = [K] - \omega^2[M] = \left[\alpha(\omega)^N\right]^{-1} \tag{3}$$

where $\left[\alpha(\omega)^N\right]$ represents real or normal FRF matrix. Using above equation, Eq. (1) becomes:

$$\left[Z(\omega)^N\right]X(\omega)^C + j[D]X(\omega)^C = F(\omega) \tag{4}$$

Multiply both sides by real or normal FRF matrix $\left[\alpha(\omega)^N\right]$. The Eq. (4) becomes:

$$X(\omega)^C + j\left[\alpha(\omega)^N\right][D]X(\omega)^C = \left[\alpha(\omega)^N\right]F(\omega) \tag{5}$$

G(ω) is a transformation matrix given by Chen and Tsuei [24]:

$$G(\omega) = \left[\alpha(\omega)^N\right][D] \tag{6}$$

The Eq. (5) becomes:

$$X(\omega)^C + jG(\omega)X(\omega)^C = \left[\alpha(\omega)^N\right]F(\omega) \tag{7}$$

Eq. (7) can be rewritten as:

$$(ID + jG(\omega))X(\omega)^C = \left[\alpha(\omega)^N\right]F(\omega) \tag{8}$$

where ID represents identity matrix. The displacement vector $X(\omega)^C$ is related to input force vector and also to complex FRF matrix $\left[\alpha(\omega)^C\right]$as:

$$x(\omega) = \left[\alpha(\omega)^C\right]f(\omega) \tag{9}$$

complex FRF matrix $\left[\alpha(\omega)^C\right]$ consist of real and imaginary parts as:

$$\left[\alpha(\omega)^C\right] = \left[\alpha(\omega)_R^C\right] + j\left[\alpha(\omega)_I^C\right] \tag{10}$$

where $\left[\alpha(\omega)_R^C\right]$ and $\left[\alpha(\omega)_I^C\right]$ represents real and imaginary part of complex FRF matrix $\left[\alpha(\omega)^C\right]$. The subscripts R and I represents real and imaginary values respectively. The complex FRF $\left[\alpha(\omega)^C\right]$ is available from experimentation. Substituting Eq. (10) in Eq. (8), we get:

$$\left(ID + jG(\omega)\right)\left(\left[\alpha(\omega)_R^C\right] + j\left[\alpha(\omega)_I^C\right]\right) = \left[\alpha(\omega)^N\right] \tag{11}$$

$$\left(\left[\alpha(\omega)_R^C\right] - G(\omega)\left[\alpha(\omega)_I^C\right]\right) + j\left(G(\omega)\left[\alpha(\omega)_R^C\right] + \left[\alpha(\omega)_I^C\right]\right) = \left[\alpha(\omega)^N\right] \tag{12}$$

since the right-hand side of the above equation is real so the imaginary part of the left-hand side must be equal to a zero matrix for all frequencies, hence:

$$G(\omega)\left[\alpha(\omega)_R^C\right] + \left[\alpha(\omega)_I^C\right] = 0 \tag{13}$$

$$G(\omega) = -\left[\alpha(\omega)_R^C\right]^{-1}\left[\alpha(\omega)_I^C\right] \tag{14}$$

using Eq. (6). The above equation becomes:

$$\left[\alpha(\omega)^N\right]\left[D\right] = -\left[\alpha(\omega)_R^C\right]^{-1}\left[\alpha(\omega)_I^C\right] \tag{15}$$

$$\left[D\right] = -\left[\alpha(\omega)_R^C\right]^{-1}\left[\alpha(\omega)_I^C\right]\left[\alpha(\omega)^N\right]^{-1} \tag{16}$$

the structural damping matrix can be identified from above equation as:

$$\left[D\right] = -\left[\alpha(\omega)_R^C\right]^{-1}\left[\alpha(\omega)_I^C\right]\left[\alpha(\omega)^N\right]^{-1} \tag{17}$$

since the normal dynamic stiffness matrix [Z(ω)^N] is inverse of normal FRF $\left[\alpha(\omega)^N\right]$ matrix $\left(\left[Z(\omega)^N\right] = \left[\alpha(\omega)^N\right]^{-1}\right)$ given in Eq. (3). Using Eq. (3), the above equation becomes:

$$[D] = -\left[\alpha(\omega)_R^C\right]^{-1}\left[\alpha(\omega)_I^C\right]\left[Z(\omega)^N\right] \tag{18}$$

The above equation is the basic equation for the structural damping identification. The identified structural damping matrix [D] is both symmetric and positive definite. The Eq. (18) is applied to frequency range of interest. This makes Eq. (18) over determined. If nf is the number of frequency points used to identify the structural damping. The Eq. (15) becomes:

$$[D]_{n\times n} = -\begin{bmatrix} \left[\alpha(\omega_1)_R^C\right] \\ \left[\alpha(\omega_2)_R^C\right] \\ \vdots \\ \vdots \\ \vdots \\ \left[\alpha(\omega_{nf})_R^C\right] \end{bmatrix}^{+}_{(n\times nf)\times n} \cdot \begin{bmatrix} \left[\alpha(\omega_1)_I^C\right]\left[Z(\omega_1)^N\right] \\ \left[\alpha(\omega_2)_I^C\right]\left[Z(\omega_2)^N\right] \\ \vdots \\ \vdots \\ \vdots \\ \left[\alpha(\omega_{nf})_I^C\right]\left[Z(\omega_{nf})^N\right] \end{bmatrix}_{(n\times nf)\times n} \tag{19}$$

superscript '+' denotes pseudo-inverse of a matrix. Eq. (19) is used to identify structural damping matrix [D] of the structure. $\left[\alpha(\omega)_R^C\right]$ and $\left[\alpha(\omega)_I^C\right]$ are the real and imaginary part of the FRF matrix, which are obtained from the experimentation. The accurate normal dynamic stiffness matrix (DSM) $\left[Z(\omega)^N\right]$ depends upon accurate knowledge of mass and stiffness matrices. Finite element model updating can be used to obtain the accurate mass and stiffness matrices of the structure. The real life measured data is always incomplete, as it is not practical to measure all the coordinates specified in the analytical finite element model. For the proposed structural damping identification method mass and stiffness matrices are reduced to the degrees of freedom measured using iterated IRS method [25] considering the measurement points.

3 CASE STUDY OF LUMPED MASS SYSTEM

4 degree- of freedom lumped mass system shown in the Fig. 1 is described by lumped masses M_1, M_2, M_3, M_4 of 5 Kg each and spring stiffnesses K_1, K_2, K_3, K_4 of 2×10^6 N/m each, the structural damping coefficients D_1, D_2, D_3, D_4 of 6×10^4 N/m, 8×10^4 N/m, 5×10^4 N/m, 9×10^4 N/m respectively. The mass and stiffness matrices for this simple system are provided. The desired damping matrix is also provided to create a better understanding of the process. Because this is a contrived example, the described result is known ahead of time. As the system has 4 DOF, 16 complex FRFs are calculated for each frequency forming a 16×16 complex FRF matrix of a function of frequency. The dynamic stiffness matrix is also calculated from the analytical mass and stiffness matrices for each frequency. As all the 4 natural frequencies are in the frequency range of 0-200 Hz. The frequency range from 0-200 is used to identify the structural damping using the proposed procedure. In this case study, it is assumed that accurate mass and stiffness matrices are known. Fig. 2 (a) shows the overlay of analytical FRF, which is undamped, and simulated experimental FRF obtained considering structural damping in the experimental data. It can be observed that the analytical FRF and simulated experimental FRF do not match with each other at resonance regions.

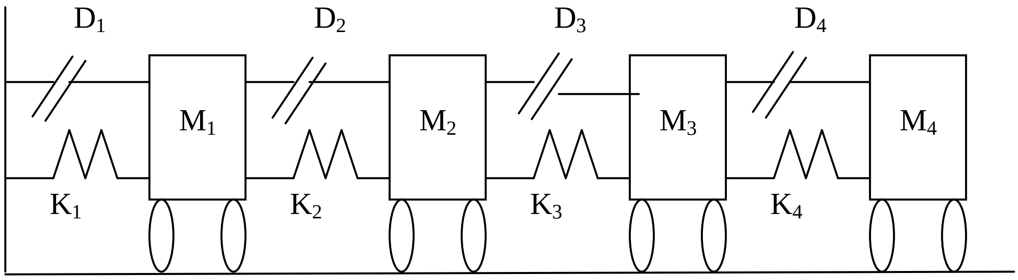
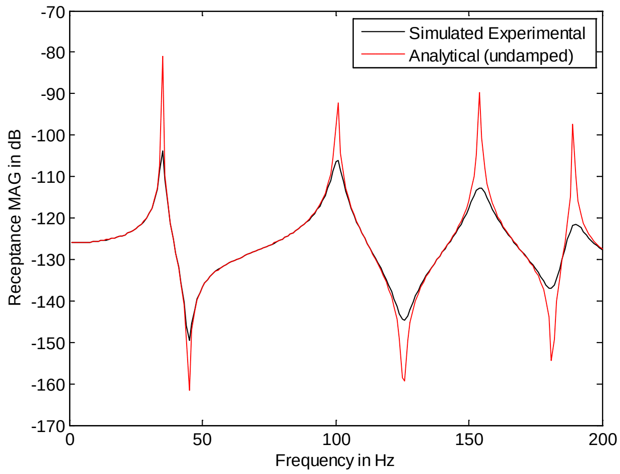
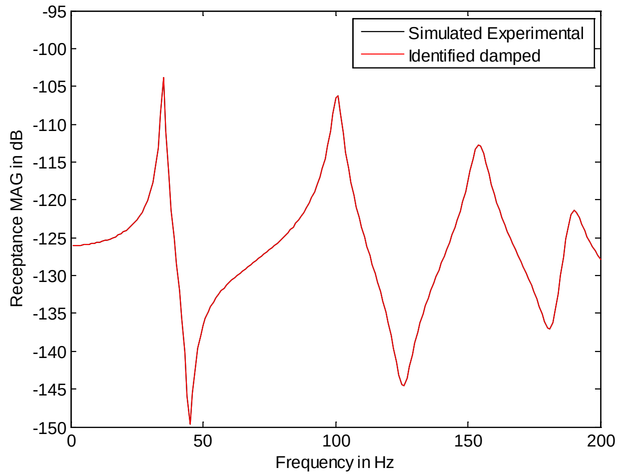


Fig. 1: Lumped mass system



(a) Overlay of simulated experimental FRF and analytical FRF of lumped mass system



(b) Overlay of simulated experimental FRF and damped FRF of lumped mass system after structural damping identification.

Fig. 2: Overlay of simulated experimental FRF and damped identified FRF of lumped mass system.

Fig. 2 (b) shows the overlay of simulated experimental FRF and identified damped FRF. It can be observed from Fig. 2 (b) that after damping identification, simulated experimental FRF and identified damped FRF match exactly and the two are virtual indistinguishable.

4 CASE STUDY OF FIXED-FIXED BEAM STRUCTURE

A simulated study on a fixed-fixed beam is conducted to evaluate the effectiveness of the proposed structural damping identification method. The dimensions of the beam are $910 \times 50 \times 5$ mm. The beam is modeled using thirty, two noded beam elements with nodes at the two ends being fixed as shown in Fig. 3.

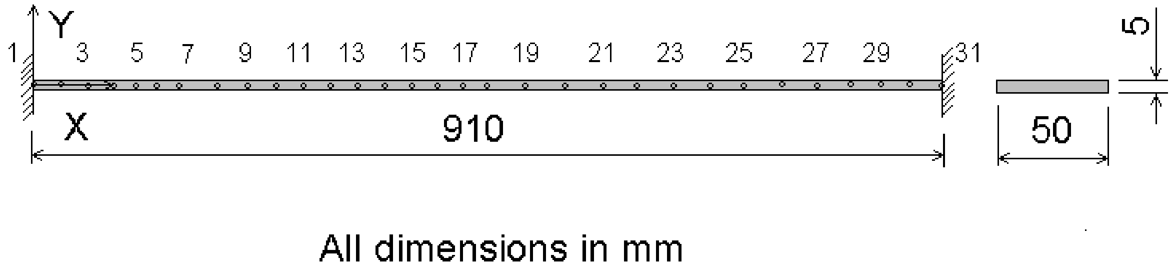


Fig. 3: Beam structure and its FE mesh

The simulated complex FRF data is treated as experimental data. The structural damping for obtaining simulated experimental FRF is considered proportional to mass and stiffness matrices as:

$$[D] = \alpha_S [M] + \beta_S [K] \tag{20}$$

where α_S and β_S are damping constants for structural damping.

Simulated experimental data is obtained by generating a damped finite element model with different levels of structural damping namely lightly damped, moderately damped and highly damped. The values of structural damping constants for the above 3 cases are given in Table 1 [21]. The performance of the proposed method is judged on the basis of accuracy with which the FRFs obtained by identified damped model match with the simulated experimental FRFs using a quality index referred to as percentage average error in FRF (AEFRF), which is calculated as:

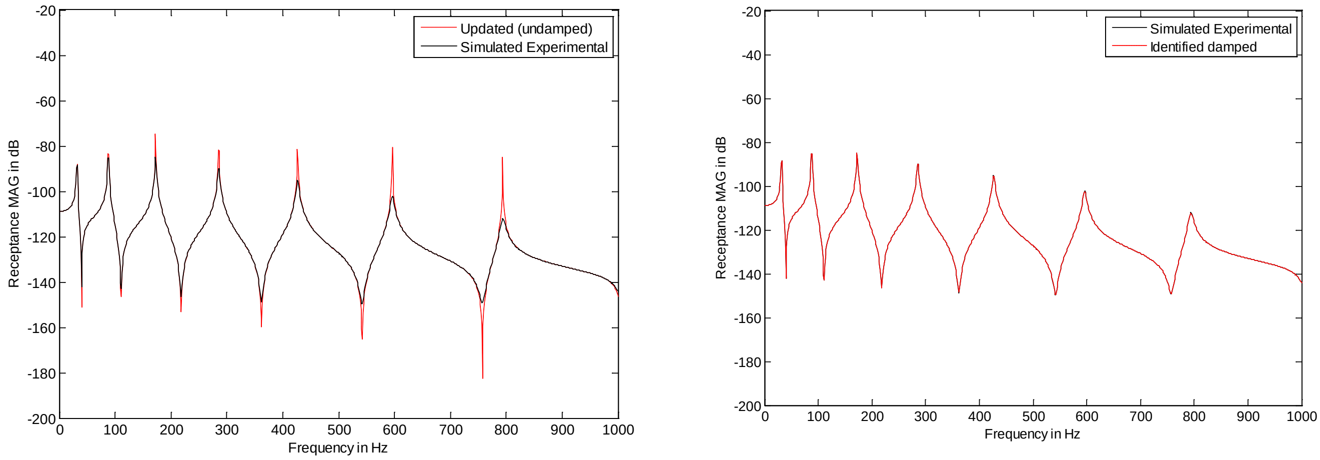
$$AEFRF = \frac{100}{n_f} \sum_{j=1}^{nf} abs\left(\frac{([\alpha(f_j)])_A - ([\alpha(f_j)])_X}{([\alpha(f_j)])_X} \right) \tag{21}$$

Where n_f is the frequency range to be considered (in present case 0 - 1000 Hz)

Levels of damping	Structural damping	
	α_s	β_s
Lightly damped	0.001	0.001
Moderately damped	0.01	0.01
Highly damped	0.1	0.1

Table 1: Damping constants values for modeling different levels of structural damping in the system [21]

Firstly, a case study is presented in which moderate structural damping is used to obtain complex simulated experimental FRF noise free data using complete data to observe the effectiveness of the proposed method in identifying the structural damping matrix. Fig. 4(a) shows the overlay of analytical FRF with simulated experimental FRF. It can be observed from Fig 4 (a) that analytical FRF matches with experimental FRF except at the resonance and anti-resonance frequencies as analytical model is considered undamped.



(a) Overlay of simulated experimental FRF and analytical FRF

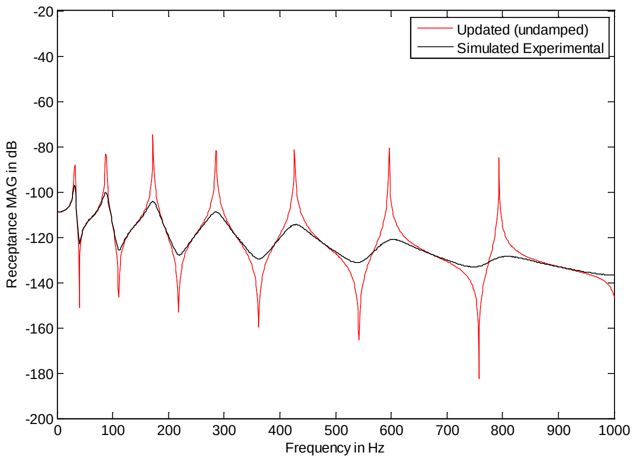
(b) Overlay of simulated experimental FRF and damped FRF

Fig. 4: Overlay of simulated experimental FRF and damped identified FRF for the case of moderate damping.

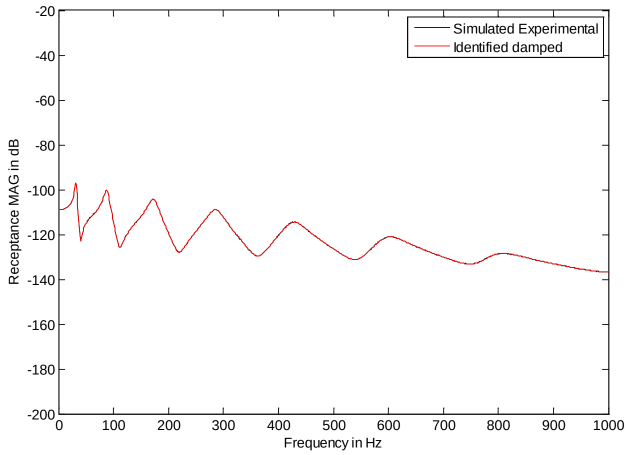
Structural damping is identified using complex FRFs. The proposed structural damping identification is applied. Fig. 4 (b) shows the overlay of the identified damped FRF and simulated experimental data. It can be observed from Fig. 4 (b) that identified damped FRF matches completely with the simulated experimental FRF and both the FRFs are virtual indistinguishable completely and the error in simulated damping and identified damping matrices is very low as shown in Fig. 8. The AEFRF for this case is 0%.

The proposed method has also been evaluated for the cases of incomplete and noisy data. The real life measured data is always incomplete, as it is not practical to measure all the coordinates specified in the analytical finite element model and always contains some measurement noise. Incompleteness is considered by assuming that only lateral degrees of freedom, at all the 29 nodes are measured. This has been referred as 50% incomplete data. For direct method of damping identification, accurate mass and stiffness matrices are reduced to the degrees of freedom measured using iterated IRS method [25] considering the measurement points. Different levels of noise i.e. noise free, 1% noise, 2% noise and 3% noise in simulated experimental data are considered.

Table 2 represents various values of AEFRF obtained for different cases and it is found to be in acceptable limits. Some typical results are discussed here. Fig. 5 (a) shows the overlays of undamped FRF and simulated experimental FRF with 3% noise which is obtained from high structural damping based system and Fig. 5 (b) shows the overlay of damped (identified) FRF and simulated experimental FRF. It can be observed from Fig. 5 (b) that the damped (identified FRF) matches completely with the simulated experimental FRF. For this case, AEFRF is 0.38%, which is an acceptable.



(a) Overlay of simulated experimental FRF and analytical FRF



(b) Overlay of simulated experimental FRF and damped FRF

Fig. 5: Overlay of simulated experimental FRF and damped identified FRF for the case of high damping.

Proposed Method				
	0% Noise	1% Noise	2% Noise	3% Noise
Lightly damped	0.00	0.012	0.041	0.08
Moderately damped	0.00	0.019	0.065	0.112
Highly damped	0.012	0.034	0.083	0.385

Table 2. AEFRF (%) after damping identification for different cases using simulating experimental data.

From the simulated experimental studies, it can be concluded that the proposed structural damping identification method is able to predict FRFs accurately various levels of structural damping in the system and also various level of noise in experimental data.

5 CONCLUSIONS

In this paper, a new method of FE structural damping identification is proposed for better FRF matching. The proposed structural damping identification method requires prior knowledge of accurate mass and stiffness matrices. This procedure is a two-steps procedure. In the first step, mass and stiffness matrices are updated and in the second step, structural damping is identified using updated mass and stiffness matrices obtained in the previous step. The proposed method doesn't require initial damping estimates. The identified structural damping matrix is both symmetric and positive definite. The proposed method addresses the difficulties for updating the damping matrices. The proposed method is working successfully for the cases of simulated numerical data. Simulated numerical case studies based on a lumped mass system and fixed-fixed beam structure with structural damping model is carried out to assess the effectiveness of the proposed method. The success of these cases has proven the feasibility of the proposed method. The results have shown that the proposed method able to identify the structural damping matrices accurately in all the cases of noisy, complete and incomplete data and with all levels of damping.

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