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CONSIDERATIONS OF VARIOUS MOVING LOAD MODELS IN STRUCTURAL DYNAMICS OF LARGE GANTRY CRANES

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Abstract. *The paper deals with the moving load problems within the structural dynamic analysis of large gantry crane as high-performance machine. The emphasis is given on combined method approach, i.e. finite element method and analytical postulations, for obtaining the mathematical model of crane. Moving trolley is considered throughout several models: moving force, moving mass, moving oscillator and moving oscillator with swinging object. Each model has characteristics which determine the responses of the crane structure, along with its dynamic properties. The title problem is solved by calculating the forced vibration responses of the two-dimensional framework with time-dependent property matrices and subjected to an equivalent moving load. Improving the moving load models increases complexity as well for postulating as for obtaining the solutions from overall crane model. Comparative presentation of models is shown here with conclusion that leads to an appropriate way of model selection prior to crane problem postulation.*

1 INTRODUCTION

Moving load problem is very important topic in structural dynamics. In the beginning, it was related with the design of railway bridges and highway structures which showed additional vibration effect from the vehicle movement. This problem triggered the research into the moving load problem, initially with Stokes and Timoshenko. Many papers are presented in excellent monograph by Fryba [1], which describes basic postulation of moving load problems and their analytical solutions.

Irrespective of the many viewpoints, moving load problem methodology has two sides: analytical approach and finite element approach. Analytical approach are limited to simple cases of structures (such as beams) and basic types of loads. Closed-form solution for the governing equations is hard to find, even for the simple cases, and involves intensive mathematics. That is why finite element approach improved the studies in moving load problems with wide range of complex structures and load models. However, it demands certain numerical integration schemes in the time-domain analysis which can be intensive computational process.

Over the years, moving load problems have gained interest in the field of machines and mechanical structures due to the fact that working parameters are increasing. Typical structures under a moving load in mechanical engineering are bridge cranes, gantry cranes, unloading bridges, tower cranes, cableways, guideways, and container cranes. Here, the quayside container cranes (QCC) and rail mounted gantry container cranes (RMG) can be pointed out in moving load analysis because of the high speeds of trolleys, large lifting capacities and overall structural dimensions.

This paper studies the dynamic responses of the RMG container crane subjected to various types of moving loads-models upon the structural design of trolleys. The inspiration is gained with the fact that container transport is increasing yearly average 8% and that up-to-date RMG cranes can have spans to 50 m with trolley speeds of 3 m/s, which place it in high-performance machine. Thus, mathematical models are needed for moving load analysis at cranes, particularly because it is very difficult and expensive in practice to do an experimental research on a real-size crane.

2 MOVING LOAD MODELS

The main distinction in moving load problems is modelling of the vehicle/trolley system. First and basic approach is moving force model (MFM). Fundamental postulation is model of structure as simple beam subjected to constant force (weight of vehicle-$m_{ss}\,g$) moving with constant speed v, [2]. Moving force model is easy to use and brings attention in contemporary studies as well [3].

Moving mass model (MMM) includes inertial effects of the vehicle/trolley, and in fundamental postulation, for vertical displacements of the beam-$w(x,t)$, can be presented as

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = [m_{ss} g - m_{ss} \frac{\partial^2 w(vt,t)}{\partial t^2}]\ \delta(x - vt) \tag{1}$$

which clearly indicates the structure-trolley interaction. Hence, the speed of moving mass, structure flexibility and the mass ratio of moving payload and structure are important factors that contribute to creating the interaction force [4]. The interaction force is highly non-linear in nature in its local and convective derivatives and changes its position and magnitude with time, which is usually represented with Dirac delta function (δ), as given in Eq. (1). MMM arises with comlexity when variable speed of moving load is included [5]. However, even for the case when more accurate results are obtained with MMM [6], the consequences of neglecting this interaction may sometimes be minor even for the extreme up-to-date QCC [7].

Moving oscillator model (MOM) assumes that payload is attached to a mass of moving vehicle through a spring. The fundamental analytical postulation is given in [8]. With high values of the spring stiffness the MOM can be reduces with MMM in most cases [9].

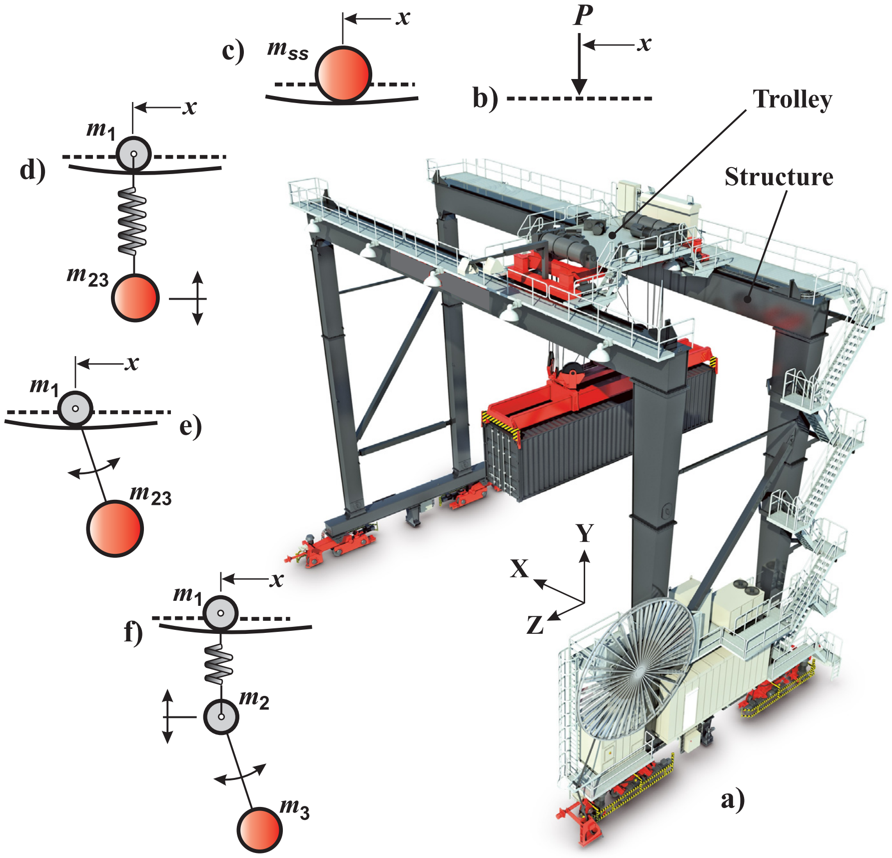

Figure 1: a) RMG container crane, b) Moving force (MFM), c) Moving mass (MMM), d) Moving oscillator (MOM), e) Moving pendulum (MPM), f) Moving oscillator with pendulum (MOPM).

In structural dynamics of cranes, the very important moving load problem is approach with model of moving trolley hoisting a swinging object or moving pendulum problem (MPM). The literature dealing with this problem is rare and only few researches can be found [10,11]. In some studies [12], it is used equivalent moving mass matrix which reduce the MPM to MMM. It is shown that influence of the swinging angle of the payload is big on horizontal responses of the structure.

The upgrade of the MPM is the model of trolley with suspended hoist with swinging object which can be named as moving oscillator with pendulum (MOPM). Compared with previous models, this model is the most complex for gaining the governing equation and obtaining the solutions [13].

The goal of this paper is to present comparative overview of the various models usage at ganty crane dynamics and their characteristics that can lead to appropriate modelling. The combined method-finite element and analytical method is adopted to gain the mathematical models [14]. Prior to gantry crane dynamics, it can be concluded that every research has to include following: (i) Both the horizontal and the vertical responses of the structure, (ii) Real cycle of trolley movement with emphasis on acceleration/deceleration periods, (iii) Lowest level of approximation when trolley inertial effects included.

3 MODEL FORMULATION

The approach used here is a combined finite element and analytical method for obtaining transverse and longitudinal vibrations of a gantry crane system subjected to a moving load. The general approach in moving load problems at cranes is also used here, thus the system of the gantry crane (Figure 1.a) is divided into two parts: the framework (structure) and the moving system. The framework is a planar (2D) discrete model consisted of top beam with length L, pier leg with height H and sheer leg with height h. The discretization of the framework (Figure 2.a) is done by using FEM, with plane-frame elements, [15]. The top beam is divided in 10 identical elements and each leg by 2 elements. Hence, framework has 41 DOF's (with extraction of the restrained displacements from supports) forming the structure displacement vector **U.**

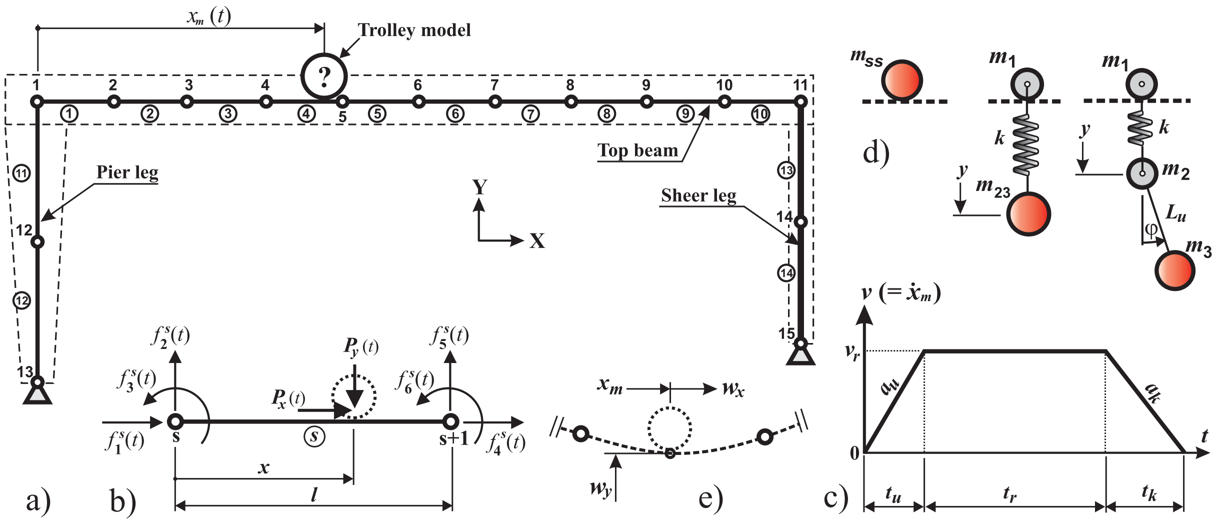

Figure 2: a) FE model of the gantry crane structure, b) Equivalent nodal forces of the elements s, c) Speed pattern of the moving load, d) Moving load models - MMM, MOM and MOPM, e) Contact point displacements.

The global position of the moving system on the top beam, Figure 2.a, is assumed to be known and defined by coordinate $x_m(t)$. Here, the acceleration/deceleration is also included in calculation because of the trolley trapezoidal speed pattern, Figure 2.c. It is assumed, by model of the gantry crane system, that a loading is symmetrically distributed on the top beam rail(s) and furthermore that relationship between the framework and the moving system can be simplified into one moving load $P(t)$, with projections in two-dimensional directions $P_x(t)$ and $P_y(t)$, Figure 2.b.

The moving system is considered as MMM, MOM and MOPM, Figure 2.d, while MFM is used as a special case of MMM when inertial effects from moving mass are neglected. The mass of the moving system is consisted of mass of trolley (m_1), hoist mass (m_2) and payload mass (m_3). Accordint to used model, additional DOF's are y-vertical displacement of oscillator and φ- swinging angle of the payload. It is assumed that trolley and structure are always in contact and that trolley is moving on the smooth surface on the top beam.

4 EQUATION OF MOTION FOR THE SYSTEM

The equation of motion for a multiple degree of freedom system is represented as follows

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}(t) \tag{2}$$

where **M**, **C**, **K** are the overall mass, damping and stiffness matrices of the system, respectively; $\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}$ are, respectively, acceleration, velocity and displacement vectors for the system and $\mathbf{F}(t)$ is the external force vector. Apart from the structural displacement vector **U**, the overall displacement vector includes the coordinates y and φ, which depends on the used load model.

4.1 Structural stiffness, mass and damping matrix

The equation of motion for a framework (structural system) is represented as follows

$$\mathbf{M_{st}}\ddot{\mathbf{U}} + \mathbf{C_{st}}\dot{\mathbf{U}} + \mathbf{K_{st}}\mathbf{U} = \mathbf{P}(t) \tag{3}$$

where $\mathbf{M_{st}}$, $\mathbf{C_{st}}$, $\mathbf{K_{st}}$ are the mass, damping and stiffness matrices of the structural system; $\ddot{\mathbf{U}}, \dot{\mathbf{U}}, \mathbf{U}$ are the respective acceleration, velocity and displacement vectors for the structural system and $\mathbf{P}(t)$ is the external force vector acting upon the structure.

According to the shown FE model of the framework, the stiffness matrix can be obtained by assembling all the element stiffness matrices [16] up to forming the square matrix $\mathbf{K_{st}}$ that corresponds with 41 DOF's of the structure. Similarly, the mass matrix of the framework $\mathbf{M_{st}}$ can be obtained. As usually practiced in FE dynamic analysis of MDOF system, the damping matrix is constructed by using the theory of Rayleigh damping in following form [17]

$$\mathbf{C_{st}} = a\,\mathbf{M_{st}} + b\,\mathbf{K_{st}} \tag{4}$$

with constants a and b that correspond to first two frequencies and their damping ratios (ξ).

4.2 Equivalent nodal forces and external load vector

The structural external force vector $\mathbf{P}(t)$ takes the form [17]

$$\mathbf{P}(t) = [0 \quad 0 \quad 0 \quad \ldots \quad f_1^s(t) \quad f_2^s(t) \quad f_3^s(t) \quad f_4^s(t) \quad f_5^s(t) \quad f_6^s(t) \quad .. \quad 0 \quad 0]^T \tag{5}$$

where $f_i^s(t),\ i = 1-6$, are the equivalent nodal forces of the element s, determined by

$$\begin{gathered} f_1^s(t) = N_1(x)P_x(t)\,,\ f_4^s(t) = N_4(x)P_x(t) \\ f_2^s(t) = -N_2(x)P_y(t)\,, f_3^s(t) = -N_3(x)P_y(t) \\ f_5^s(t) = -N_5(x)P_y(t)\,, f_6^s(t) = -N_6(x)P_y(t) \end{gathered} \tag{6}$$

Noting that l is the element length and x is the distance along the element s to the point of the application of the forces (Figure 2.b), the relative distance is given by $\xi = x/l$, thus, the shape functions, $N_i = N_i(x) = N_i(\xi)$ (i =1-6), take the following form

$$N_1 = 1-\xi\,, N_4 = \xi\,, N_2 = 1-3\xi^2+2\xi^3\,, N_3 = l(\xi-2\xi^2+\xi^3)\,, N_5 = 3\xi^2-2\xi^3\,, N_6 = l(-\xi^2+\xi^3) \tag{7}$$

The trolley moves from left end of the beam, at time t = 0, to the right end with defined speed pattern and its position on the top beam at time t is known - $x_m(t)$. The element number s on which the moving forces are applied, at any time t ($t \neq 0$), is

$$\mathrm{s} = IntegerPart[\frac{x_m(t)}{l}] + 1 \tag{8}$$

The nodal forces can be calculated in terms of the global position, by

$$\xi = \frac{x_m(t) - (\mathrm{s}\text{-}1)l}{l} \tag{9}$$

According to the Eq. (6) one may represent the element nodal force vector as

$$\{f^s(t)\} = \mathbf{N_x}^T P_x - \mathbf{N_y}^T P_y \tag{10}$$

where

$$\mathbf{N_x} = [N_1 \quad 0 \quad 0 \quad N_4 \quad 0 \quad 0],\ \mathbf{N_y} = [0 \quad N_2 \quad N_3 \quad 0 \quad N_5 \quad N_6] \tag{11}$$

Adjustment with total DOF's give the following matrices

$$\mathbf{N_X} = [0 \quad 0 \quad \dots \quad \mathbf{N_x} \quad \dots \quad 0],\ \mathbf{N_Y} = [0 \quad 0 \quad \dots \quad \mathbf{N_y} \quad \dots \quad 0] \tag{12}$$

These matrices are with non-zero elements which correspond to the DOF's of element s where the moving load is located at, while other elements are zero. The submatrices $\mathbf{N_x}$ and $\mathbf{N_y}$ are calculated by (9,8,7). One may see that these matrices engage only 6 values which *move along* in the (12), as moving load changes the position on the top beam.

The axial displacement at any location within the finite element, $w_x = w_x(x,t)$, and transversal displacement at any location within the finite element $w_y = w_y(x,t)$, Figure 2.e, can be presented in matrix form as

$$w_x = \mathbf{N_X U},\ w_y = \mathbf{N_Y U} \tag{13}$$

Second derivates of the expression (13) can be presented [13], in following form

$$\ddot{w}_x \approx \mathbf{N_X}\ddot{\mathbf{U}},\quad \ddot{w}_y = \mathbf{N''_Y U}\,\dot{x}^2 + \mathbf{N'_Y U}\,\ddot{x} + 2\mathbf{N'_Y}\dot{\mathbf{U}}\,\dot{x} + \mathbf{N_Y}\ddot{\mathbf{U}} \tag{14}$$

where the superscripts ('), ('') are representing the first and second derivate of expressions (7) with respect to x, while $\dot{x}$ is the velocity and $\ddot{x}$ is the acceleration of the moving system. The structural external force vector takes the matrix form

$$\mathbf{P}(t) = \mathbf{N_X}^T P_x - \mathbf{N_Y}^T P_y \tag{15}$$

which can be used, in combination with (3), for forced vibrations of the framework.

The interaction forces between the structure and the moving load model are, in general,

$$P_{x,y} = f(\ddot{w}_x, \ddot{w}_y, m_{i,}\ddot{y}, \varphi, \ddot{\varphi}) \tag{16}$$

which is detailed presented in [13] for MMM, MOM, MOPM. The additional equations for oscillator and pendulum are obtained according to the Second Newton's law. Combination of these equations with Eqs. (3-16) up to the form of the Eq. (2) give the equation of motion for the used system.

The general procedure is to divide the total time τ, needed for moving system to travel from the left end to the right end of the top beam, into p steps with a time interval Δt and to calculate all the given matrices for each time step r (r =1-p).

5 NUMERICAL RESULTS AND DISCUSSION

Dynamic behaviour of the gantry crane subjected to various moving loads is obtained by solution of the Eq. (1). Original in-house software is created to solve the title problem with direct integration method based on the Newmark algorithm [18]. The maximal time interval Δt is 0.005 s. The gravitational acceleration g is taken to be 9.81 m/s^2. The crane structure is made of steel with density 7850 kg/m^3 and modulus of elasticity 2.1 10^{11} Pa. Initial model includes structural damping with $\xi=\xi_1=\xi_2=0.5$ %.

Geometric characteristics of the gantry crane are L=40 m and H=h=15 m. Element properties are: A_n=0.09 m^2, I_n=0.041 m^4 (n=1-10), A_{11}=0.085 m^2, I_{11}=0.036 m^4, A_{12}=0.07 m^2,

I_{12}=0.024 m^4 and A_n=0.048 m^2, I_n=0.01 m^4 (n=13-14). The moving system consists of payload of 52 t, trolley of 3 t and hoist of 5 t, thus m_{ss}=60 t. Initial spring stiffness in moving system is k=10^9 N/m, while rope length is L_u=12 m. The system moves with 2 speed patterns, where pattern v_1 assumes v_r=3 m/s, a_u=a_k=0.6 m/s^2, which corresponds to characteristics of nowadays systems of trolleys, while the pattern v_2 assumes v_r=5 m/s, a_u=a_k=1.25 m/s^2 which is extreme assumption, but expected in the close future.

First, the MMM model is analysed. As expected from physical intuition, the biggest values of vertical displacements are for node 6, i.e. the middle point. Horizontal displacements for all the top beam points are the same, because of high axial stiffness of the elements. Figure 4 shows the results for both the speed patterns. The responses are higher for pattern v_2. This influence is very significant for horizontal displacement U_{X1} where values reach the maximum of 5.41 cm in the deceleration period, while maximum value with pattern v_1 is 4.1 cm, Figure 3. The difference is much smaller for vertical displacement of middle point, Figure 4.b. Maximum values occur when trolley is at midspan, and for v_1 is 5.2 cm and 5.4 cm for pattern v_2.

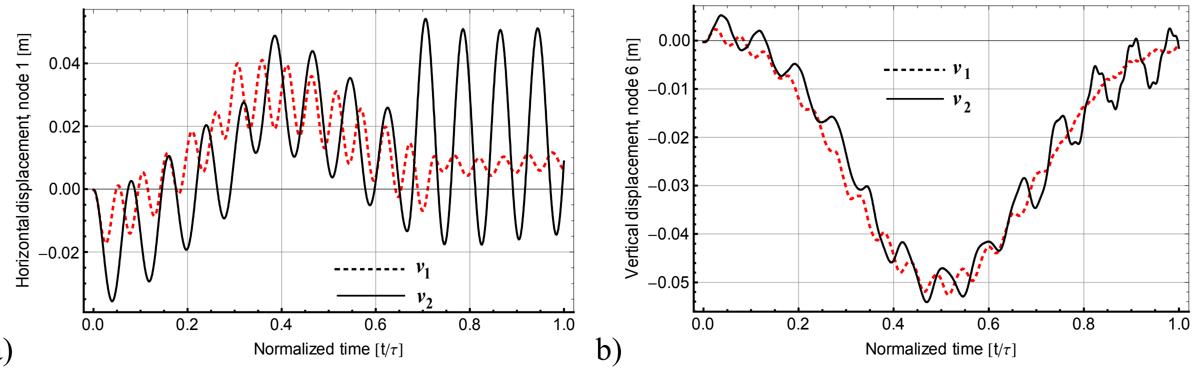

Figure 3: a) Horizontal displacement U_{X1}, b) Vertical displacement U_{Y6}; v_1,v_2.

When inertial effects of the moving mass are neglected, MFM is analyzed and comparison of results show big difference in values for horizontal displacement, Figure 4.a. MFM simplifies the mathematical algorithm but when MMM is included one may calculate the frequencies of the whole system at each time step. The results for first 3 frequencies are shown at Figure 4.b. It is obvious that frequencies are dependent on position of mass moving on the top beam.

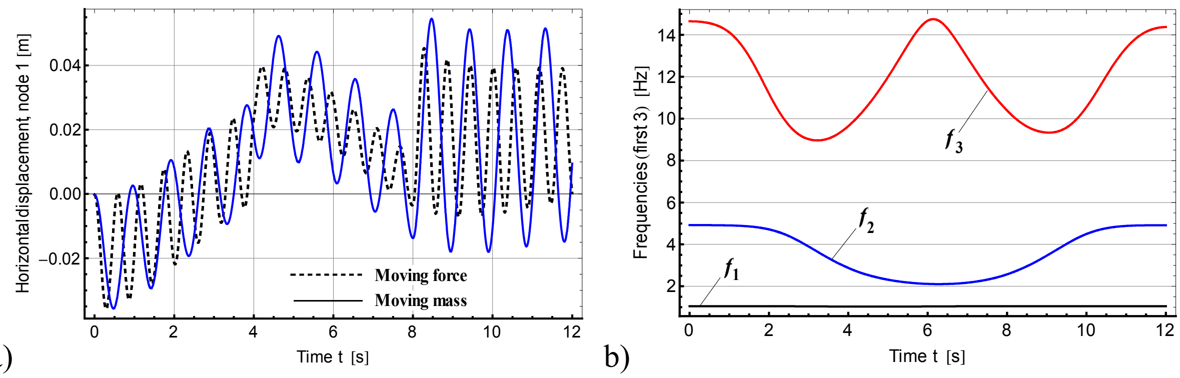

Figure 4: a) MMM/MFM Horizontal displacement Ux1, v_2, b) MMM first 3 frequencies

The vertical and horizontal displacements of node 6, for MOM model with speed pattern v_2, are presented in Figure 5.a. When compared with values from Figure 3, for speed pattern v_2, one can find negligible difference because the spring stiffness of k=10^9 N/m is high enough to comply with moving mass model. Here, it is convenient to find the values of vertical displacement of the beam over the point of contact $-w_y$. With decrease of stiffness to $k=10^7$ N/m, one may see slight change of displacement y in relation to w_y, Figure 5.b. Thus, up to these values there is no significant influence in dynamic responses of structure.

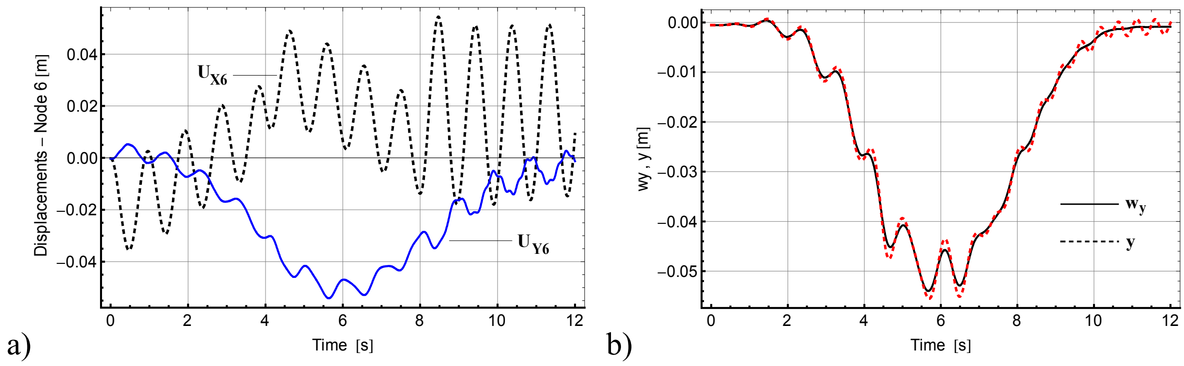

Figure 5: MOM, a) Vertical and horizontal displacement-node 6, b) Displacement y, $k=10^7$ N/m; v_2.

Finally, the MOPM model is used. One can find that increase of acceleration has high influence on the swinging angle of the payload, which should bring attention in design. The swinging angle reaches the value of 14.3°, Figure 6.a, in this case. Moreover, additional inertial effects due to swinging have influence on horizontal displacements of the structure, Figure 6.b, while vertical displacement are similar to the values with previously presented models.

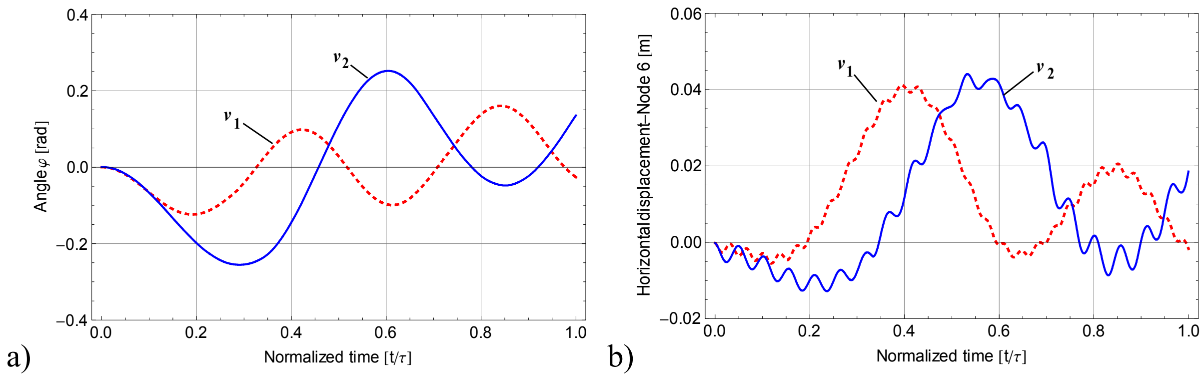

Figure 6: MOPM, a) Swinging angle φ, b) Horizontal displacement-node 6; v_1,v_2.

6 CONCLUSIONS

The results from Figure 4.a show that MFM is less accurate then MMM, considering horizontal displacements of the structure. However, the influence of moving load acceleration/deceleration is included with MFM which approve this approach in some cases.

MMM includes inertial effects of the moving mass, with additional effects due to convective derivatives as a result, which assures postulation of the problem as in analytical case with Eq. (1). Main advantage is possibility to adjust the algorithm to calculate frequencies of the system with moving mass included, Figure 4.b. This is important at cranes because the payload is usually heavier then the structure, therefore calculation of structural frequencies with MFM are not dealing with satisfactory results.

MOM can analyze the influence of the flexibility of the trolley structure. However, there is no significant differences from MMM with realistic crane parameters, Figure 5.a. This is in correlation with results from all the models that show very small influence on vertical displacements of the structure.

The MOPM is the most complex model as for postulation of governing equation as for obtaining the results. In this case, CPU time required to obtain the solution is much greater but this should be the cost to pay if additional parameters are included. The influence of payload swinging is very important at gantry cranes, especially when expected to achieve high performances for the trolley speed pattern, Figure 6.

From the given models, authors would like to give emphasis on MMM and MOPM. According to previously stated, MMM can stand for initial approach in moving load problems at gantry cranes, while MOPM is extended approach for more satisfactory results in cases where anty-sway systems are not developed.

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