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APPLICATION OF THE BIFURCATION THEORY FOR COMPLETE ANALYSIS OF DYNAMICAL SYSTEMS WITH MDOF, NON-IDEAL VIBRATIONS, CHAOTIC AND RARE ATTRACTORS

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Abstract. *Some new applications of the so-called* bifurcation theory *[1-2] for forced nonlinear dynamical periodic systems (NDS) with several degrees of freedom are presented. Among them are a nonlinear mechanical system with double-well potential, pendulum-like systems, and a system with non-ideal vibrations [3].The bifurcation theory is established for direct global complete bifurcation analysis for essentially nonlinear dynamical periodic systems, described by models of ODE equations or by map-based models with discrete-time equations. Our approach is based on ideas of Poincaré, Andronov and many other modern scientists, concerning global nonlinear dynamics, structural stability and bifurcations, topological properties and chaotic responses.*

The idea of the new bifurcation theory (BT) goes from a fact that the NDS in a given parameters and state spaces has finite number (usually not so many) of independent bifurcation groups S(p) with their own complex topology and bifurcations, chaotic behavior, and as a rule, with rare regular and chaotic rare attractors (RA). For each point of parameter space it is possible to find all essential stable and unstable fixed points of the periodic orbits This periodic skeleton, with stable and unstable orbits, allows to mark out the main and bifurcation groups and to start global analysis in parameter and state spaces using each the orbit continuation in parameter space. The main concepts of the BT are: complete bifurcation group (BG); unstable periodic infinitiums subgroups (UPI-subgroups), responsible for chaos in the system; complex protuberances; and periodic skeletons for a system with parameter p or for a some restricted parameter space. For realization of the bifurcation theory in applications we use our own software Spring [2].

For illustration of the advantages of the new bifurcation theory we use in this paper three typical nonlinear models with several DOF mentioned above. Special attention is paid for building complete bifurcation groups of the mechanical non-ideal system with multiplicity regions, chaotic attractors and rare subharmonic attractors and complex topology of their basins of attractors. Some open problems concerning using of the method of complete bifurcation groups and the bifurcation theory for new applications are also planned to discuss in this presentation

1 INTRODACTION

Our paper is arranged in such a way. In section 2 we show how to use periodic skeleton for finding unusual periodic attractors for a rotor system with asymmetric nonlinear suspension [2]. In section 3, we illustrate the usefulness of periodic skeletons and complete bifurcation analysis for finding new unusual bifurcation groups (fully unstable submerged subharmonic isles). In section 4 we discuss some new problems of complete bifurcation analysis for non-ideal dynamical systems.

2 UNUSUAL PERIOD-1 ATTRACTORS IN ROTOR SYSTEMS WITH ASYMMETRIC SUSPENSION

Our model (Figure 1) described by such equation:

$$\begin{cases} \ddot{x} + b\dot{x} + (1+k)(c_1 x + c_2 x(x^2 + y^2)) = e\omega^2 \cos \omega t, \\ \ddot{y} + b\dot{y} + c_1 y + c_2 y(x^2 + y^2) = e\omega^2 \sin \omega t, \end{cases} \qquad (1)$$

where k is a coefficient of asymmetry of nonlinear suspension. Our aim is to build bifurcation diagram changing frequency of excitation ω for period-1 orbits.

Periodic skeleton. For the system (1) with parameters: $c_1 = c_2 = 1$, $b = 0.2$, $e = 0.15$, $k = 0.3$, $\omega = 1.6$, we have found periodic skeleton for period-1 orbits. For this aim we use software Spring [2] starting from 500 initial conditions. Five period-1 orbits found are shown in Table 1. with their passports (fixed point and stability evaluation). Starting from found fix points (Table 1) the bifurcation diagram S(ω) was built (Figure 2). The system has not only two P1 attractors but three P1 attractors. It is a new fact for nonlinear rotor systems, we spouse.

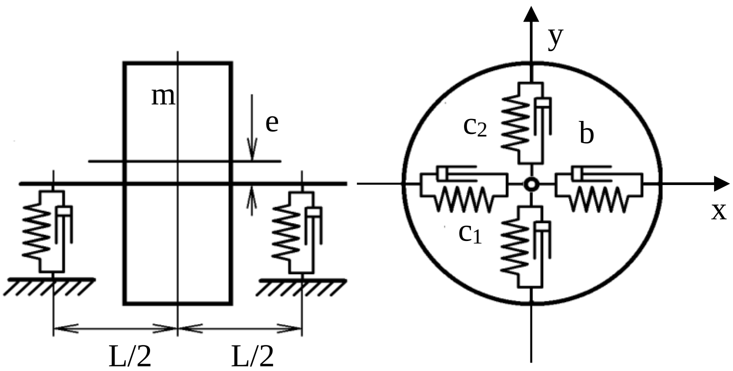

Figure 1: Rotor model.

N	Order	x1	v1	x2	v2	Ro_{max}	Alpha
1	P1	0.144982	1.460202	-1.362769	0.167427	0.675	14.7°
2	P1	0.467466	1.342594	-0.391212	-0.976606	0.675	14.9°
3	P1	-0.31288	0.143133	-0.052776	-0.396005	0.675	80.6°
4	P1	-0.174975	1.264633	-1.235709	-0.794507	1.22	0°
5	P1	-0.173508	1.436648	-0.111281	-0.8059	1.21	0°

Table 1: Periodic skeleton with P1 orbit for rotor asymmetry system. Three first P1 orbits are stable, but two last – unstable. Parameters: $c_1 = c_2 = 1$, $b = 0.2$, $e = 0.15$, $k = 0.3$, $\omega = 1.6$.

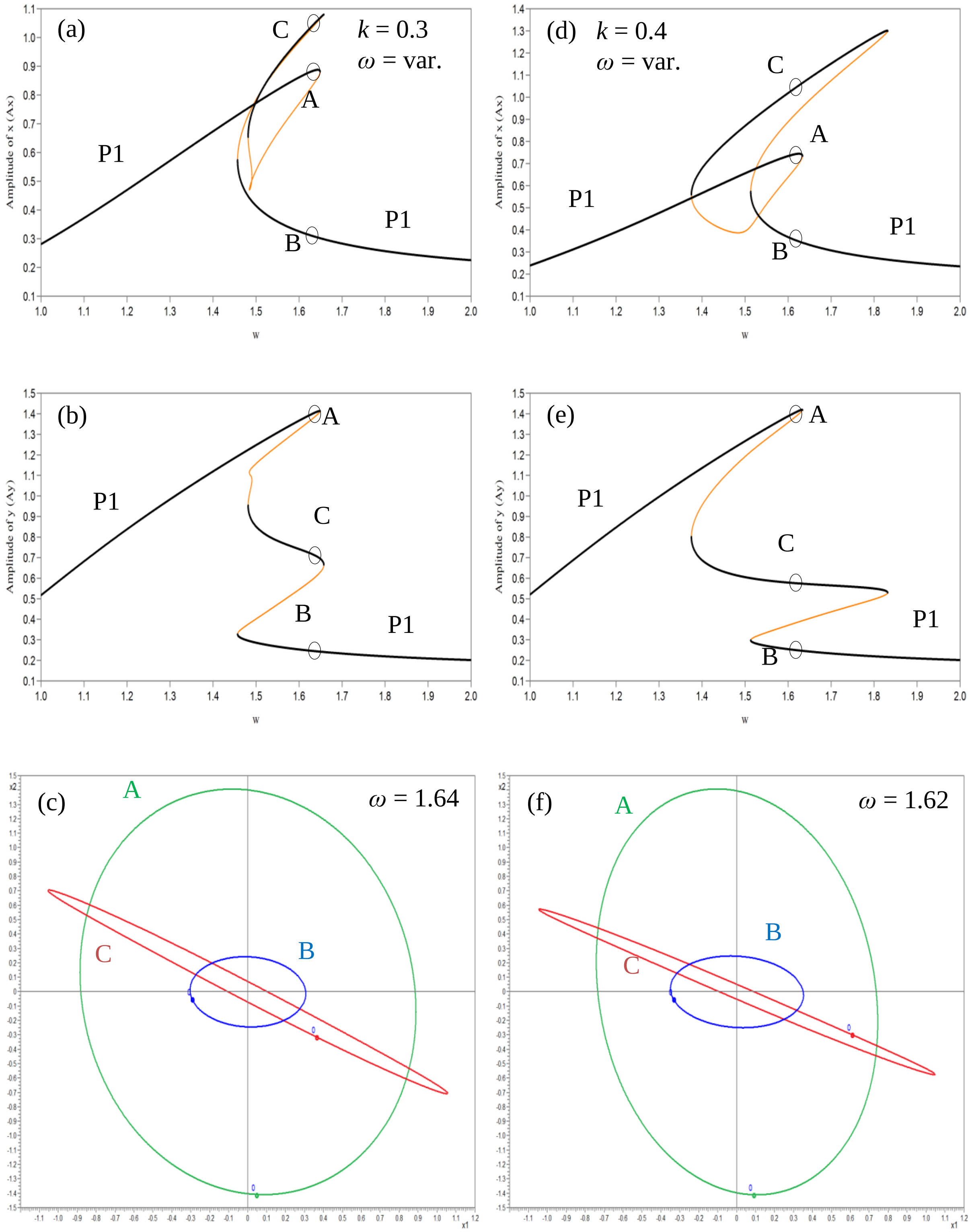

Figure 2: Bifurcation diagrams and trajectory of rotor for the system with parameter asymmetry (a-c) k =0.3, (d-f) k = 0.4. Other parameters ca be found in the text.

3 DOUBLE-WELL POTENTIAL SYSTEM WITH TWO DEGREES-OF-FREEDOM WITH UNUSUAL FULLY UNSTABLE 2T ISLES [2]

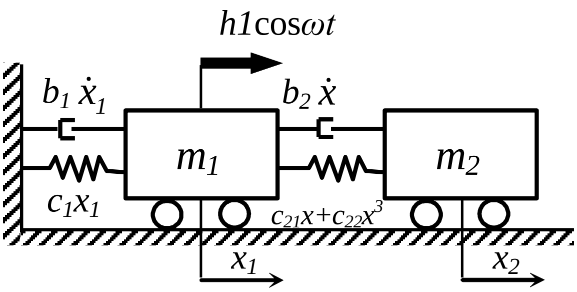

Figure 3: Rotor model.

The double-well system (Figure 3) described by equations:

$$\begin{cases} m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 - b_2\dot{x} - c_{21}x - c_{22}x^3 = h1cos(\omega t + \varphi_0), \\ m_2\ddot{x}_2 + b_2\dot{x} + c_{21}x + c_{22}x = 0, \end{cases} \tag{2}$$

where x_1 and x_2 are generalized coordinates and $x = x_2 - x_1$. Dynamical characteristics are shown in Figure 4.

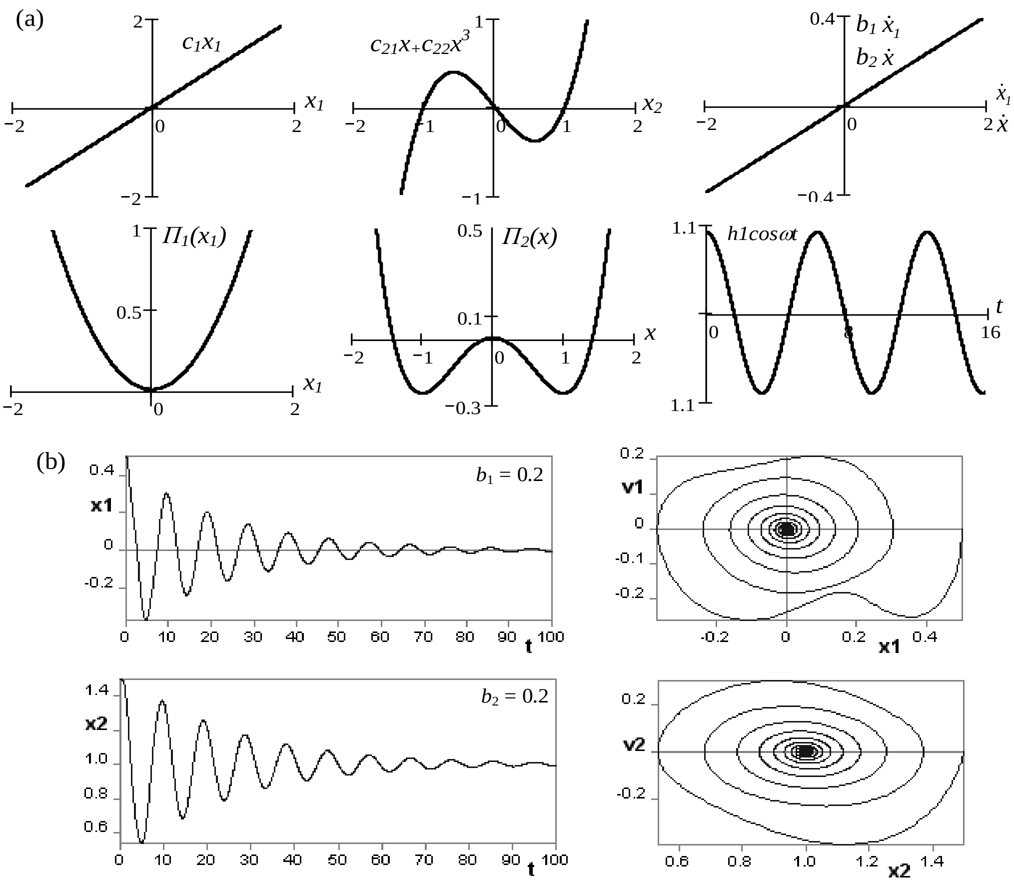

Figure 4: Driven damped nonlinear system with two degrees-of-freedom with double-well potential.

Our task is to do complete bifurcation analysis changing the amplitude of excitation h_1 near h_1 = 0.65.. We start the investigation from building periodic skeleton (Table 2) for orbits P1, P2, P3, P4 and P5. Parameters of the system: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h_1 = 0.65$.

N	Ord	x1	v1	x2	v2	Ro	Alpha
1.	P1	-0.129101	-0.082511	-1.587515	0.270691	1.865	180°
2.	P1	0.323869	1.001202	-0.44025	-0.239861	1.554	132.9°
3.	P1	-0.271892	0.251026	0.105925	0.06813	14.84	0°
4.	P1	0.066284	0.500056	-1.379934	-0.208921	14.84	0°
5.	P1	-0.388875	0.344091	0.296773	-0.01447	1.86	180°
6.	P2	-0.152448	0.874192	-1.201265	-0.634519	3.03	138.8°
7.	P2	-0.151861	0.316856	-0.172822	-0.198257	40.03	0°
8.	P2	-0.332329	0.251465	0.191861	0.121648	7.73	180°
9.	P2	-0.275199	0.801363	0.159292	-0.117275	20.47	0°
10.	P2	-0.802298	0.33394	-0.257702	-0.372004	21.28	0°
11.	P2	-0.2466	0.822948	0.197581	-0.083833	3.45	100.6°
12.	P2	-0.012823	0.176438	0.18046	-0.02271	617.04	180°
13.	P3	-0.381402	0.31673	0.332407	0.031314	25.06	0°
14.	P3	-0.625043	0.377746	-0.08156	-0.25167	6.69	0°
15.	P3	-0.31089	0.138164	0.059531	-0.016837	507.6	0°
16.	P3	-0.595076	0.27309	-0.254582	-0.117196	777.3	0°
17.	P4	-0.239352	-0.158382	-1.640796	0.343924	36.51	180°
18.	P4	0.129593	0.196659	0.343394	0.03922	45.46	180°
19.	P4	-0.174384	0.251678	0.097085	0.051498	110.3	0°
20.	P4	0.207933	0.977746	0.413483	0.463058	161.9	180°
21.	P5	-0.003878	0.654633	-0.619691	-0.487624	24.84	0°
22.	P5	-0.215413	0.414365	-0.346901	-0.264937	6.96	107.4°
23.	P5	0.192434	0.986601	0.273883	0.403505	85.87	180°
24.	P5	0.19376	0.811889	-0.428947	-0.389263	10.49	0°
25.	P5	0.32961	0.785141	-0.069514	-0.110303	4.199	143.0°
26.	P5	-0.894345	0.528571	-0.634449	-0.180239	52.02	180°
27.	P5	-0.299483	0.24787	0.156235	0.103283	72.18	0°

Table 2: Periodic skeleton with P1, P2, P3, P4 and P5 orbits. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h_1 = 0.65$.

For a given parameter point we have found five period-1 orbits, seven period-2 orbits, four fixed point for period-3, four orbits for period-4 and seven fixed points for period-5.

It is interesting that all found fixed points are unstable and so the system has near h_1 = 0.65 several different unstable periodic infinitiums (UPI) and chaotic behavior. Some of found fixed points are direct saddle points but some are inverse saddle points. Some of found fixed point has rather great multipliers (for example see points Nr. 12, 14,15). Now we use the found fixed point of the period-2 orbits for building 2T bifurcation diagram (2T subharmonic isle) ,Figure 5. It is wonderful that 2T subharmonic isle contents only unstable orbits.

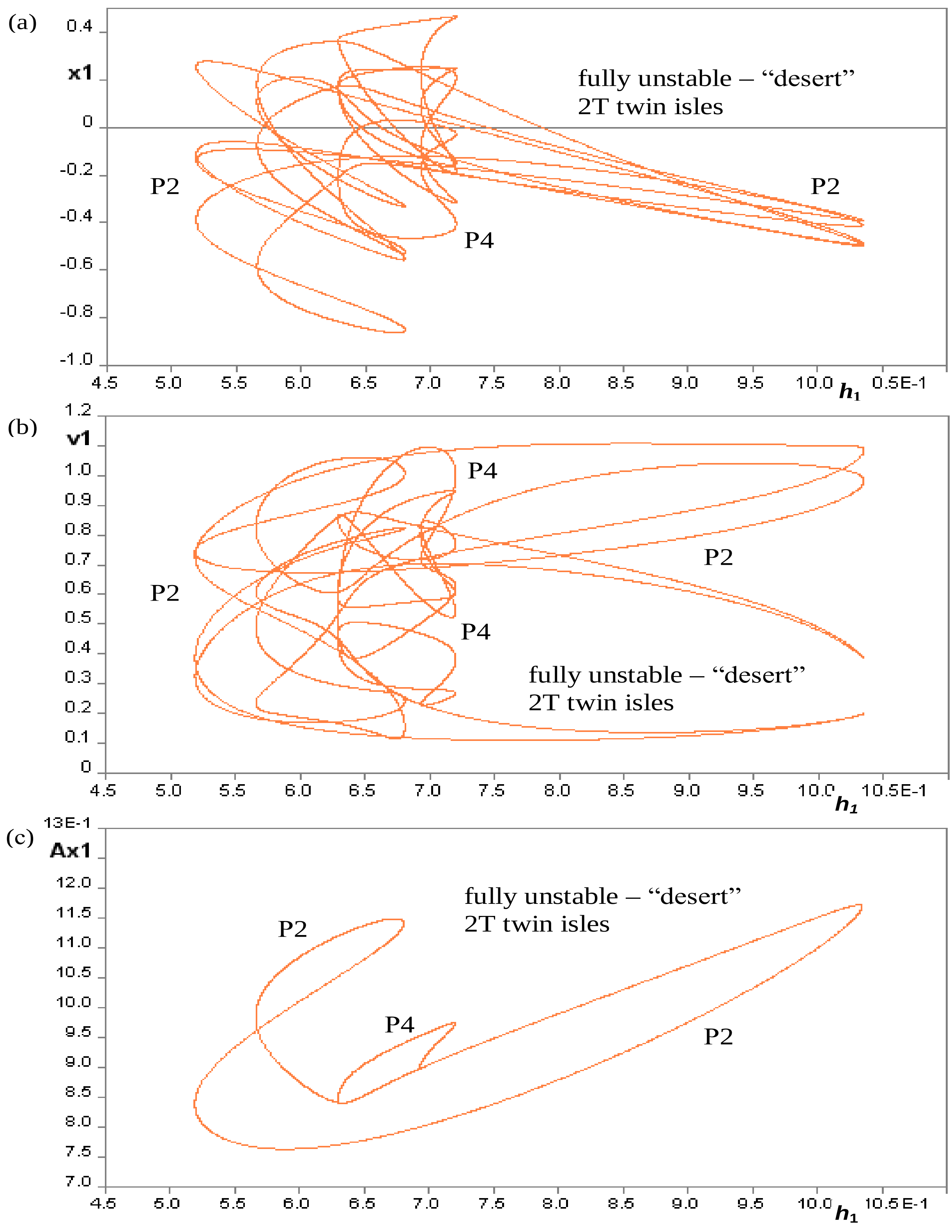

Figure 5: Bifurcation diagram 2T (subharmonic 2T isle) for the double-well driven damped potential system with two degrees-of-freedom. On the 2T bifurcation diagrams are shown period-2 and period-4 orbits. All found period-2 and period-4 orbits are unstable. Changing some parameters of the system its possible to see the birth of rare attractors from this fully unstable 'submerged' or 'desert' isles. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, $h_1 =$ var.

4 CHAOS IN NON-IDEAL DOUBLE-WELL POTENTIAL SYSTEM [4]

The bifurcation theory is very good for analysis of periodical dynamical systems (DS) where at list one periodic orbit exist. It is an open problem how to find all bifurcation groups for quasi-periodic DS dynamical systems or for a systems with random parameters. Such class of problems may are in DS with non-ideal excitation [3-4]. From J.M. Balthazar [4] we know that in the double-well potential systems there are chaotic attractors (ChA). The aim of this section is to prepare starting information for parameter continuation and for global analysis of such kind of DS. Here we change only one parameter δ_1 from Balthazar's model (3). It is clear from Figure 6 that increasing parameter δ_1 from 2 up to δ_1 =10 that the system change its behaviour. For $\delta_1 = 2$ the system has quasi-periodic small orbit (QP SO) attractors in each potential well. After increasing δ_1 up to $\delta_1 = 3$ large chaotic orbit attractor (LO ChA) appears. After increasing δ_1, two twin QP large orbit appear. Then for $\delta_1 = 6$ again LO ChA exists (similar Balthazar's, at delta = 8.373). For δ_1 =10 we again see twin QP attractors. These obtained results we plan use for future complete bifurcation analysis of the non-ideal system and for finding new bifurcation groups and rare attractors.

$$\begin{aligned}&(M+m)\ddot{x}+b\dot{x}-k_1 x+k_{nl}x^3 = mr(\ddot{\theta}\sin(\theta)+\dot{\theta}^2\cos(\theta))\\ &(J+r^2 m)\ddot{\theta}-rm\ddot{x}\sin(\theta)=L(\dot{\theta})=d-e\dot{\theta};\ \text{where}\end{aligned} \qquad (3)$$

$$\tau=\omega_1 t;\qquad x_1=\frac{rx}{\omega_1^2};\qquad \omega_1=\sqrt{\frac{k_1}{M+m}};\qquad \alpha_1=\frac{b}{(M+m)\omega_1};\qquad \delta_1=\frac{m\omega_1^2}{(M+m)};$$

$$\rho_1=\frac{mr^2}{(J+r^2m)\omega_1^2};\qquad \rho_3=\frac{e}{(J+r^2m)\omega_1};\qquad \beta_1=\frac{k_l}{(M+m)\omega_1^2};\qquad \beta_2=\frac{k_{nl}r^2}{(M+m)\omega_1^6};$$

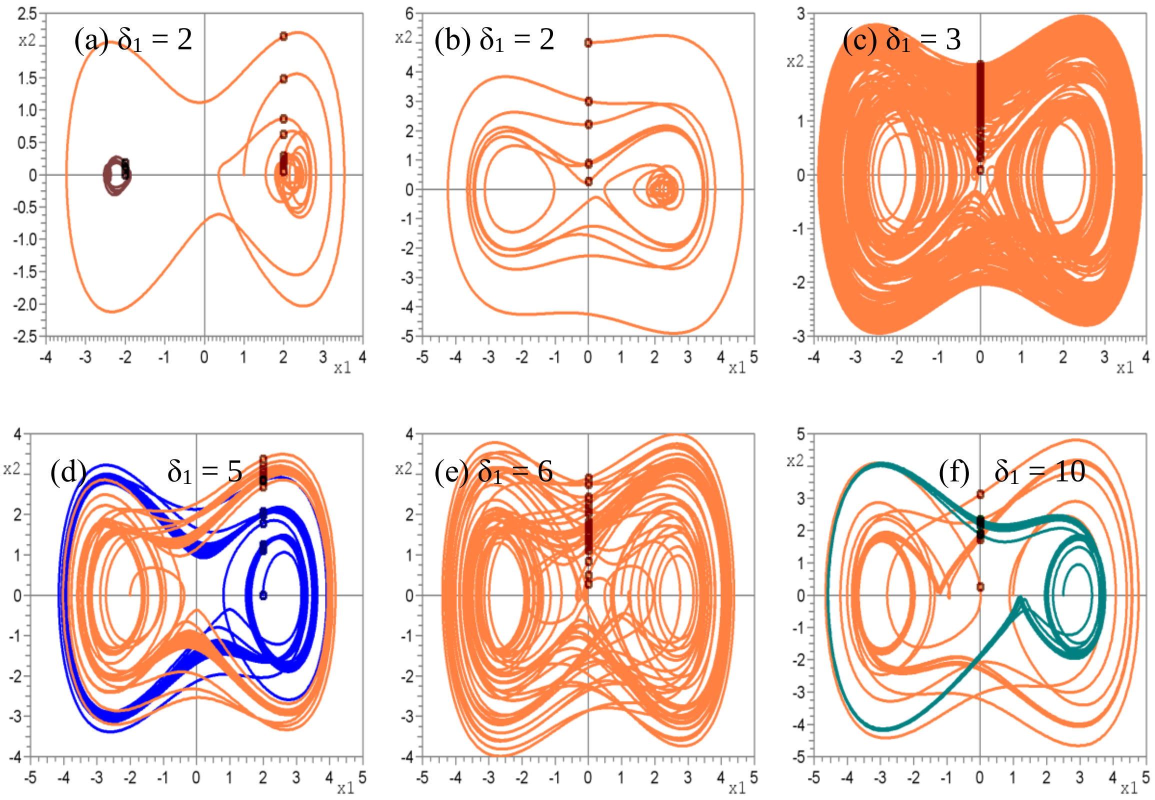

Figure 6: Phase diagrams with quasi-periodic and chaotic behaviour with small and large orbits for different parameter δ_1. Psrsmeters:$\alpha_1 = 0.1$, $\beta_1 = 1$, $\beta_3 = 0.2$, δ_1 = var., $\rho_1 = 0.05$, $\rho_2 = 100$, $\rho_3 = 200$.

5 CONCLUSIONS

- In the paper we give examples of application of novelty bifurcation theory for the global analysis of strongly nonlinear dynamical systems.
- For asymmetric rotor system and double-well potential driven damped system with 2DOF new qualitative nonlinear behaviour have been found. The important role for building complete bifurcation diagrams are periodic skeletons with passport data for stable and unstable periodic orbits.
- The open problems of quasi-periodic orbits continuation and bifurcation analysis of non-ideal nonlinear systems also are discussed in the presentation.

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