11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

UNKNOWN BIFURCATION GROUPS WITH CHAOTIC AND RARE ATTRACTORS IN THE DUFFING-UEDA AND DUFFING - VAN DER POL ARCHETYPAL OSCILLATORS

Mikhail V. Zakrzhevsky[1], **Igor T. Schukin**[2], **Raisa S. Smirnova**[2], **Eduard Shilvan***[1]

[1]Institute of Mechanics, Riga Technical University, Latvia
mzakr@latnet.lv, ed_shilvan@inbox.lv

[2]Riga Technical University Daugavpils Affiliation, Latvia
igor@df.rtu.lv, raja@df.rtu.lv

Keywords: Duffing-Ueda Oscillator, Duffing-van der Pol Oscillator, Complete Bifurcation Groups, One- and Two parameters Bifurcation Diagrams, Rare Attractors, Chaos.

Abstract. *Recently proposed so-called method of complete bifurcation groups (MCBG) [3-4] allows finding new previously unknown qualitative results in many essentially nonlinear dynamical systems. In this presentation we illustrate the advantage of the MCBG for the well-known Duffing-Ueda and Duffing-van der Pol archetypal driven damped oscillators [1-2]. About 50 unknown different bifurcation groups have been found with rare regular and chaotic attractors.*

In the study of the Duffing-Ueda and Duffing-van der Pol equations, we use concepts of the novelty Bifurcation Theory (BT), periodic skeletons, complex protuberances theory, rare periodic and rare chaotic attractors [3-4]. Our new results are presented in one- and two-parameter bifurcation diagrams, time histories and phase diagrams and domains (basins) of attractions. It is shown that there are many new unknown bifurcation groups with complex topology with subgroups with several unstable periodic infinitiums (UPI), different rare periodic and chaotic attractors in the same bifurcation group.

Some results obtained by Ueda in his famous two parameter bifurcation diagrams [1-2] are essentially supplemented and corrected by our new ones. Some important new bifurcation subharmonic groups with UPI and rare attractors are found for the Duffing-van der Pol model as well. Using these results, a new explanation of the birth of the first Ueda's "broken-egg" chaotic attractor in the Duffing-van der Pol equation [1], based on our complete bifurcation analysis, are given in the presentation. The same new results have been obtained for the archetypal Duffing-Mathieu equation (not included in this paper).

Obtained new results for the "well-known" archetypal nonlinear oscillators are good illustrations for importance of the new Bifurcation Theory of Nonlinear Dynamical Systems [3-4]. The main cornerstone of this Bifurcation Theory is a physical fact that all essentially nonlinear dynamical periodic systems in a parameter region have many periodic (stable and especially unstable) solutions which belong to different, not mutually connected, simple and complex bifurcation groups, as a rule, with unstable periodic infinitiums and rare attractors.

1 INTRODUCTION

This paper covers the new results in Duffing-Ueda and Duffing-van der Pol models. The main aim of the research is to find new bifurcation groups in Duffing-Ueda and Duffing-van der Pol models and to demonstrate how the method of complete bifurcation groups is applied to the global analysis of typical nonlinear dynamical systems.

In the second section of the paper we discus some new results of bifurcation analysis for Duffing-Ueda model. In this section we show in Figure 1(a) a new two-parameter bifurcation diagram with narrow strip of period – five (P5) rare attractors and chaotic attractors (ChA-5) inside the diagram (‘chaotic jacket’). Figure 1(b) show one-parameter diagram for the same system with so-called submerged P5 isles with tip P5 and ChA-5 attractors. This bifurcation diagrams are typical for many nonlinear dynamical systems.

In the third section we show an example of one parameter bifurcation diagram with several regions of multiplicity and with new sub-harmonic isles with unknown topology. In the paper we continue investigation made by Ueda and others scientists.

.

2 DUFFING-UEDA EQUATION AND IT’S BIFURCATION ANALYSIS

A well-known the Duffing-Ueda model [1] with nonlinear forced oscillations and cubic nonlinearity are shown in Eq. (1):

$$\ddot{x} + b\dot{x} + x^3 = h\cos\omega t \tag{1}$$

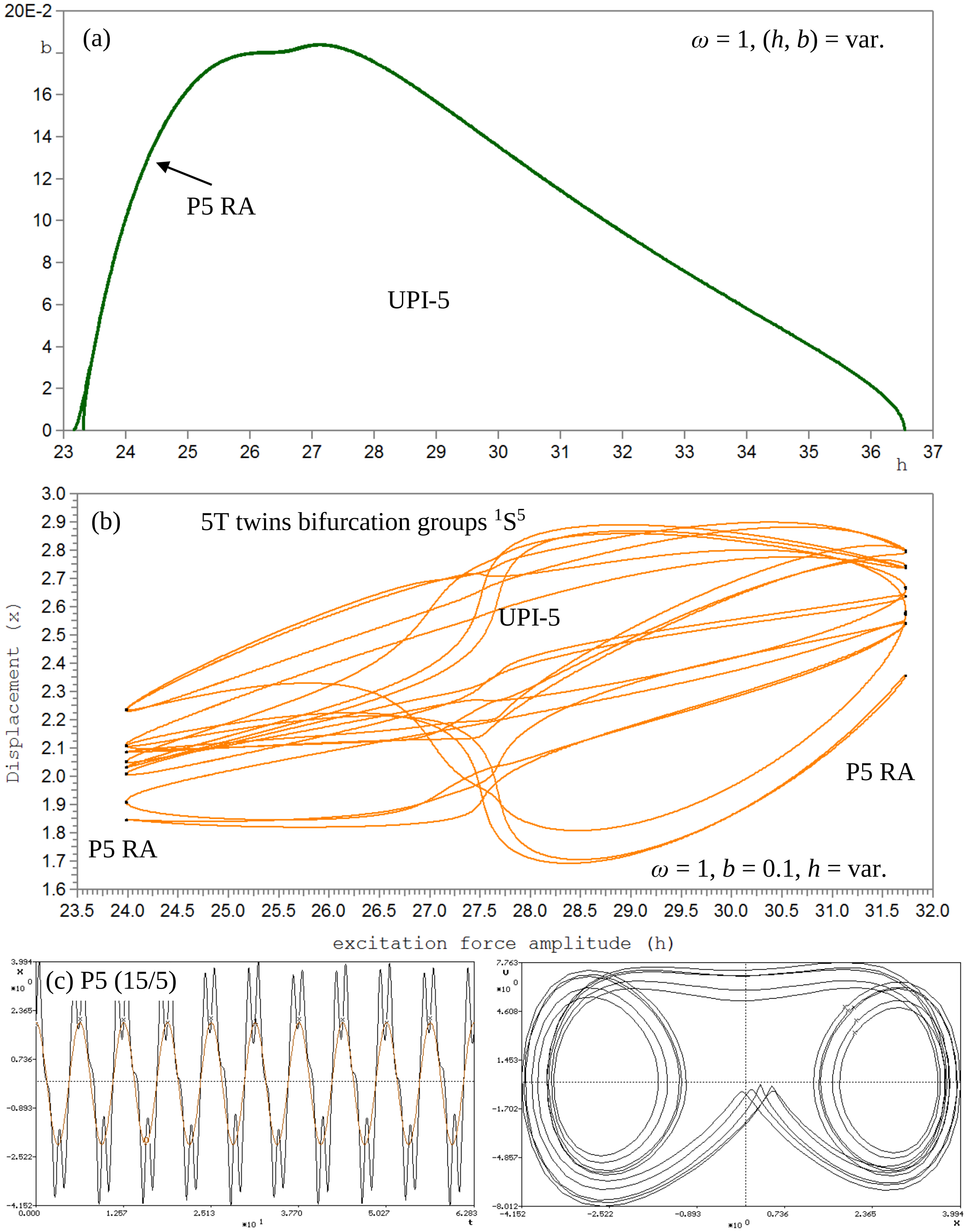
For this equation we have found many new bifurcation diagrams with rare attractors and UPI [3-4]. For this paper we took only new unknown bifurcation results with different 5T bifurcation groups (subharmonic period – five isles). The complex, but typical bifurcation diagram with period – five, with different topology, with rare attractors (RA) and UPIs are shown in Figures 2-4. With bold black line on one-parameter diagram are shown stable periodic orbits and with reddish line – unstable periodic orbits. The same complete bifurcation analysis are done for other bifurcation groups 1T, 2T, 3T, 4T and others, but we have no space for these results in this paper.

3 DUFFING-VAN DER POL MODEL IT’S BIFURCATION ANALYSIS

An equation for archetypical Duffing-van der Pol oscillator is shown below Eq. (2).

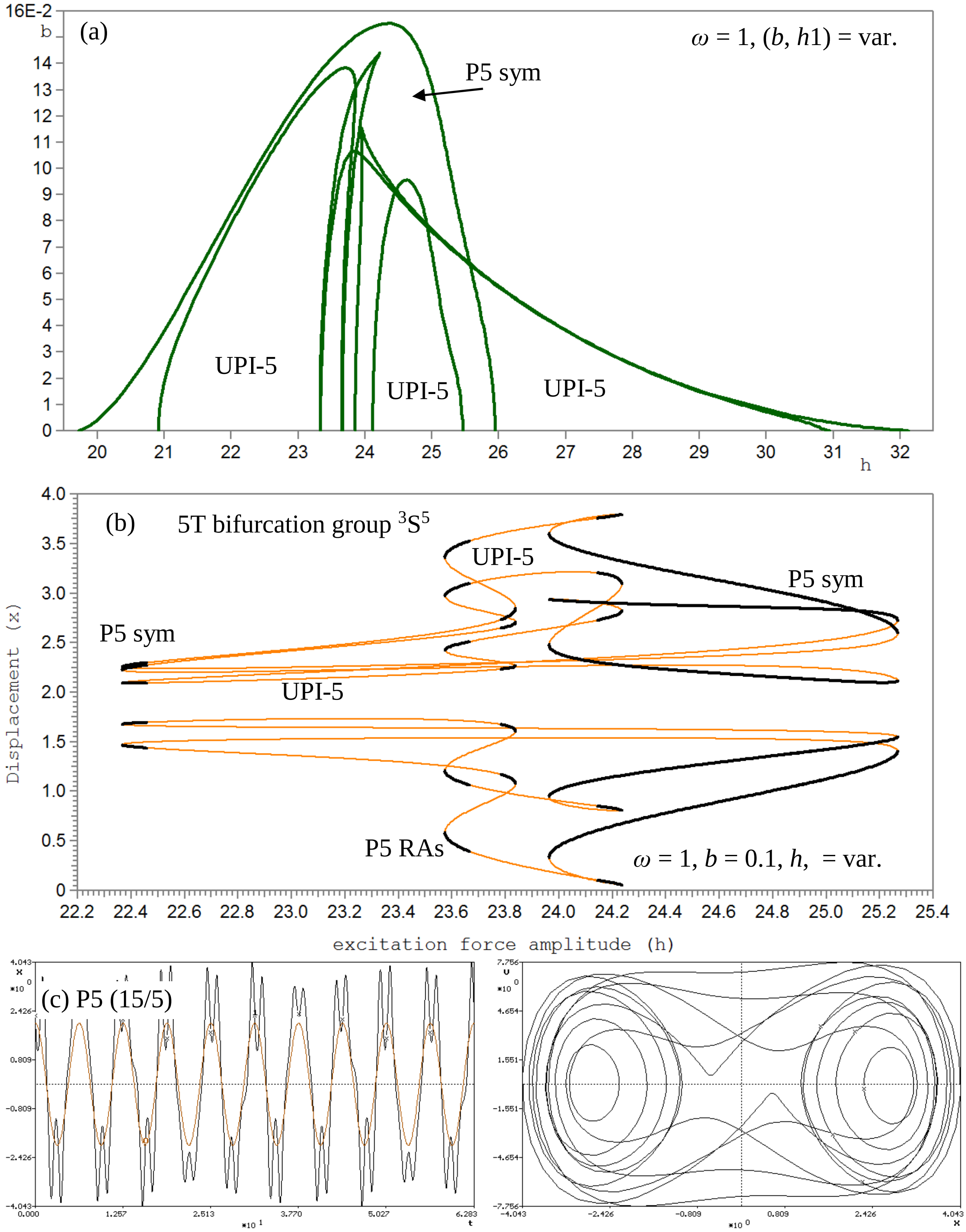
$$\ddot{x} - \mu\left(1 - \gamma x^2\right)\dot{x} + x^3 = h\cos\omega t \tag{2}$$

Example of using the method of complete bifurcation group for the Duffing-van der Pol are shown in Figure 5, where variable parameter is harmonic exaltation frequency ω. Several bifurcation groups (1T, 2T, 3T, 4T, 5T) are shown, some of them has rare attractors and chaos. Some subharmonic isles (2T, 3T) has fully unstable fold from one side of the isle. This model has quash-periodic attractors as well (shadow areas in Figure 5).



P5 (15/5) fixed point (1.847043, 4.817090), ρ_i = 0.253, Alp = 149.97, h = 23.984

Figure 1: Duffing-Ueda symmetric oscillator with linear dissipation. (a-b)Two-parameter (chaotic jacket) and one-parameter (submerged isle) bifurcation diagrams for bifurcation group 5T with the RA and UPI-5. (c) time history and phase diagram for stable regime P5. Parameters: $f(x) = x^3$, $b = 0.1$, $\omega = 1$, h = var.



P5 (15/5) fixed point (1.847043, 4.817090), ρ_i = 0.253, Alp = 149.97, *h* = 23.984

Figure 2: Duffing-Ueda symmetric oscillator with linear dissipation. (a-b)Two-parameter and one-parameter bifurcation diagrams for bifurcation group 5T with the RA and UPI-5. (c) time history and phase diagram for stable regime P5. Parameters: $f(x) = x^3$, $b = 0.1$, $\omega = 1$, h = var.

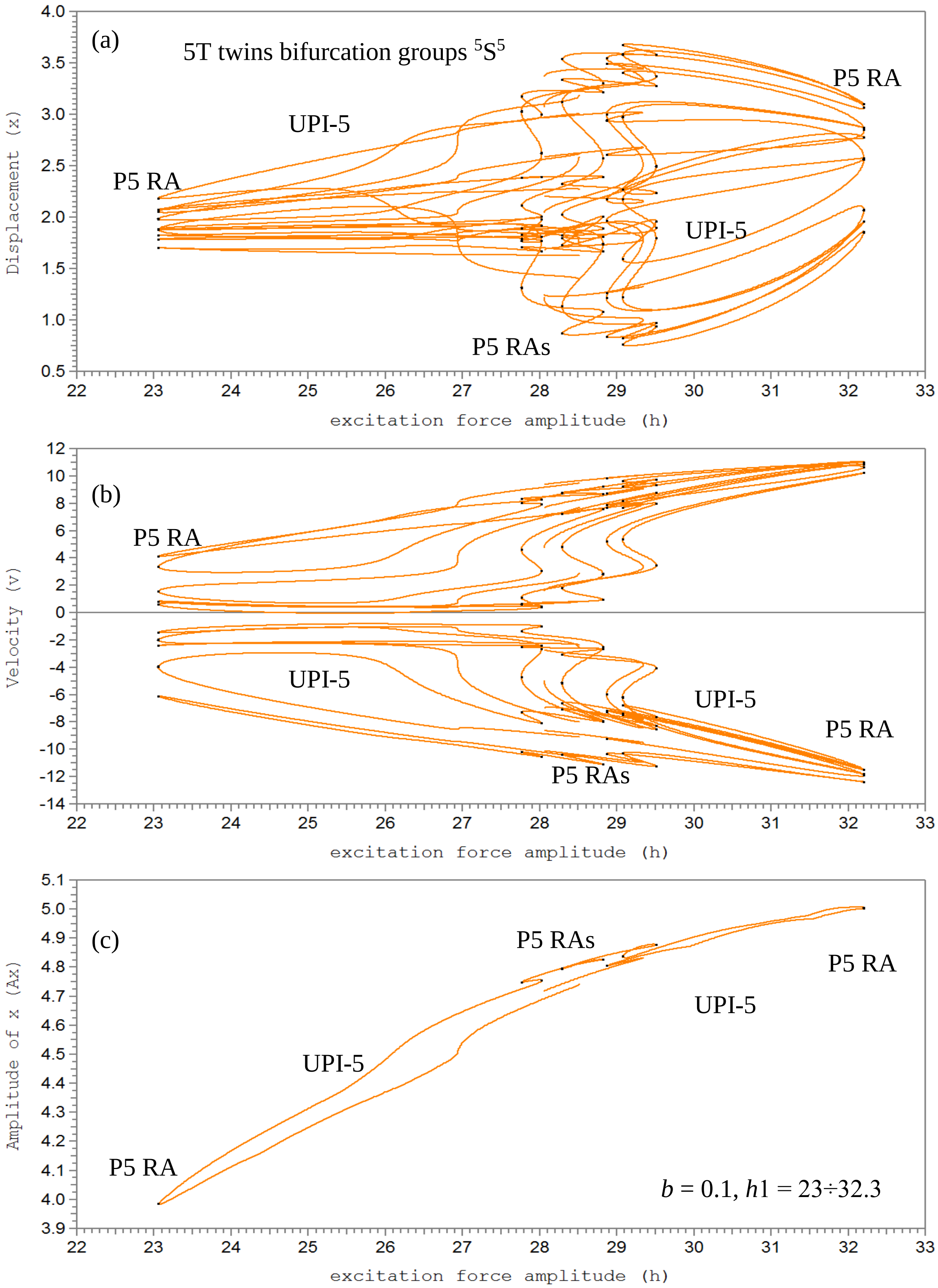


Figure 3: Duffing-Ueda symmetric oscillator with linear dissipation. Complex one-parameter bifurcation diagram for bifurcation group 5T with the RA and UPI-5. Parameters: $f(x) = x^3$, $b = 0.1$, $\omega = 1$, h = var.

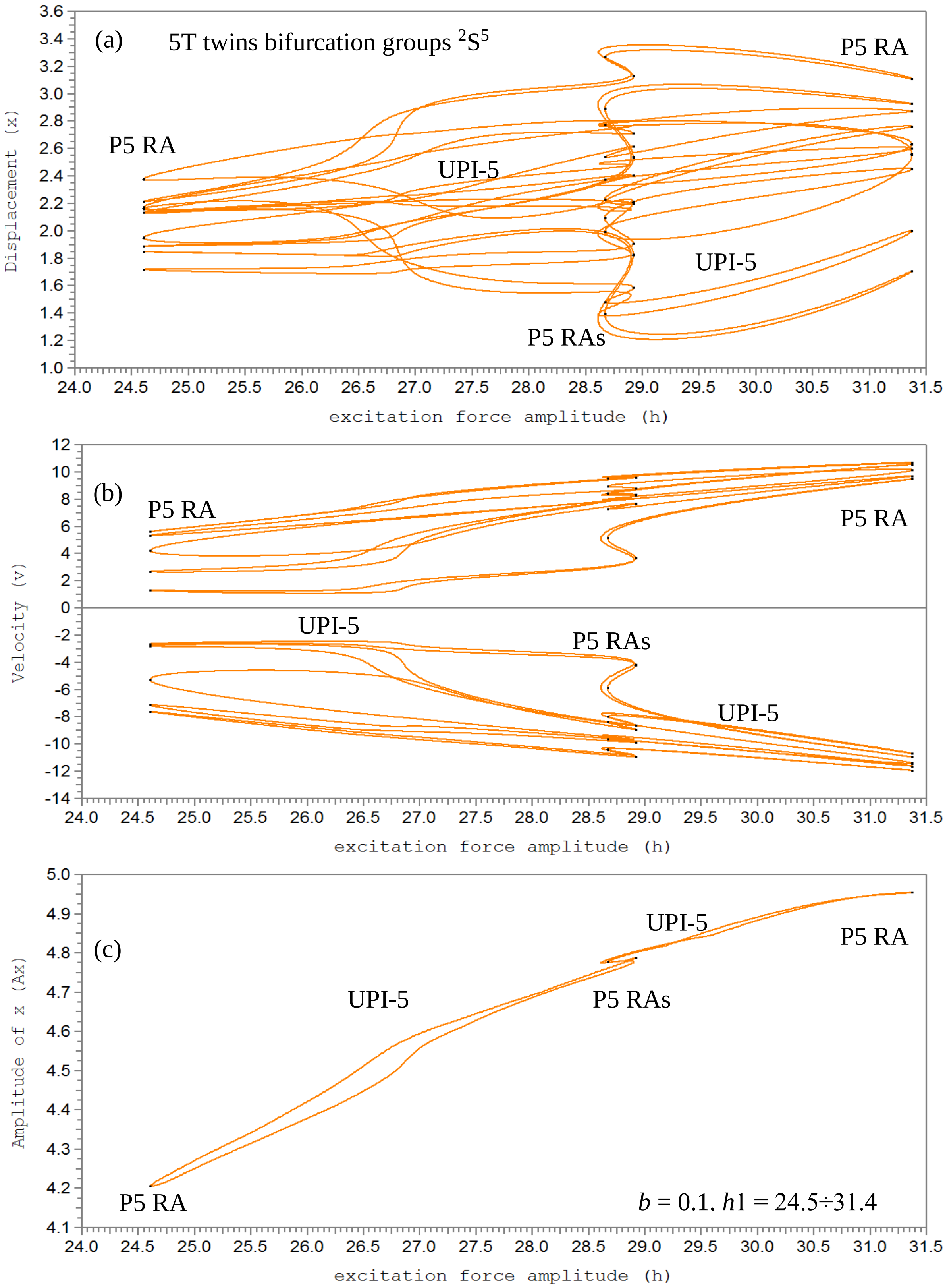


Figure 4: Duffing-Ueda symmetric oscillator with linear dissipation. Complex one-parameter bifurcation diagram for bifurcation group 5T with the RA and UPI-5. Parameters: $f(x) = x^3$, $b = 0.1$, $\omega = 1$, h = var.

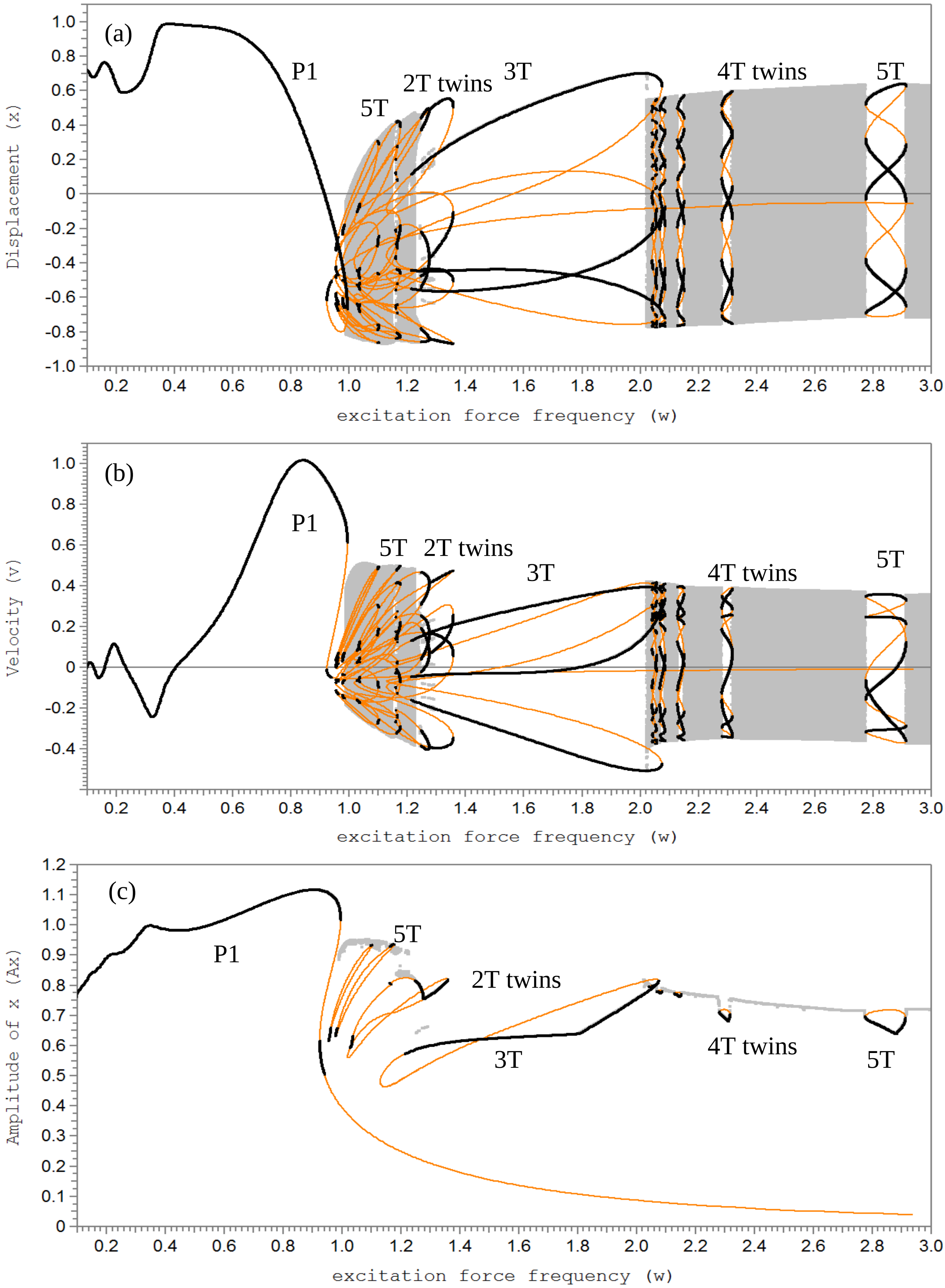


Figure 5: Duffing-van der Pol symmetric oscillator with linear dissipation. one-parameter bifurcation diagrams for bifurcation groups 1T,2T,3T,4T, 5T with the RA and UPIs. (c) time history and phase diagram for stable regime P5. Parameters: $f(x) = x^3$, $\mu = 0.2$, $\gamma = 8$, $h = 0.35$ ω = var.

4 CONCLUSIONS

In the paper it is shown that using the method of complete bifurcation groups allows finding new, previously unknown, qualitative results in archetypal nonlinear dynamical systems. For the Duffing-Ueda four unknown 5T bifurcation group have been found in the region amplitude of excitation h = 22 .. 32. This bifurcation groups have their own previously unknown rare and chaotic attractors. Build two-parameter bifurcation diagrams (h, b) for 5T witch has typical topology for different bifurcation groups (2T, 3T, 4T). We name this two-parameter bifurcation diagrams as chaotic jacket, because it has chaotic attractor inside the diagram. This diagram show the birth some submerged isles on one-parameter diagram.

The bifurcation analysis of the Duffing-van der Pol model show, that this system has unknown bifurcation groups with periodic, quasi-periodic and rare attractors. Some subharmonic isles have new topology with fully unstable folds from one side.

Obtained new results for the archetypal nonlinear oscillators are good illustrations for the new bifurcation theory of nonlinear dynamical systems [3-4]. This bifurcation theory allows to find new complex bifurcation groups with unknown periodic and chaotic attractors.

5 ACKNOWLEDGMENT

This work has been supported by the European Social Fund within the project «Support for the implementation of doctoral studies at Riga Technical University».

REFERENCES

[1] Y. Ueda, *Steady motions exhibited by Duffing's equation: a picture book of regular and chaotic motions, in New Approaches to Nonlinear Problems in Dynamics.* Edited by P.J. Holmes, pp. 311 – 322, SIAM, Philadelphia (1980). Y. Ueda, The Road to Chaos, 1992, Aerial Press, Inc.

[2] I. Kovacic , M.J. Brennan (Eds.), *The Duffing Equation: Nonlinear Oscillators and their Behaviour.* Wiley, 2011.

[3] M. Zakrzhevsky, *New concepts of nonlinear dynamics: complete bifurcation groups, protuberances, unstable periodic infinitiums and rare attractors.* Journal of Vibroengineering 10, Issue 4, 12 (2008), 421-441.

[4] M. Zakrzhevsky, I. Schukin, Yu. Ivanov, V. Frolov, R. Smirnova, V. Yevstignejev, A. Klokov, and E. Shilvan, *Nonlinear Dynamics and Chaos. Complete Bifurcation Analysis and Rare Attractors*. Riga Technical University, Institute of Mechanics, Riga, Latvia, 2nd edition, 2013, p. 210.