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# NONLINEAR DYNAMICS OF SHROUDED TURBINE BLADE SYSTEM WITH IMPACT AND FRICTION

**B. Santhosh, S. Narayanan*, C. Padmanabhan**

Machine Design Section,
Department of Mechanical Engineering,
Indian Institute of Technology Madras, Chennai, India
santhoshbpillai@gmail.com, narayans@iitm.ac.in, mouli@iitm.ac.in

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**Abstract.** *Dry friction damping is an effective way of reducing the resonant amplitudes in turbine bladed system where the inherent structural damping is very small. One method of inducing damping in such systems is by providing shrouds at the tip of the blades. In this case, the shrouds constrain the blade motion not only along the contact plane but also along the direction normal to the contact plane. The tangential motion along the contact plane results in friction between the contact surfaces and the interface undergoes stick slip motion. The motion normal to the contact plane results in intermittent separation and impact of the contacting surfaces.*

*In this paper the nonlinear dynamics of dry friction damped systems is investigated with application related to the vibration damping in shrouded turbine blade systems. A contact element proposed by Yang and Menq is used to model the contact interface, the stick slip motion induced by friction and the intermittent separation and impact due to variable normal load. The dynamics of these systems are governed by differential equations of discontinuous nature which are treated as Filippov systems. Assuming periodic motion of the interface the transition times corresponding to different states are computed and the hysteresis plots generated. A multi-harmonic balance procedure in combination with path following based on an arc length continuation is used to generate the periodic solutions.*

## 1 INTRODUCTION

The inherent structural damping in the case of turbo machinery bladed system is very small. The blade system is subjected to fluid forces which have a wide spectrum of frequencies and the spectrum of the natural frequencies of the blade system will also be very dense. Therefore the resonant regime is very large in these structures and avoiding all of them is practically impossible. Dry friction damping is the common mechanism used to damp the vibration in turomachinery bladed system to avoid high cycle fatigue failure. This is a passive way of vibration damping and can be achieved mainly by three mechanisms. They are the friction between the blade disk interface, under platform dampers and shrouded bladed system. In the case of blade disk interface, the relative motion between the blade and the disk interface produces the damping effect. Under platform dampers are metallic pieces of different geometry kept at the root of the blades and are held in position by the centrifugal force produced during the rotation of the blade disk system. The dry friction existing at the contact interface produces the vibration damping. In shrouded bladed system, the tips of the blades are connected by shrouds and again the relative motion between the contact interface leads to vibration reduction. There are mainly two challenges associated with the solution of dry friction damped turbine bladed system. Dry friction is inherently a nonlinear problem and computationally efficient tools are to be developed for its analysis. The second challenge is in the modeling of the contact interface. The interface will be either in a stick or slip state or with alternate stick slip motion if the normal load is constant. If the normal load is varying during the course of motion, the interface undergoes intermittent separation in addition to the stick slip motion. The system is thus found to undergo a combination of impact and friction nonlinearities during its motion. The aim is thus to obtain the exact transition times in steady state at which the interface undergoes various conditions then to calculate the interface frictional forces during each state and incorporate them in a computationally efficient framework to obtain the steady state periodic solutions and to predict the optimum parameters for the friction damper which gives maximum damping.

Griffin [1] provided a review on friction damping in turbine blade system with emphasis on the modeling and computational methods. Number of contact models have been proposed in the literature and modified later to include more features for the evaluation of contact forces. A one dimensional contact model with constant normal load was proposed by Griffin [2]. Coulomb friction element is added in series with a spring in order to take care of the elastic properties of the contact surface during deformation. The blade is modeled as a single degree of freedom system and this contact model is used by a number of researchers to study the effect of dry friction damping [3, 4] at constant normal load. This model is modified by Yang and Menq [5] to include the case of variable normal load. Single term harmonic balance method is used by them to study the frequency response under normal load variation. Petrov and Ewins [6]used the same contact model along with a multi harmonic balance framework to obtain the steady state periodic solutions. Analytical expression for the friction force vector and tangent stiffness matrices are provided for the faster computation of steady state response. Firrone et.al [7] modified the contact model proposed by Yang and Menq and used a coupled static/ dynamic harmonic balance method to study the effect of underplatform dampers on the forced response of bladed disk .

In this paper, the one dimensional contact model which can take care of the normal load variation proposed by Yang and Menq is used for the modeling of the friction interface. This model can also take care of the case of interface having constant normal load. Multi harmonic balance framework along with arc length continuation is used to obtain the frequency response.

Single degree of freedom model with constant and variable normal load and two degree of freedom system with constant normal load is investigated.

## 2 CONTACT INTERFACE MODEL

The contact interface model used in the paper is one proposed by Yang and Menq [5] which can take care of the variation of the normal load. The interface contact model is shown in Figure 1(a). The model consists of two springs of stiffness $k_u$ along the tangential direction

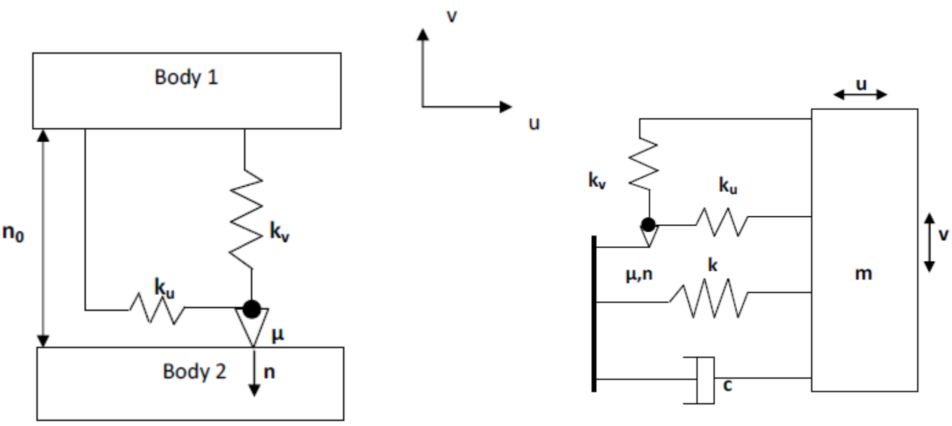



Figure 1: (a) Friction interface model (b) Sdf system with the interface model

and $k_v$ along the normal direction. The friction element is modeled based on the Coulomb friction law and is characterized by the frictional coefficient $\mu$. The initial preload is given by $n_0$ when the two bodies are in contact. If the two bodies are separated initially, then the gap can be represented using a negative value of the initial preload $n_0$. $u$ and $v$ represent the relative motions of the interface along the normal and tangential directions. The variable normal load along the contact interface can be expressed as

$$n = n_0 + k_v v \tag{1}$$

For the case of one dimensional model with constant normal load, the value of $k_v = 0$. During vibration, the interface can be in either of the four states, stick, positive slip, negative slip or separation. The state of the system and the transition times at which the state changes in one period of vibration are to be determined.

## 3 MULTI HARMONIC BALANCE METHOD

The single degree of freedom (sdf) model with the contact interface model is shown in figure1(b). In multi harmonic balance method, the steady state solution of the harmonically excited system is assumed to be periodic and displacements $u$ and $v$ are expressed in the form of a truncated Fourier series as

$$u(\tau) = U_0 + \sum_{j=1}^{N} [U_{2j-1}\cos(j\tau) + U_{2j}\sin(j\tau)] \tag{2}$$

$$v(\tau) = V_0 + \sum_{j=1}^{N} [V_{2j-1}\cos(j\tau) + V_{2j}\sin(j\tau)] \tag{3}$$

where the $U_i$ and $V_i$ are the Fourier coefficients and $\tau$ is a time parameter. The equation of motion for a multi degree of freedom system with nonlinear contact forces is given by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{F}(t) \tag{4}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are the mass, damping and stiffness matrices, $\mathbf{q}$ is the generalized displacement vector, $\mathbf{f}(\mathbf{q}, \dot{\mathbf{q}})$ is the nonlinear contact force vector and $\mathbf{F}(\mathbf{t})$ is the external excitation vector. The generalized displacement vector in the case of the sdf system shown in figure 1(b) is $\{u \quad v\}^T$. The equation of motion for the system shown in figure 1(b) considering the force balance in the tangential and normal direction is given by

$$m\ddot{u} + c\dot{u} + ku + f_u = F_u(t) \tag{5}$$

$$m\ddot{v} + f_v = F_v(t) \tag{6}$$

where $f_u$ and $f_v$ are the contact forces in the tangential and normal directions. $F_u(t)$ and $F_v(t)$ are the external excitation in the tangential and normal directions. Expressions for $f_u$ and $f_v$ depending on the state of the contact interface is given in equation (9) and (10). Substitute expressions (2) and (3) in (4) and apply the Galerkin procedure. This results in a residue vector $\mathbf{R}$ in terms of the Fourier coefficients of the displacement vector and can be expressed as

$$\mathbf{R}(\mathbf{Q}) = \mathbf{J}\{\mathbf{Q}\} + \mathbf{F_{NL}}(\mathbf{Q}) - \mathbf{P} \tag{7}$$

where $\mathbf{J}$ is the linear Jacobian matrix, $\mathbf{Q}$ is the Fourier coefficients of the displacement vector, $\mathbf{F_{NL}}(\mathbf{Q})$ is the vector of Fourier coefficients of the nonlinear contact forces and $\mathbf{P}$ is the vector of Fourier coefficients of the external excitation. The nonlinear equation (7) can be solved by Newton-Raphson method till a specified convergence lilmit of the residue vector. The Newton-Raphson procedure is applied as

$$\mathbf{Q}^{i+1} = \mathbf{Q}^i - \left[\mathbf{J} + \frac{\partial \mathbf{F_{NL}}(\mathbf{Q})^i}{\partial \mathbf{Q}}\right]^{-1} \mathbf{R}(\mathbf{Q})^i \tag{8}$$

where $\frac{\partial \mathbf{F_{NL}}(\mathbf{Q})}{\partial \mathbf{Q}}$ is the nonlinear Jacobian matrix.

## 4 DETERMINATION OF THE STATES AND TRANSITION TIMES

The next step is to find out the states and the transition times at which the state change takes place for one period of the steady state periodic motion. The interface can be in any one of the four conditios, positive slip (PS), negative slip (NS), stick or in full separation. The time instants at which the transition takes place and the state that follows can be found out which are explained in the subsections. The input parameters available for this purpose are $\mu, n_0, k_u, k_v, U$ and $V$.

### 4.1 Transition time and states after separation

If the interface is initially separated it can either go to a contact state or will remain separated for the full period. The procedure discussed below can be used to identify the states and the transition times. Solve the equation $n = n_0 + k_v v = 0$ for one time period to determine the $\tau$ values for which $n$ becomes zero. If there exist no root then look for the sign of $n$ for the entire period to identify whether the interface is in a contact state or fully separated state. If there exist a root of $n = 0$ for which $\dot{v} > 0$,$(\tau_s)$, then find out $f_1 = \dot{u}|_{\tau_s}$ and $f_2 = \mu \frac{k_v}{k_u} \dot{v}|_{\tau_s}$. If $f_1 > f_2$ then the interface is in a positive slip (PS) state. If $f_1 < -f_2$ then the interface is in a negative slip (NS) state else if $-f_2 < f_1 < f_2$, then the interface goes to a stick state.

### 4.2 Transition time and states after positive slip

From PS the system can either go to a stick state or separation and analytical conditions can be established for its determination. Solve the equation $\dot{u} - \mu \frac{k_v}{k_u} \dot{v} = 0$ and find out the $\tau$ values for which $\ddot{u} - \mu \frac{k_v}{k_u} \ddot{v} < 0$. These values give the time instants at which stick occurs. The time instants of separation can be established from the $\tau$ values obtained by solving the equation $n = 0$ for which $\dot{v} < 0$.

### 4.3 Transition time and states after negative slip

From NS the system can go to the stick state or to separation. To determine the transition time solve the equation $\dot{u} - \mu \frac{k_v}{k_u} \dot{v} = 0$ and find the values of $\tau$ for which $\ddot{u} - \mu \frac{k_v}{k_u} \ddot{v} > 0$ which gives the time instants at which stick occurs. A condition for the separation can be established as discussed in the previous subsection.

### 4.4 Transition time and states after stick

From stick state the interface can go to PS, NS or separation. The condition for the interface to go to separation can be established as discussed in the previous subsections. To determine the conditions for the interface to go to PS solve the equation $k_u u - \mu k_v v + (f_0 - \mu n_0 - k_u u_0) = 0$ and determine the time instants at which $k_u \dot{u} - \mu k_v \dot{v} > 0$. Similarly for the inetrface to go to NS, solve the equation $k_u u + \mu k_v v + (f_0 + \mu n_0 - k_u u_0) = 0$ and determine the time instants for which $k_u \dot{u} + \mu k_v \dot{v} < 0$. $f_0$ and $u_0$ are the values of frictional force and tangential displacement at the beginning of the stick state.

The conditions explained above are for the case of variable normal load and can be suitably modified for the case of constant normal load.

## 5 EVALUATION OF INTERFACE FORCES AND TANGENT STIFFNESS MATRIX

The expressions for the tangential and normal contact forces at the interface for different states of the system in time domain is given by

$$f_u = \begin{cases} k_u(u - u_0) + f_0 & \text{stick} \\ \mu(n_0 + k_v v) & \text{positive slip} \\ -\mu(n_0 + k_v v) & \text{negative slip} \\ 0 & \text{separation} \end{cases} \tag{9}$$

$$f_v = \begin{cases} n_0 + k_v v & \text{contact} \\ 0 & \text{separation} \end{cases} \tag{10}$$

Fourier coefficients for the tangential and normal forces are given by $F_{NL} = [F_U \quad F_V]^T$ and can be obtained from

$$\begin{Bmatrix} F_U \\ F_V \end{Bmatrix} = \frac{2}{\pi} \sum_{i=1}^{n} \int_{\tau_i}^{\tau_{i+1}} \begin{Bmatrix} S(\tau) f_u \\ S(\tau) f_v \end{Bmatrix} d\tau \tag{11}$$

where $S(\tau)$ is related to the displacements $u$ and $v$ by $u = SU^T$ and $v = SV^T$, therefore $S$ is the vecotr of basis functions of the Fourier series. Complete analytical formulation of $F_U$ and $F_V$ for different states of the system are given in [6].

## 6 RESULTS

The above explained procedure is applied to three examples which include a sdf system and a two dof system with constant normal load and a single degree of freedom model with the contact interface modeled with a variable normal load.

### 6.1 Example 1: Single dof model with constant normal load

The model is obtained by considering the value of $k_v = 0$ in figure 1(b) which is a repersentative model for a turbine blade with a blade to ground friction damper.The equations of motion for the system can be expressed as

$$m\ddot{u} + c\dot{u} + ku = f(t) - f_u \tag{12}$$

where $f(t) = F_0\cos(\omega t)$ is the external excitation and $f_u$ is the nonlinear frictional force which is expressed as

$$f_u = \begin{cases} k_u(u-y) & \text{when} \quad k_u|u-y| \leq \mu n \\ \mu n \quad sgn(\dot{y}) & \text{when} \quad k_u|u-y| \geq \mu n \end{cases} \tag{13}$$

where $y$ is the displacement of the damper and $sgn$ is the signum function. The parameters used for simulation are $m = 1kg, c = 0.3N/m/s, k = 3.55e4N/m, k_u = 1e4N/m, \mu = 0.5, F_0 = 1N$. The number of harmonics used in the simulation are $7$. The frequency response of the system for different values of the normal load are shown in figure2. When $n = 0$, the

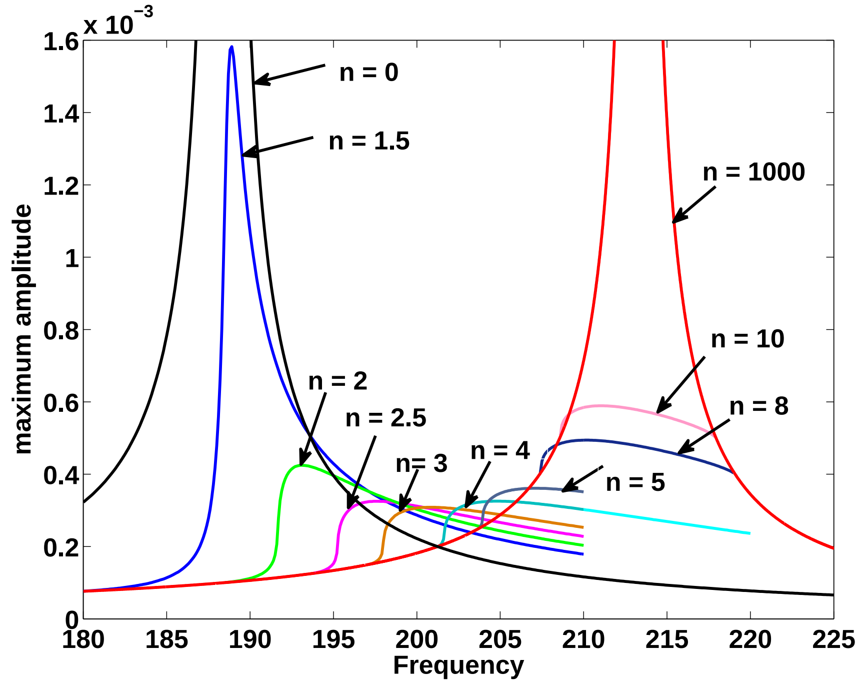



Figure 2: Frequency response of dry friction damped sdf system

system behaves like a sdf system with stiffness $k$ and when the value of $n$ is large say 1000, the interface is in a fully stuck state and the stiffness of the system is the combined value of $k + k_u$. For moderate values of $n$, the interface undergoes both stick and slip motion and this is the time when actual damping of the system takes place. From the frequency response it can also be observed that there is an optimum value of normal load for which the amplitude of the mass is minimum.

### 6.2 Example 2: Two dof system with friction damper under constant normal load

The second example considered is the case of a two dof system with a friction damper attached to the first mass which is shown in figure3. The first mass is subjected to a harmonic excitation. The system parameters are $m_1 = m_2 = 1kg, k_1 = k_2 = 40kN/m, c_1 = c_2 = 4Ns/m, F_0 = 100N, k_u = 30kN/m$. The natural frequencies of the linear system without

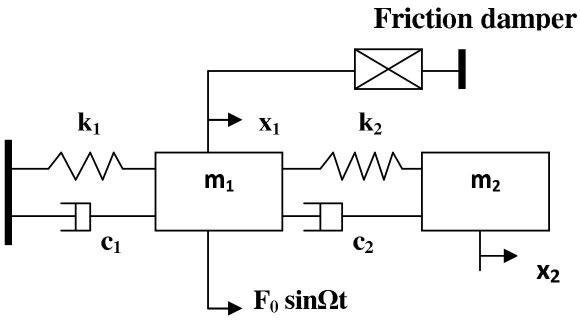



Figure 3: Two dof system with dry friction damper

including the damper stiffness are $19.67Hz$ and $51.50Hz$. After including the damper stiffness the natural frequencies becomes $23.52Hz$ and $56.97Hz$. The frequency response of the first mass for different values of the normal load is shown in figure 4. It can be observed that there is an optimum value for the normal load for which the amplitude is a minimum.

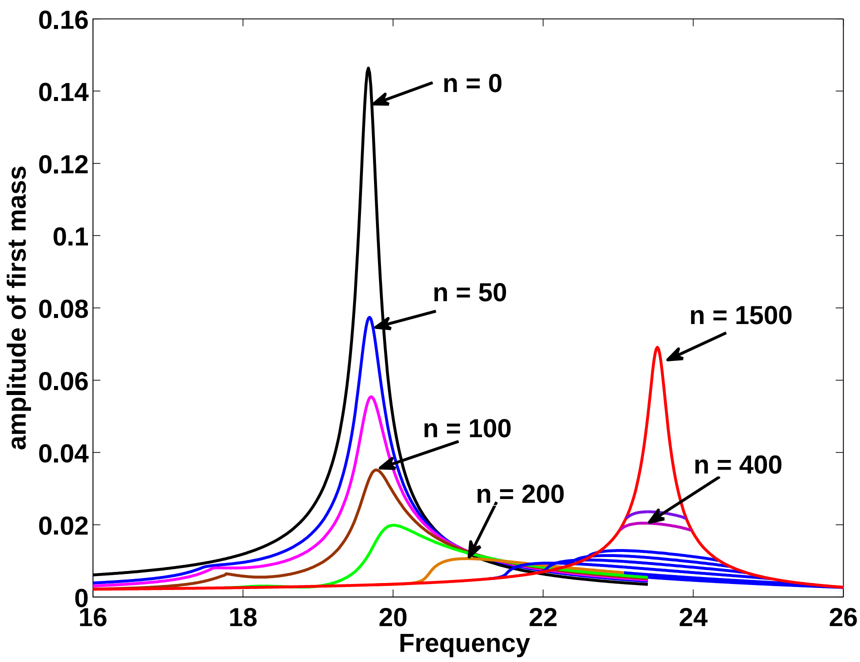
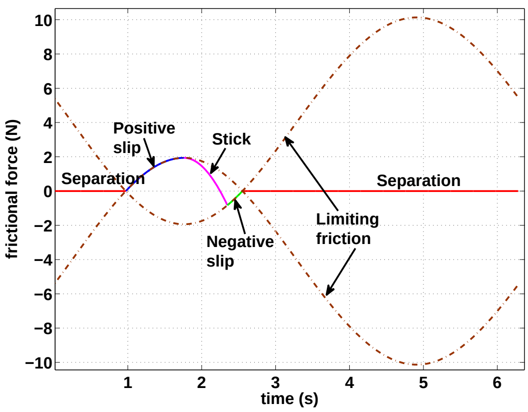



Figure 4: Frequency response of dry friction damped two dof system

### 6.3 Example 3: Single dof with variable normal load

The third example considered is a sdf system with a friction damper with variable normal load. This is the typical scenario which occurs between two adjacent blade tips in the case of a shrouded blade system. The model of the sdf system with the contact model is shown in figure1(b). Essentally it has to be considered as a two dof model. The equations of motion are given in (5)-(6). The modal informations relating to the dynamics of the system are given by $m_1 = m_2 = 1$, the modal frequencies $\omega_{n1} = 1, \omega_{n2} = 10$, modal damping ratio $\zeta_1 = \zeta_2 = 0.01$ and the mode shapes $\phi_1 = [0.707 \quad 0.707]^T, \phi_2 = [1 \quad -0.5]^T$. The value of friction coefficient $\mu = 0.4$, the normal and tangential stiffness value $k_v = k_u = 1$ and the amplitudes of the external harmonic excitation is split into $f_u = f_v = 1$. Typical plots showing the time variation of the frictional force and the hysteresis plots along with the possible transitions are shown in figure 5(a) and (b). The frequency response of the system for different values of the normal



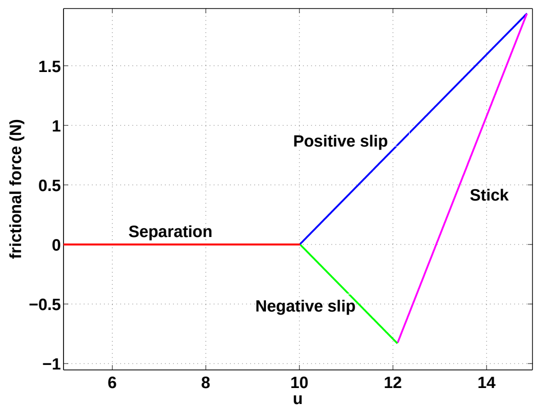




Figure 5: (a) Time variation of frictional force (b) Hysteresis plot

loads are shown in figure 6.

When the interface is fully separated throughout the cycle $(n_0 = -100)$ the frequency response corresponds to the linear system . When $n_0$ is given a negative value say $-20$, it indicates that there exists a gap between the contact surfaces. During the course of vibration, the the interface again comes in contact and undergoes stick-slip motion. This can be observed from the bending of the frequency response curve to the right indicating the hardening behavior. In this case multivalued solutions exist for the response for a particular frequency. When $n_0$ becomes a positive value $(n_0 = 10)$, the interface is initially at contact and during the course of vibration it separates and comes back into contact later. The frequency response in this case bend towards the left and this indicates a softening behavior. Multivalued responses exist for this case also. For larger values of $n_0$ say $500$, the interface always remains in a stuck state. Both in hardening and softening behavior of the response curve jump phenomenon can be observed. This contact interface model can be implemented on a multi dof shrouded blade system to predict the response of the system.

## 7 CONCLUSIONS

- A contact interface model which is capable of taking care of the variation of normal load proposed in the literature is used to study the vibratory response of single dof and two dof systems with constant and variable normal load.

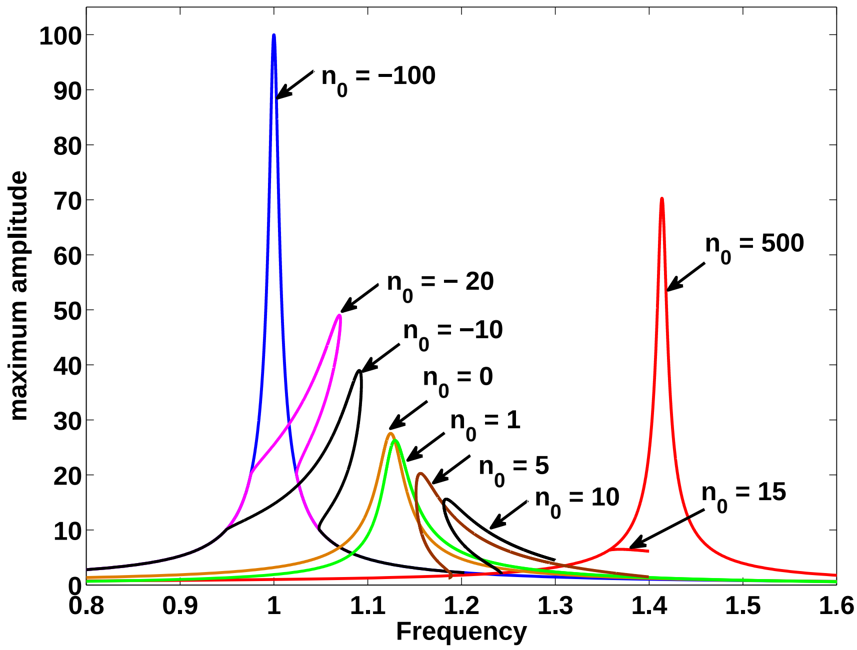



Figure 6: Frequency response of dry friction damped sdf under variable normal load

- Analytical expressions are given for the calculation of transition times and the states of the system for one period of motion.
- Multi harmonic balance in combination with arc length continuation is used to plot the frequency response.
- With variable normal load, the problem represents the combined effect of impact and friction which is a typical scenario observed in the case of shrouded turbine blade system.
- The frequency response shows jump phenomenon with multivalued response for certain frequency ranges.
- The system can be treated as a Filippov system with three distinct states, stick, slip and separation and can be integrated using an event driven numerical integration technique.

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