11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

AN INFLUENCE OF BALANCING ON NON-UNIFORM MOTION OF MECHANISMS

Vlastimil Votrubec* [1]

[1] VÚTS, a.s., Svárovská 619, 460 01 Liberec XI, Czech Republic
vlastimil.votrubec@vuts.cz

Keywords: Balancing, Non-Uniform Motion, Crank-Slider Mechanism

Abstract. *The purpose of balancing is reducing the shaking force and shaking moment that negatively influence the behaviour of machines. The balancing principles and methods have been researched for a long time but results of the investigation aren´t commonly applied. This can be caused by the lack of information and some drawbacks that balancing brings. Disadvantages of balancing are usually large addition of mass and inertia, design problems and others. Every added mass causes changes of energy and thus changes of angular velocity of the drive shaft. The aim of this paper is to describe this unexplored influence of balancing on non-uniform motion.*

Each balancing method affects the uniformity of motion variously. For a comparison, a crank-slider mechanism was chosen to demonstrate that influence of several different balancing methods varies. The biggest interest was put on static balancing by counterweights which is the most used method and very often the only one applied. Also the influence of balancing using the counter-rotary elements, idler loop and opposite movements is presented. The influence of balancing depends on parameters of balancers - mass and distance between the counterweight and axis of rotation, moment of inertia and speed ratio of counter-rotary elements. Appropriate setting of parameters leads to more uniform rotation of the crank shaft, whereas the best results can be achieved by static balancing. However, requirements for uniform motion are often at odds with requirements for low mass and inertia addition.

1 INTRODUCTION

A change of energy in moving links causes non-uniform rotation of the driving shaft and thus influences motion of other mechanisms. Especially links which energy decreases to zero have got a great impact on the non-uniform motion, e.g. sliding mass of crank-slider mechanism. This change requires great energy changes and thus input torque changes of the driving shaft which causes its non-uniform rotation and subsequent negative phenomena. The change is especially noticeable in heavy links or links with a large change in velocity (e.g. the effect of slay mechanism of weaving machine on shedding mechanism).

Balancing of mechanisms is usually done in order to reduce the inertia forces and torques on the base, thereby reducing vibrations and suppressing other undesirable phenomena. Today there exist a lot of balancing methods and their design and influence on the machine behavior is therefore different [1]. Apart from the desired effects of balancing such as a reduction of shaking force and moment, balancing has got also some undesirable effects. It is mainly the added mass and moment of inertia or larger work space [2]. Rarely surveyed area is the influence of balancing on the non-uniformity of the links motion, especially on the rotation of the driving shaft of the machine.

Fluctuation of rotation speed of the driving shaft doesn`t affect balancing of the shaking force and moment (if the compliance and clearance are neglected). But it doesn`t works vice versa. The added balancing mass usually significantly affects the rotation of the driving shaft. The next chapters describe the influence on non-uniform motion of the driving shaft for several more common and known methods of balancing (rotary balancers, counter-rotary balancers, opposite movements, added dyad). The particular methods are compared on the example of the crank-slider mechanism balanced by different configurations of the mentioned balancing methods.

2 BALANCING

The goal of balancing is reducing or better eliminating of shaking force and moment of mechanisms. Conditions for balancing can be obtained from the conservation of linear and angular momentum law

$$\boldsymbol{P} = \sum_{\mathrm{i}=1}^{\mathrm{n}} m_i \dot{\boldsymbol{r}}_i = \boldsymbol{const}. \tag{1}$$

$$\boldsymbol{B} = \sum_{\mathrm{i}=1}^{\mathrm{n}} (\boldsymbol{I}_i \dot{\boldsymbol{\varphi}}_i + \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i) = \boldsymbol{const}. \tag{2}$$

where m_i is the mass of moving link i, $\boldsymbol{I_i}$ the moment of inertia matrix, $\boldsymbol{r_i}$ the position vector of the centre of mass m_i and $\dot{\boldsymbol{\varphi}}_i$ the vector of the angular velocity.

The mechanism is statically balanced if the Eq. (1) is fulfilled. It implies the centre of mass of the mechanism moves with constant velocity or is stationary which is more practical in calculations. Constant or zero angular momentum in the Eq. (2) is a condition for dynamic balancing. Dynamically balanced mechanism is always balanced also statically.

There are many balancing methods and principles researched in the past, some of them are described here [1, 3, 4]. The most common method of balancing is using rotary balancers, e.g. counterweight on the crank shaft which balances centrifugal force of the crank and part of the connecting rod. Widespread balancing method is also force harmonic balancing. A counterweight attached to the input shaft of the mechanism produces chosen harmonic component of the balanced shaking force. Other principle uses opposite movement of a mechanism to balance original mechanism. All of these three methods which provide only partial balancing are typically applied in engines.

The balancing certainly has some effects on balanced mechanisms. The positive impact is clear – the reduction of shaking force and moment and thus reduction of vibration and noise.

The negative impact is addition of mass and moment of inertia, bigger work space, design problems and others. These drawbacks of balancing of double-pendulum were described here [5]. The balancing also has influence on non-uniform motion of mechanism, especially rotation of the drive shaft. This influence wasn´t researched much; therefore some results of the investigation are presented in this paper.

It can be assumed that each balancing method has got different influence on non-uniform motion of mechanisms. This non-uniform rotation of the drive shaft is compared for several balancing methods on the example of crank-slider mechanism. The calculation of that influence is done for static balancing, dynamic balancing using counter rotating balancers, idler loop and duplicate mechanism.

3 NON-UNIFORM MOTION OF CRANK-SLIDER MECHANISM

The non-uniform angular velocity can be derived from the total kinetic energy of the mechanism or equation of motion. If the mechanism is substituted by equivalent system that makes rotational motion, the angular velocity of the crank shaft can be calculated from this equation

$$I_{eq}\ddot{\varphi} + \frac{1}{2}\frac{d\,I_{eq}}{d\varphi}\dot{\varphi}^2 = M_{eq} \tag{3}$$

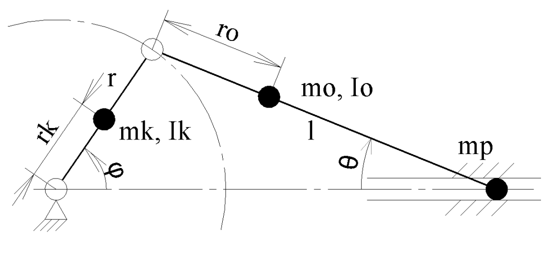
where I_{eq} is equivalent moment of inertia, M_{eq} is equivalent moment and φ is angle of rotation of the crank shaft. It is convenient to rewrite Eq. (3) to the following formula, where the variable ω depends on angle φ

$$I_{eq}\frac{d\omega}{d\varphi}\omega + \frac{1}{2}\frac{d\,I_{eq}}{d\varphi}\omega^2 = M_{eq} \tag{4}$$

The non-uniform motion which depends on total equivalent moment of inertia of the mechanism is then calculated by dimensionless number σ

$$\sigma = \frac{2(\omega_{max} - \omega_{min})}{\omega_{max} + \omega_{min}} \tag{5}$$

The comparison of the influence of balancing on non-uniform motion is accomplished on the crank slider mechanism according to the Figure 1. The parameters of the mechanism correspond to the real crank-slider mechanism of the needle bar in an industrial sewing machine.



	Crank	Connecting Rod	Slider
Mass	0.4 kg	0.024 kg	0.056 kg
Inertia	132 kg·mm^2	15 kg·mm^2	-
Length	18 mm	56 mm	-
CoM	3 mm	24 mm	-

Figure 1: The scheme of the crank-slider mechanism (left) and parameters of the real mechanism from sewing machine (right). CoM means a position of the centre of mass of the link (rk, ro).

The equivalent moment of inertia has to be derived for each link which is slightly complicated for the connecting rod and the resultant relation isn´t therefore shown. The numerical solution for parameters in the Figure 1 provides the value of non-uniformity 0.079 which means that the angular velocity of the crank shaft fluctuates about 8%. The next chapter will compare this value with the value for the mechanism balanced by various balancing methods.

4 THE NON-UNIFORM MOTION OF BALANCED MECHANISMS

The main focus of this investigation is aimed at static balancing using counterweights and its influence of balancing on non-uniform rotation of the crank shaft. This balancing is the most frequent and very often is the only one that is used for balancing mechanisms. There are compared other methods based on counter rotating counterweights, idler loop and opposite movements.

4.1 Static balancing

The crank slider mechanism is statically (forced) balanced by two counterweights mounted on the crank and the connecting rod opposite its centre of mass. The principle is obvious from the Figure 3 (if the counter rotating masses are not considered). The condition of balancing are derived from the vector equation of zero linear momentum of the mechanism

$$m_k \dot{\boldsymbol{r}}_1 + m_o \dot{\boldsymbol{r}}_2 + m_p \dot{\boldsymbol{r}}_3 + m_{v1} \dot{\boldsymbol{r}}_{v1} + m_{v2} \dot{\boldsymbol{r}}_{v2} = \boldsymbol{0} \tag{6}$$

This equation can be edited with use of angles φ and θ (angle between the crank, connecting rod and the base) to the following formulas

$$\left(lm_p - l_{v2}m_{v2} + m_o r_o\right)\cos\theta\,\dot{\theta} + \left[r\left(m_o + m_p + m_{v2}\right) + m_k r_k - l_{v1}m_{v1}\right]\cos\varphi\,\dot{\varphi} = 0 \tag{7}$$

$$\left(lm_p - l_{v2}m_{v2} + m_o r_o\right)\sin\theta\,\dot{\theta} + \left[r\left(m_o + m_p + m_{v2}\right) + m_k r_k - l_{v1}m_{v1}\right]\sin\varphi\,\dot{\varphi} = 0 \tag{8}$$

The conditions of balancing result from both Eqs. (7) and (8)

$$lm_p - l_{v2}m_{v2} + m_o r_o = 0 \tag{9}$$

$$r\left(m_o + m_p + m_{v2}\right) + m_k r_k - l_{v1}m_{v1} = 0 \tag{10}$$

The equivalent moment of inertia and thus the non-uniform rotation of the crank shaft depend on the four parameters from the Eqs. (9) and (10). They are the distances between the centre of mass of the counterweight and the axis of rotation l_{v1}, l_{v2} and the mass m_{v1}, m_{v2}. The distance or mass is always chosen and the second parameter is enumerated. On the Figure 2 there is a 3D diagram of non-uniform motion of the mechanism σ. On the horizontal axis there are distances l_{v1}, l_{v2} and the fluctuation of angular velocity of the crank shaft σ is on the z-axis.

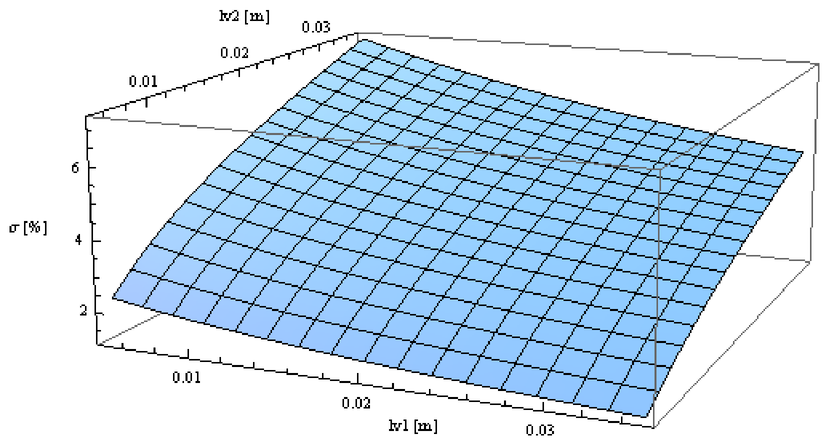


Figure 2: 3D plot of the non-uniform rotation σ of the crank shaft of the statically balanced crank mechanism.

From the Figure 2 it is clear that the uniformity of the rotation is the best for maximal value of parameter l_{v1} and minimal value l_{v2}. It is caused by different character of motion of the counterweights. The counterweight on the connecting rod rotates with variable angular velocity so its equivalent moment of inertia contributes to higher non-uniformity of motion. For enhancement the uniformity of the rotation it is necessary to reduce the equivalent moment of inertia. It is achieved by shorter distance l_{v2} of the counterweight. The situation on the counterweight of the crank is contrary. The uniformity of motion is better for higher moment of inertia and the distance l_{v1} should be longer.

The non-uniform motion σ is in all cases in the Figure 2 better than σ of an unbalanced mechanism. The angular velocity of the crank shaft for several settings of parameters and input velocity 2π rad/s is in the Figure 3. As it was said the uniformity improves with higher distance l_{v1} and smaller distance l_{v2}.

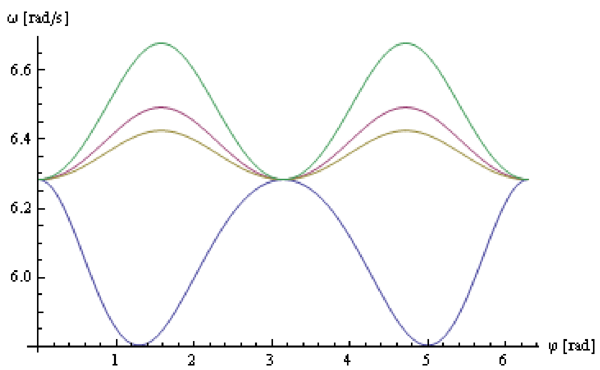


Figure 3: Angular velocity of the crank shaft of the unbalanced mechanism (blue) and balanced mechanism with parameters l_{v1}=l_{v2}=0.01 m (red), l_{v1}=0.03 m and l_{v2}=0.01 m (yellow), l_{v1}=0.01 m and l_{v2}=0.03 m (green).

Appropriate settings of parameters of balancing leads among the diminishing of shaking force to enhancement of uniformity of motion. However these parameters can be at odds with the requirements for minimal addition of mass and equivalent moment of inertia (Figure 4). For a low mass addition, counterweights have to be placed far from the axis of rotation. For a low maximal value of equivalent moment of inertia, counterweights should be placed in the contrary way than for the best uniform rotation of the crank shaft. The designer has to make a trade off between these and other options and qualify particular drawbacks and advantages.

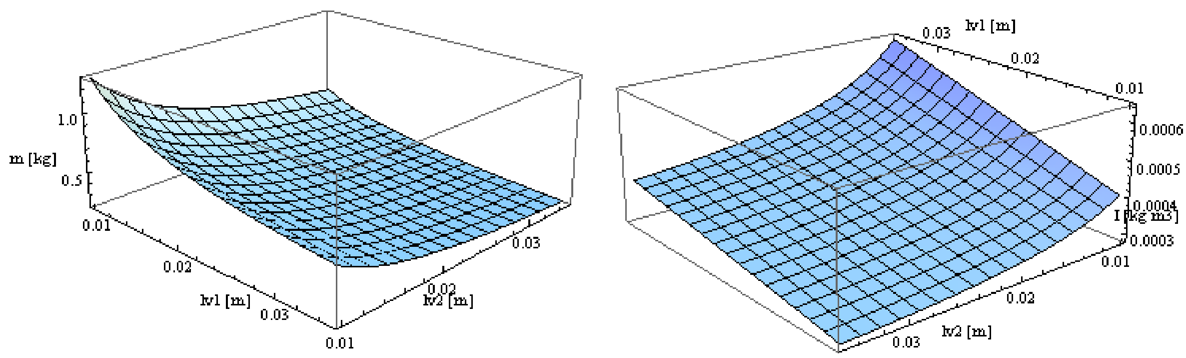


Figure 4: Added mass of counterweights (left) and maximal value of the equivalent moment of inertia of the mechanism (right) in dependence on distance l_{v1}, l_{v2} of the counterweights from the axis of rotation.

4.2 Balancing by counter-rotating balancers

This type of balancing provides complete (dynamic) balancing. Force balancing is achieved by counterweights described in the previous section. Balancing of shaking moment is realized by counter-rotating elements (Figure 5). The configuration of these counter-rotating links can be various with corresponding advantages. The two basic approaches are using the counter-rotating elements separately on the frame or using them mounted directly on the links that they balance. The opposite rotation is done by gears, chains or belts.

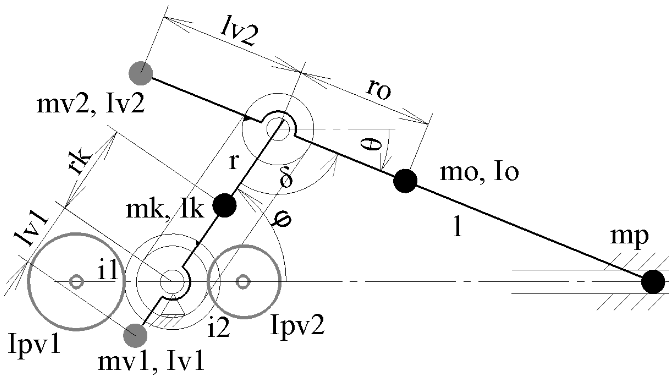


Figure 5: Complete balancing of the crank-slider mechanism. The shaking force is balanced by counterweights m_{v1}, m_{v2} and the shaking moment by counter-rotating elements I_{pv1}, I_{pv2}. The counter-rotating links can be mounted separately on the frame (left) or directly on the link that they balance (right).

The conditions of balancing are the same for static balancing, conditions for moment balancing using separately mounted counter-rotating links (Figure 5 left) are derived from equation of zero angular momentum of the mechanism

$$I_k\dot{\boldsymbol{\varphi}} + I_{v1}\dot{\boldsymbol{\varphi}} + I_o\dot{\boldsymbol{\theta}} + I_{v2}\dot{\boldsymbol{\theta}} + \boldsymbol{r}_1 \times m_k\dot{\boldsymbol{r}}_1 + \boldsymbol{r}_{v1} \times m_{v1}\dot{\boldsymbol{r}}_{v2} + \boldsymbol{r}_2 \times m_o\dot{\boldsymbol{r}}_2 + \boldsymbol{r}_{v2} \times m_{v2}\dot{\boldsymbol{r}}_{v2} + \boldsymbol{r}_3 \times m_p\dot{\boldsymbol{r}}_3 + i_1 I_{pv1}\dot{\boldsymbol{\varphi}} + I_{pv2}\left[i_2\dot{\boldsymbol{\theta}} + (1 - i_2)\dot{\boldsymbol{\varphi}}\right] = \boldsymbol{0} \qquad (11)$$

where I_{v1}, I_{v2}, I_{pv1}, I_{pv2} are moments of inertia of counterweights and counter-rotating links and i_1, i_2 are speed ratios. The conditions of complete balancing result from the sole non-zero vector component of the Eq. (11)

$$I_k + I_{v1} + i_1 I_{pv1} + I_{pv2}(1 - i_2) + r^2\left(m_o + m_p + m_{v2}\right) + l_{v1}^2 m_{v1} + r_k^2 m_k = 0 \qquad (12)$$

$$I_o + I_{v2} + i_2 I_{pv2} + l^2 m_p + l_{v2}^2 m_{v2} + r_o^2 m_o = 0. \qquad (13)$$

The non-uniform motion of the crank shaft again depends on the parameters of the balancers. Beside the parameters of the counterweights (distance and mass) they are moments of inertia and speed ratios of the counter-rotation links. One of them is chosen, the second one is calculated. Counter-rotary balancers have the main influences. Similarly to previous case, equivalent moment of inertia of balancer I_{pv1} only increases the constant part of total equivalent moment of inertia of the mechanism and thus makes the rotation of the crank shaft more uniform. For this purpose the balancer should have small moment of inertia and high speed ratio i_1. The second counter-rotation link should have the speed ratio i_2 small.

This principle is demonstrated in the Figure 6. The non-uniform rotation σ is a function of speed ratios i_1 and i_2 of the counter-rotation balancers, the parameters of counterweights are chosen for $l_{v1} = l_{v2} = 0.01$ m ($m_{v1} = 0.93$ kg, $m_{v2} = 0.37$ kg). If the angular velocity of the counter-rotary links is the same but with opposite rotation with the balanced links, then the non-uniform motion σ = 0.14.

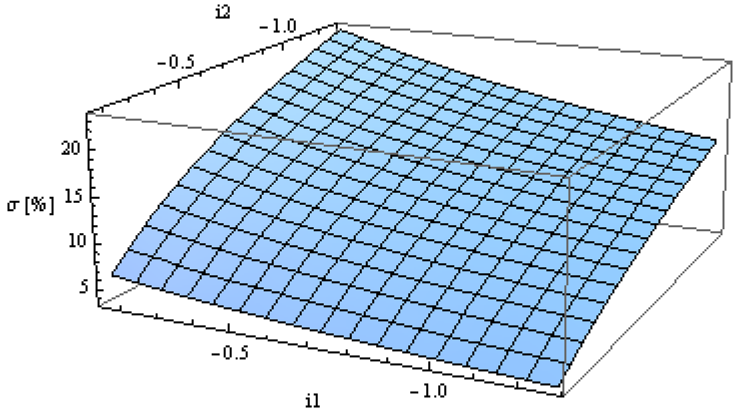


Figure 6: The non-uniform motion of the crank shaft σ of the real crank slider mechanism completely balanced by two counterweights and separately mounted counter-rotary links. Non-uniformity of motion before balancing was 7.9 %.

Moments of inertia of the counter-rotation links are severalfold bigger than moments of inertia of other links and significantly increase the total equivalent moment of inertia. It is possible to reduce these moments by increasing speed ratios. Balancing by counter-rotation links makes the uniformity of motion worse in most of the cases, but applying of appropriate parameters of counter-rotation links improves the uniformity. The disadvantage is that this setting of balancers parameters raises the total equivalent moment of inertia.

The second mentioned approach of balancing is using the counter-rotation links directly mounted on moving links (Figure 5 right). The moment of inertia of the counter-rotary balancer equals the moment of inertia of counterweights so the balancing conditions are more simple. Also the added mass is lower, because the mass of the counterweight is used for counter-rotary balancer. However, the influence on non-uniform motion of the crank shaft is almost the same, because the main source of non-uniform motion, counter-rotary link balancing the connecting rod, remains. This conclusion applies to many other configurations with counter-rotary links that exist.

4.3 Balancing by idler loop and opposite movements

The idler loop transfers the inertia torque of the connecting rod to the axis on the frame where it is balanced by counter-rotary link. The other link balances the inertia torque of the crank. The transfer of motion from connecting rod using gears is needless. Shaking force is balanced by two counterweights (Figure 7).

The conditions of balancing are similar to the previous. The influence of balancing on non-uniform motion is worse due to the less mass m_{v1} of the counterweight on the crank. According to the Figure 7 and settings of the parameters of the mechanism (a = 0.02 m, b = 0.009 m, m_5 = 0.02 kg), the non-uniform motion σ is 0.177. The uniformity can be improved by appropriate values of the parameters.

The principle of balancing by opposite movements is addition of mechanism to the origin mechanism so their links will generate the same inertia forces and torques with opposite direction. For static balancing one mechanism has to be added and for complete balancing another two mechanisms. The total equivalent moment of inertia and mass are fourfold bigger but the non-uniformity of the motion is the same as before balancing.

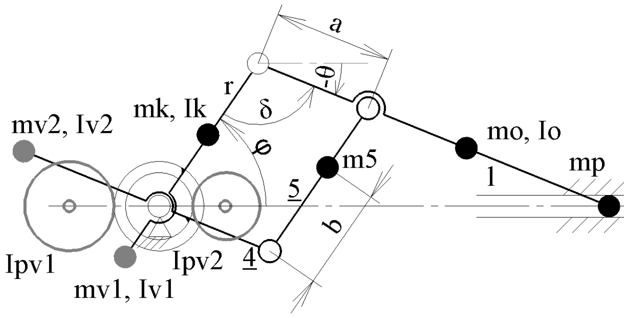


Figure 7: The crank-slider mechanism balanced by idler loop and two counter-rotary links mounted on the base.

5 CONCLUSION

Generally it can be said that the influence of balancing crank-slider mechanism on non-uniform rotation of the crank shaft depends on the parameters of balancing. The exception is balancing by opposite movements which doesn´t affect the angular velocity of the crank shaft. The fluctuation of that angular velocity is a function of total equivalent moment of inertia. The assessment of balancing methods regarding the uniformity of motion is done by evaluation of particular balancing parameters.

The static balancing using counterweights usually improves the uniformity of motion or it deteriorates the uniformity only a little. Balancing by counter-rotary balancers (idler loop) then again in most cases has a bad influence on the uniformity. The uniformity of the motion goes better when the equivalent moment of inertia of the balancer moving with constant velocity is raised and the moment of inertia of the balancer moving with variable velocity is reduced.

For minimal fluctuation of angular velocity of the crank shaft the counterweight on the crank should be far from the axis of rotation and the counterweight on the connecting rod should be close to the axis of rotation. The counter-rotary link which balances the rotation of the crank should have high speed ratio, while the counter-rotary link that balances motion of the connecting rod should have low speed ratio. These recommendations are often at odds with requests for low mass addition, moment of inertia addition and others.

REFERENCES

[1] V. H. Arakelian and M. R. Smith, Shaking force and shaking moment balancing of mechanisms: A historical review with new examples. *Journal of mechanical design*, **127**, 334-339, 2005.

[2] V. Van der Wijk, J. L. Herder, B. Demeulenaere, Comparison of various dynamic balancing principles regarding additional mass and additional inertia. *Journal of mechanisms and robotics*. **1**, 2009.

[3] G. G. Lowen and R. S. Berkof, Survey of investigations into the balancing of linkages. *Journal of mechanisms*, **3**, 221-231, 1968.

[4] I. S. Kochev, General theory of complete shaking moment balancing of planar linkages: A critical review. M*echanism and machine theory*, **35**, 1501-1514, 2000.

[5] V. Van der Wijk and J. L. Herder, Synthesis of dynamically balanced mechanisms by using counter-rotary countermass balanced double pendula. *Journal of mechanical design*. **131**, 2009.