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EXACT NONLINEAR DYNAMICS FOR DAMPED PENDULUM SYSTEMS UNDER PARAMETRICALLY OR FORCED EXCITATION

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Abstract. *A new type of the simplest models of essentially nonlinear damped parametrically-excited or forced systems, suggested by the author, is presented. The main idea of the so-called "exact model" is using for its description a qualitatively "equivalent" piece-wise linear model with only constant forces on each its linear sub-system. For "exact" model all restoring and driven forces are changed into piece-wise constant linear forces. Also for periodic excitation may be used instant periodic impulses, if necessary. Damping in the exact models is introduced by models of impact damping or by models with dry friction. So to obtain the exact solution in a linear subsystem of the exact model, and to find its switching times, only quadratic equations are necessary. It's important to underline that the simplicity of finding switching times stays the same for systems with several or even many degrees-of-freedom.*

Exact models allow investigating all main nonlinear phenomena in typical nonlinear models using Poincare ideas, well-known theory of piecewise linear systems and their stability evaluation The main advantages of "exact" models are their simplicity, speed and exactness. Some results are qualitatively compared with known results for different nonlinear systems with parametrical or external exaltation .

In this presentation we illustrate the main ideas of the exact nonlinear dynamics for three damped pendulums one DOF models: under vertical and horizontal parametrically excitation, and a pendulum model under forced excitation. The birth of different chaotic attractors from subharmonic bifurcation groups and the birth of chaotic different rotations are good examples for described new approaches. New possibilities of the exact nonlinear dynamics for several and many degrees of freedom systems and for stochastically excited systems also are discussed in the paper.

The most interesting and important result of this presentation, to my mind, is the possibility of obtaining the exact simple analytical formulae for chaotic dynamics.

1 INTRODUCTION. EXACT NONLINEAR DYNAMICS (END). MAIN IDEAS.

We illustrate the main idea of the so-called “exact model” is using pendulum models. For its description it is necessary to include a qualitatively “equivalent” piece-wise linear model with only constant forces on each its linear sub-system. For “exact” model all restoring and driven forces are changed into piece-wise constant linear forces. Also for periodic excitation may be used instant periodic impulses, if necessary. Damping in the exact models is introduced by models of impact damping or by models with dry friction.

An example of transformation of usual nonlinear pendulum models into exact pendulum model is shown on Figure 1. It is possible to prepare exact models for different kinds of parametric and external pendulum’s equations.

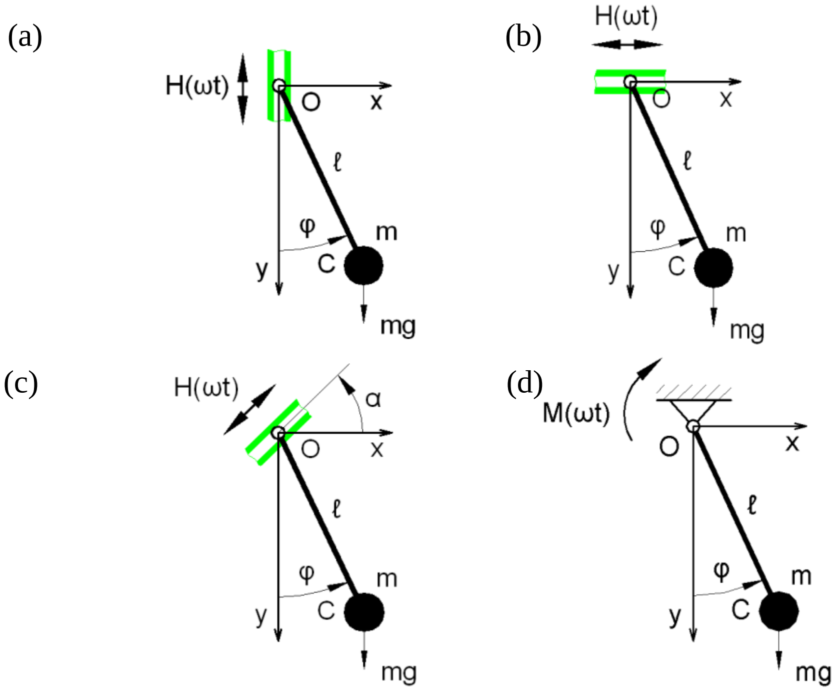

Figure 1: Usual pendulum models.

Below we show a change of the usual pendulum eq. (left column) into exact (right column)

Usual pendulum’s equations		“Exact” pendulum’s equations
$\ddot{\varphi}+b\dot{\varphi}+(1+h\cos\omega t)\sin\pi\varphi=0$	(1)	$\ddot{\varphi}+(1+h\cos r(\omega t))\sin r(\pi\varphi)=0$
$\ddot{\varphi}+b\dot{\varphi}+a_1\sin\pi\varphi=h\cos\omega t\cos\pi\varphi$	(2)	$\ddot{\varphi}+a_1\sin r(\pi\varphi)=h\cos r(\omega t)\cos r(\pi\varphi)$
$\ddot{\varphi}+b\dot{\varphi}+a_1\sin\pi\varphi=h\cos\omega t$	(3)	$\ddot{\varphi}+a_1\sin r(\pi\varphi)=h\cos r(\omega t)$

Here sinr(α), cosr(α),… mean rectangle sin-function, rectangle cos-function, etc. For exact pendulum’s models dissipation introduced by impact: if $\varphi = d_i$ $\dot{\varphi}^+ = R\dot{\varphi}^-$, $0 < R < 1$.

Foundation of the exact nonlinear dynamics may be found in [1-5]. We illustrate application of ‘exact’ nonlinear dynamical models by two models: bilinear driven system (Figure 2-3) with subharmonic regimes P3 and P5 and by a pendulum exact model (Figure 4) with a chaotic rotational attractor. For these examples we use papers [6-10].

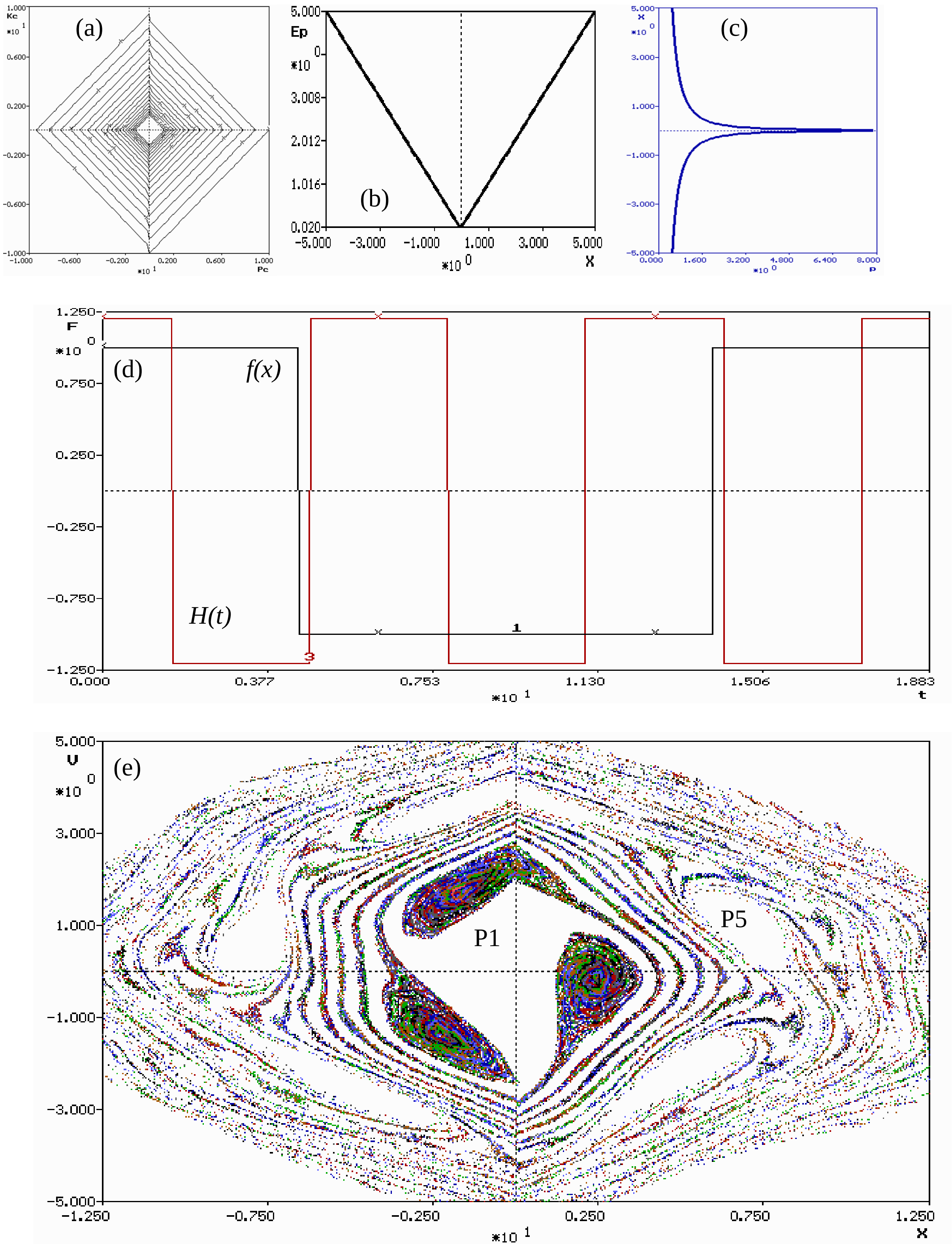

Figure 2: Exact bilinear driven system $\ddot{x} + f\mathrm{sign}(x) = h\mathrm{sign}(\sin(\omega t))$ if $x = 0$, $v^+ = Rv^-$. (a) free oscillation in energy coordinates, (b) – potential well, (c) – backbone curve, (d) – rectangle restoring and rectangle external forces, (e) – basins of attraction period – 1, period – 3, period – 5. Comparative analysis of usual and “exact” models are shown in Figure 3.

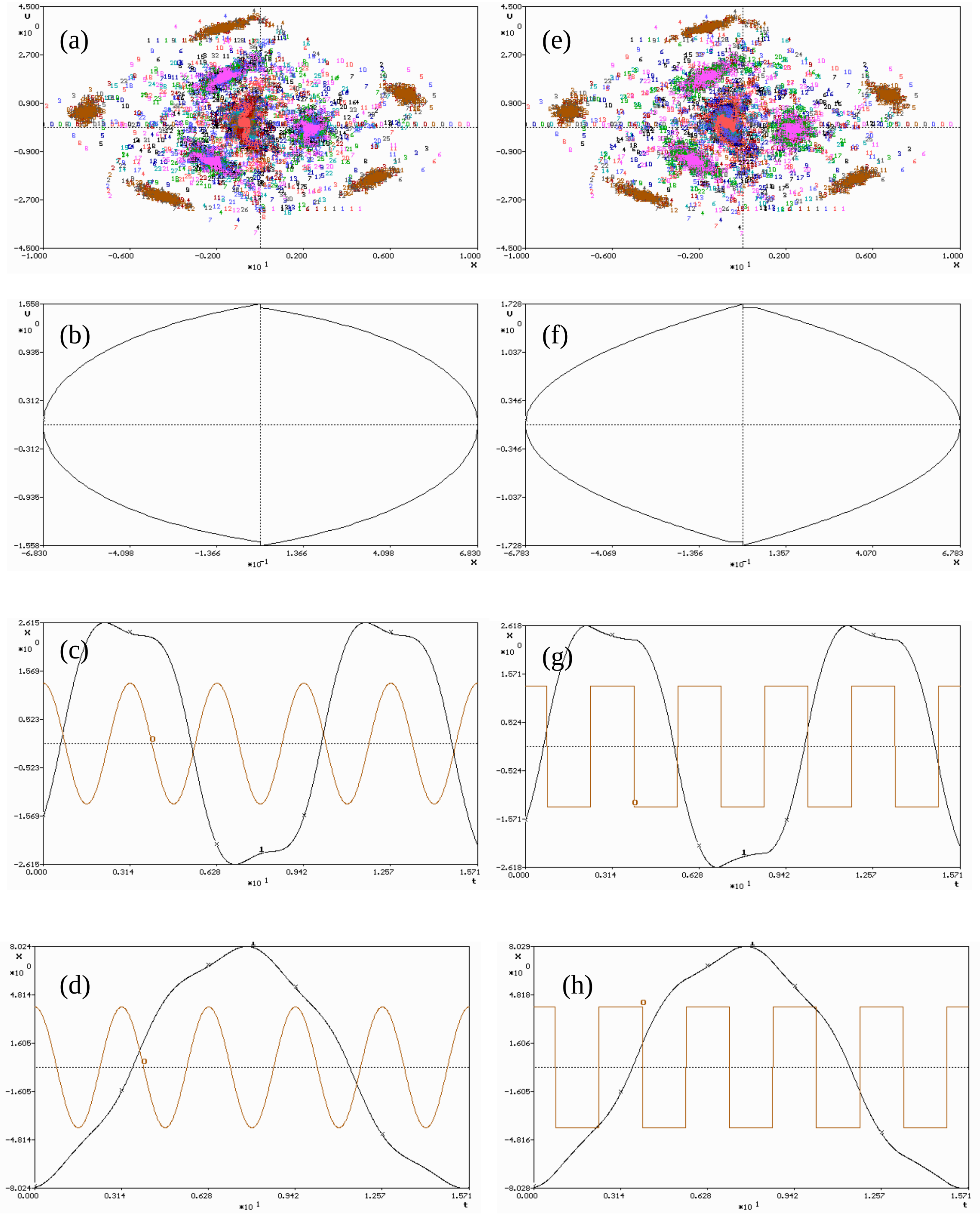

Figure 3: Comparative analysis of usual and "exact" models. $\ddot{x} + f\,\mathrm{sign}(x) = h\,\mathrm{sign}(\sin(\omega t))$ if $x = 0$, $v^+ = Rv^-$. For the usual models (left column) we have harmonic exaltation *H(ωt) = hcos(ωt), h = 1.5*. For exact models (right column) we change harmonic excitation to rectangle excitation with *h = 1.2*. Other parameters: $\omega = 1$, coefficient of restitution R = 0.97, f = 1. The figure demonstrates isomorphism of two dynamical systems: usual and "exact" one.

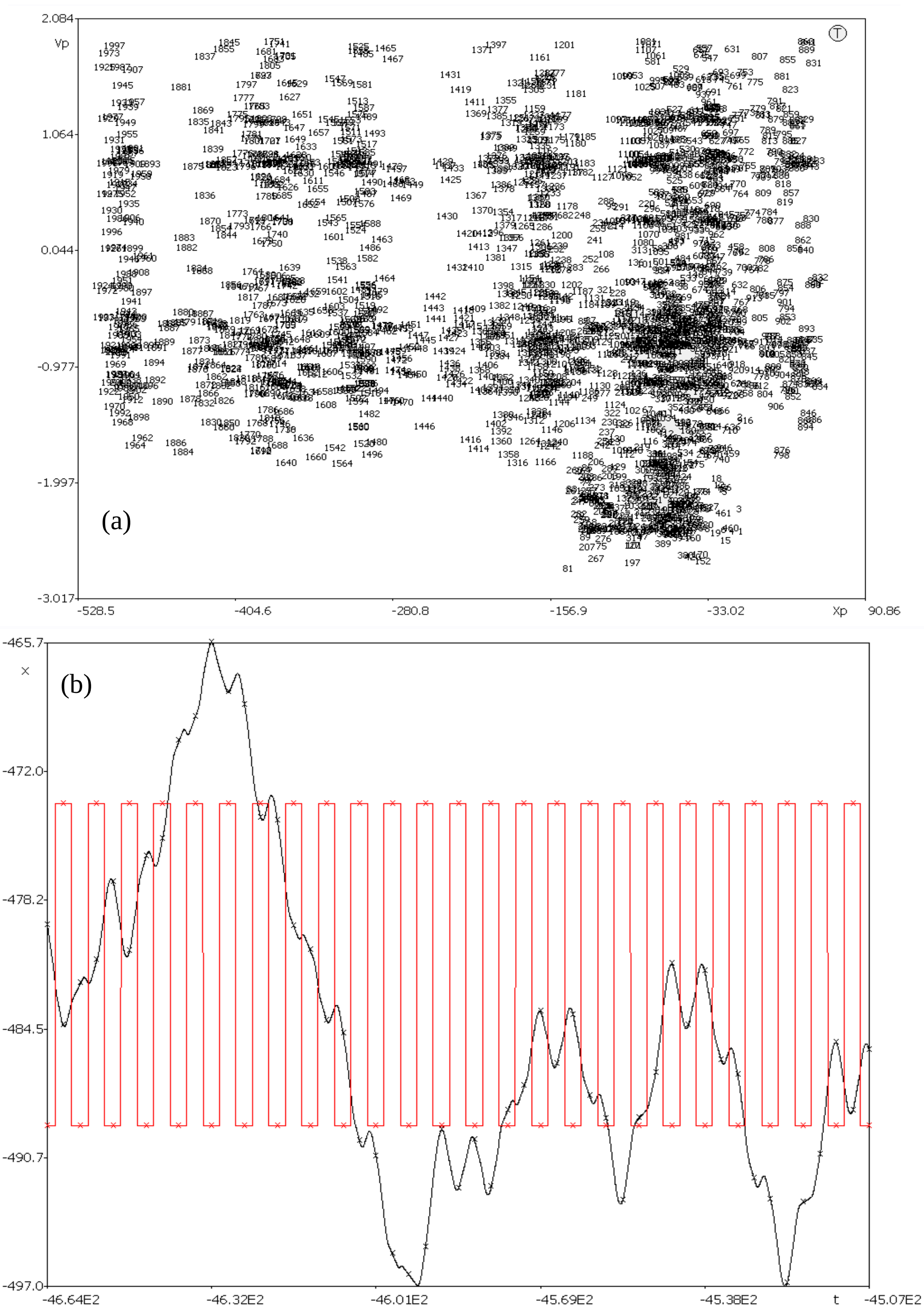

Figure 4: Forced chaotic rotational oscillations of the exact pendulum system with rectangle excitation. Parameters: f1 = 1.0, L = 1.0, R = 0.9; h = 1.0, w = 1.0,
a) Pouncaré mapping NT = (0 – 2000)T, b) time history for , NT = (1950-2000)T x0 = 0.5, v0 = 0. This example shows the possibility of quick and exact investigation of chaotic motion in pendulum systems.

2 CONCLUSIONS

- The new approach, which we name "Exact Nonlinear Dynamics" (END), is proposed for the global qualitative analysis of strongly nonlinear dynamical systems. For this aim we change usual models into special piece-wise linear models which allow obtaining exact results using only quadratic algebraical equations.
- Changing usual models by 'exact' models is possible due to the isomorphism of qualitative behavior of dynamical systems.
- The birth, life and the death of subharmonic orbits of different types in parameter and state spaces may be illustrated by exact models e.g. by bilinear model.
- The approach allows doing exact analytical stability evaluation and complete bifurcation analysis using novelty bifurcation theory [9-10].
- We spouse that the birth, life and the death of chaotic attractors of different types in parameter and state spaces may be well done by the exact nonlinear dynamics.
- The approach allows to obtain qualitative results with high exactness and high speed.
- We suppose that the END-approach allows to find new unknown phenomena and rare attractors in such complex dynamical systems as many links pendulum, Toda-like chains, nano- and micro-dynamical structures and chemical systems.

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