TEST-RIG FOR IDENTIFICATION AND CONTROL APPLICATIONS

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Abstract. In this paper a test-rig for identification and control applications is presented. The experimental apparatus is an autonomous Unmanned Ground Vehicle (UGV). The experimental apparatus is composed of of three parts: structure, actuators and sensors. The structure is a frame made of methyl methacrylate on which are mounted the aluminium supports for the wheel-motors group, drives and the microcontroller. The actuators are DC motors, provided with gearboxes, governed by two drives, respectively. Ultrasonic sensors and accelerometers are used to feedback signal for control. Open-loop and closed-loop algorithms were implemented on Arduino microcontroller. Arduino was created by Olivetti in 2005 and has gained a worldwide success. It is suitable for many uses. The development environment (IDE) is called Sketch and is written in Java. The programming language is based on C and C++. Arduino, once programmed, can operate autonomously, powered by a battery or connected to PC by USB. It is suitable for many applications, such as robotics, design, art, advanced programming. The N4SID (Numerical algorithms for Subspace State Space System Identification) method was used to obtain a dynamical model of the UGV from experimental data. The obtained results show the effectiveness of the test-rig for control and identification of mechanical systems.
1 INTRODUCTION

Unmanned ground vehicles (UGVs) are, nowadays, widely used in many fields. Autonomous UGV, uses its sensors to localize its position in the environment. Such information will be processed by the onboard controller, and, on the basis of the mission, it will properly manage its actuators [1, 2].

In this paper an autonomous UGV is presented [3]. This system proposed as a test-rig for identification procedures and control laws [4, 5]. Afterwards, it is described an experimental activity that highlights the potential of such a system. In particular, it is described the implementation, on the system, of a control law [6, 7, 8, 9, 10] based on a "state observer" [11, 12, 13, 14]. Finally, the results of this experimental activity are shown.

2 DESCRIPTION OF TEST RIG

The experimental apparatus (see Figure 1) is an autonomous Unmanned Ground Vehicle, UGV. The system is composed of a mobile platform, with a methyl methacrylate chassis. On which can be mounted four fixed driven wheels or two fixed driven wheels and a third unpow- ered castor wheel. The motorized wheels are driven by DC electric gear-motors with digital incremental encoders. Each pair of motors is powered by a drive. In this work it is used the platform configured with three wheels. On the platform has been installed an Arduino Mega2560 microcontroller, receives data and sends commands. The two motorized wheels are controlled by the microcontroller. It is possible to drive the vehicle for carrying out trajectories and also to rotate it on itself. The mobile platform is equipped with various sensors that make the system autonomous. On the vehicle are present two ultrasonic sensors to detect the presence of obstacles on the trajectory, a triple-axis accelerometer, a gyroscope triple-axis and a magnetometer triple-axis for a total of 11 transducers.

The outputs signals were acquired in real time by using Matlab software [15].

Figure 1: Unmanned Ground Vehicle
3 MATHEMATICAL BACKGROUND

3.1 State-Space model

The equations of motion for a finite-dimensional linear (or linearised) dynamic system are a set of \( n \) second-order differential equations, where \( n \) is the number of independent coordinates:

\[
M\ddot{x}(t) + R\dot{x}(t) + Kx(t) = F(t)
\]

where \( M, R \) and \( K \) are the mass, damping and stiffness matrices, respectively, \( \ddot{x}(t), \dot{x}(t) \) and \( x(t) \) are vectors of generalized acceleration, velocity and displacement, respectively, and \( F(t) \) is the forcing function.

On the other hand, if the response of the dynamic system is measured by the \( m \) output quantities in the output vector \( y(t) \), then the output equations can be written in a matrix form as follows:

\[
y(t) = C_a\ddot{x}(t) + C_v\dot{x}(t) + C_d x(t)
\]

where \( C_a, C_v \) and \( C_d \) are output influence matrices for acceleration, velocity and displacement, respectively. These output influences matrices describe the relation between the vectors \( \ddot{x}(t), \dot{x}(t), x(t) \) and the measurement vector \( y(t) \).

Let \( z(t) \) be the state vector of the system:

\[
z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}
\]

If the excitations of the dynamic system is measured by the \( r \) input quantities in the input vector \( u(t) \), the equations of motions and the set of output equations can both be respectively rewritten in terms of the state vector as follows:

\[
\dot{z}(t) = A_c z(t) + B_c u(t), \quad y(t) = C z(t) + D u(t)
\]

where \( A_c \) is the state matrix, \( B_c \) is the state influence matrix, \( C \) is the measurement influence matrix and \( D \) is the direct transmission matrix. These matrix can be computed in this way:

\[
A_c = \begin{bmatrix} 0 \\ -M^{-1}K \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ M^{-1}B_2 \end{bmatrix} \quad (5)
\]

\[
C = \begin{bmatrix} C_a - C_a M^{-1}K & C_v - C_a M^{-1}R \end{bmatrix}, \quad D = C_a M^{-1} B_2 \quad (6)
\]

where \( B_2 \) is an influence matrix characterizing the locations and the type of inputs according to this equation:

\[
F(t) = B_2 u(t)
\]

In the case, very common, in which the measured output is just a linear combination of the state, the output equation becomes:

\[
y(t) = C z(t)
\]

The equations (4) and (8) constitute a continuous-time state-space model of a dynamical system. On the other hand, consider a discrete-time state-space model of a dynamical system:

\[
z(k + 1) = A z(k) + B u(k), \quad y(k) = C z(k) \quad (9)
\]
Because experimental data are discrete in nature, the equations (9) form the basis for the system identification of linear, time-invariant, dynamical systems. The state matrix $A$ and the influence matrix $B$ of the discrete-time model can be computed from the analogous matrices $A_c$, $B_c$ of the continuous-time model sampling the system at equally spaced intervals of time:

$$A = e^{A_c \Delta t}, \quad B = \int_0^{\Delta t} e^{A_c \tau} d\tau B_c$$

where $\Delta t$ constant interval.

### 3.2 LQ Optimal Regulation

In optimal control one attempts to find a controller that provides the best possible performance with respect to some given index of performance [16][17]. E.g., the controller that uses the least amount of control-signal energy to take the output to zero. In this case the index of performance would be the control-signal energy.

When the mathematical model of the system to be controlled is linear and the functions that appear in the index of performance are quadratic forms, we have a problem of LQ optimal control.

Considering the linear model stationary, stabilizable and detectable:

$$z(k + 1) = Az(k) + Bu(k), \quad y(k) = Cz(k)$$

the problem of regulation (LQ) in infinite time is to determine the optimal feedback control law which minimizes the performance index:

$$J_{LQ} := \sum_{k=1}^{\infty} \left[ y^T(k)Qy(k) + u^T(k)Ru(k) \right]$$

where $Q = Q^T > 0$ and $R = R^T$ and the term

$$\sum_{k=1}^{\infty} u^T(k)Ru(k)$$

corresponds to the energy of the control signal and the term

$$\sum_{k=1}^{\infty} y^T(k)Qy(k)$$

corresponds to the energy of the controlled output.

The solution is given by the control law:

$$u(k) = -Kz(k), \quad K = R^{-1}B^TS$$

where the symmetric matrix $S$ is the unique positive semidefinite solution of the algebraic Riccati equation (ARE). The matrix $Q$ and $R$ can be chosen by applying the following rule:

$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } y_i^2}$$

$$R_{ii} = \frac{1}{\text{maximum acceptable value of } u_i^2}$$

$Q$ and $R$ are choosing in order to penalize more the non-zero position.
3.3 State-Space Observer Model

As seen in section 3.2 the optimal solution of a regulation problem (LQ), consists in the feedback of the state variables of the model of the system to be controlled. Often, however, the state variables of the system to be controlled are not directly measurable, but we have available to measure of a limited number of their linear combinations, for this reason, it becomes essential in the design of a control system, the use of a device estimation of the state, said observer [18].

These devices allow to obtain an estimate of the state variables from the knowledge of past inputs and measures available of the system [19].

The observer of the state must have in order to be acceptable, two properties fundamental. If we denote by \( z(k) \) the true state, with \( \hat{z}(k) \) the predicted state and \( e(k) := z(k) - \hat{z}(k) \) the estimates error, we can say that an estimate must be asymptotic in the sense that the estimation error should converge to zero for \( k \to \infty \) and with arbitrary dynamic in the sense that the speed of convergence must can be arbitrarily chosen by the designer.

We consider the following linear system, stationary and fully observable:

\[
\begin{align*}
    z(k+1) &= Az(k) + Bu(k), \\
    y(k) &= Cz(k), \\
    z(1) &= z_0
\end{align*}
\] (18)

Suppose that the system is of order \( n \) and the output vector is of dimension \( m \) with \( m < n \), and that the initial state \( z_0 \) is not known. Our intention is to build a state estimator for the system (18) of the form:

\[
\begin{align*}
    \hat{z}(k+1) &= A\hat{z}(k) + Bu(k) + G[y(k) - C\hat{z}(k)], \\
    \hat{z}(1) &= \hat{z}_0
\end{align*}
\] (19)

i.e. using the error on the estimate of the extent to modify the behaviour of the observer. The above equation can also be written as:

\[
\begin{align*}
    \hat{z}(k+1) &= (A - GC)\hat{z}(k) + Bu(k) + Gy(k), \\
    \hat{z}(1) &= \hat{z}_0
\end{align*}
\] (20)

Note that the estimate of the initial state must be chosen on the basis of any information that you have available on the system and in the absence of these, the obvious choice is \( \hat{z}_0 = 0 \).

From equations (18) and (19) it is possible to derive the following model of the estimation error:

\[
\begin{align*}
    e(k+1) &= (A - GC)e(k), \\
    e(1) &= z_0 - \hat{z}_0
\end{align*}
\] (21)

From which it follows that, in order to make effective the estimate, it must choose the eigenvalues of \( A - GC \) with negative real part sufficiently lower than the smallest real part of the eigenvalues of the matrix \( A \). Since the system (18) is fully observable is always possible to determine the matrix \( K \) in such a way that the matrix \( A - GC \) has arbitrary eigenvalues. The matrix \( G \) is called the gain matrix of the observer. The speed with which it extinguishes the error depends on the dynamics of the observer and thus is greater as much as is great (in form) the real part (negative) of the eigenvalues of the matrix \( A - GC \). Make the observer more ready also means increasing the bandwidth and thus make the estimate more sensitive to noise inevitably present on the measures available.

4 IDENTIFICATION AND CONTROL OF THE UGV

The multibody model of the UGV has been used to calculate the feedforward control [20, 21, 22]. Then, it has been developed an experimental activity to verify the theoretical feedforward
law. It has been observed that the feedforward control is sensible to noise. To make robust
the control law it has been developed an identification procedure for obtaining the state matrix
and the state observer (see Eq. [19]). This procedure is based on N4SID method [25]. On
the basis of identified system it has been developed an LQ regulator that has been added to
feedforward control in order to make more effective the control system [26].

4.1 Identification

In this paper it has been used a N4SID method for the identification system.
N4SID numerical method allows to obtain the state matrix and the state observer by input-
output data [27, 28].
The state space matrices are not calculated in their canonical forms (with a minimal number
of parameters), but as full state space matrices in a certain, almost optimally conditioned basis
(this basis is uniquely determined, so that there is no problem of identifiability). This implies
that the observability (or controllability) indices do not have to be known in advance.
Input and output data, indicated in figure 2, formed by voltage applied to the motors and angles
of the wheels, fed to the identification algorithm N4SID, in order to calculate the matrices
$A, B, C, D$ and the observer matrix $G$, Eqs. (18) and (19). These matrix are indicated in Eqs.
(22-25). These matrices have been obtained assuming the sampling period of the input and
output equal to $T_s = 0.01$ s.

$$A = \begin{bmatrix}
1 & -4.740e-05 & 2.665e-06 & -3.708e-05 \\
-0.0005266 & 1.001 & -0.01689 & 0.0006557 \\
0.001857 & 0.01783 & 0.9946 & -0.01416 \\
0.001423 & 0.003359 & 0.008817 & 0.9834 \\
\end{bmatrix}$$ (22)
\[ B = \begin{bmatrix} -1e - 07 & -4.574e - 08 \\ 1.43e - 05 & -5.01e - 05 \\ 1.4e - 05 & -2.524e - 05 \\ 0.0001159 & -4.996e - 05 \end{bmatrix} \] (23)

\[ C = \begin{bmatrix} 1.263e + 05 & -461.7 & 4.65 & -2.088 \\ 9.051e + 04 & 663.9 & -3.721 & -0.5209 \end{bmatrix} \] (24)

\[ G = \begin{bmatrix} 2.215e - 06 & 1.565e - 06 \\ -3.385e - 04 & 4.117e - 04 \\ 0.0049 & -0.0067 \\ -0.0108 & -0.0037 \end{bmatrix} \] (25)

### 4.2 LQR control

To the feedforward control is added a feedback compensator that is proportional to the deviation \( \delta \hat{z}(k) = \hat{z}(k) - \bar{z}(k) \):

\[ \delta u(k) = -K \delta \hat{z}(k), \] (26)

where \( \hat{z} \) is estimated state and \( \bar{z} \) is imposed state, respectively.

The equation for estimating the deviation is like to the Eq. (19):

\[ \delta \hat{z}(k + 1) = A \delta \hat{z}(k) + B \delta u(k) + G (\delta y(k) - \delta \hat{y}(k)) \] (27)

with

\[ \delta \hat{y}(k) = C \delta \hat{z}(k) \] (28)

in which:

\[ \delta \hat{y}(k) = \hat{y} - \bar{y} \quad \text{and} \quad \delta y(k) = y - \bar{y} \] (29)

Where \( A, B, C \) and \( G \) are (22), (23), (24), and (25) and where \( \hat{y} \) is estimate output, \( \bar{y} \) is imposed output, \( ad \ y \) is the actual output.

The control gain matrix \( K \) has been calculated by applying LQR optimal control method in which the penalization matrices \( Q \) and \( R \) (see Eq. (3.2)) are the following:

\[ Q = \begin{bmatrix} \frac{1}{100^2} & 0 \\ 0 & \frac{1}{100^2} \end{bmatrix}, \quad R = \begin{bmatrix} \frac{1}{100^2} & 0 \\ 0 & \frac{1}{100^2} \end{bmatrix} \] (30)

In this way, it’s been obtained the following matrix \( K \):

\[ K = \begin{bmatrix} -1.5119e + 05 & 444.8 & -81,875 & 136.34 \\ 1.2608e + 04 & -650.39 & 87.417 & -9.2388 \end{bmatrix} \] (31)

### 5 RESULTS AND DISCUSSION

In figure 3 are indicated three UGV trajectories. By the black line, it is indicated the target trajectory. By the blue line it is indicated the UGV trajectory obtained by means a feed-forward control law. By the red line, the UGV trajectory by applying to the motors the feedforward and the feedback control law calculated by means an identification procedure.

In figure 4 the motor inputs are reported for FFC law and for FFC + FBC law, respectively. In summary:
Figure 3: Target, FFC, FFC + FBC trajectory of the UGV

Figure 4: Motor Inputs
1. The feedforward control law, obtained by multibody model, is not effective for imposing the target trajectory. This is due to noise and parameter uncertainty.

2. In order to overcome this drawback, it has been identified a model system, via N4SID, and designed an LQ regulator by means to drive UGV on the target trajectory.

3. The results have been very good.

REFERENCES


