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ON THE NONLINEAR DYNAMICS OF A THERMOMECHANICAL SHAPE MEMORY ALLOY OSCILLATOR EXCITED BY IDEAL OR NON-IDEAL ENERGY SOURCES

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Abstract. *When the excitation is not influenced by the response, it is called ideal or ideal energy source. On the other hand, when an excitation is influenced by the system response, it is known as non-ideal. This paper discusses the nonlinear responses of a shape-memory non-ideal oscillator, with an unbalanced motor of excitation with limited power supply. The restoring force provided by the shape-memory device is described by a thermomechanical model capable to describe hysteretic behavior via the evolution of a suitable internal variable. Computer simulations are carried out via a numerical approach showing qualitative results concerned with regular and non-regular motions, and comparing with those obtained for the reference ideal system.*

1 INTRODUCTION

Shape memory alloy (SMA) are a group of metallic materials that exhibit special properties like thermal shape memory effect, pseudoelasticity and high damping capacities. Due to these properties, the nonlinear dynamics of oscillators with restoring force provided by a shape memory material (SMM) and subject to harmonic mechanical excitation has been investigated during the recent years [1-2]. In these studies, the SMA has been described using different constitutive models for the restoring force, for example, polynomial non-hysteretic models and various types of thermomechanical internal variables hysteretic models.

The study of problems that involve the coupling of several systems, was explored widely in the last years, in function of the change of constructive characteristics of the machines and structures. In this way, some phenomena are observed in dynamical systems composed by supporting structures and rotating machines, where the unbalancing of the rotating parts is the main cause of vibrations. In the study of these systems, for a more realistic formulation one has to consider the action of an energy source with limited power (non-ideal), that is, to consider the influence of the oscillatory system on the driving force and vice versa. Recently a number of works has been done, in order to investigate the resonant conditions of non-ideal vibrating oscillator systems [3], and a number of non-ideal vibrating systems has been studied, for some examples, see [4-6], besides others.

In this work the attention is focused on the nonlinear dynamics of an oscillator with a Shape Memory Device with pseudoelastic behavior, subject to different external force, generated by ideal and non-ideal energy sources. The pseudoelastic restoring force is modeled by a thermomechanical constitutive law proposed in [7] for the SMM and then adapted in [1] for use in nonlinear dynamics.

2 DYNAMICAL MODELLING OF THE VIBRATING SYSTEM

2.1 Ideal Excitation (Harmonic excitation)

A Shape Memory Oscillator (SMO) is characterized by a Shape Memory Device (SMD) that provides a restoring force against the relative displacements of a pair of points of a main structure [8]. A SMD is composed by an arrangement of Shape Memory Materials (SMM) that may be designed to yield various kinds of behavior. In this work the attention is focused on SMDs with pseudoelastic behavior (Fig. 1).

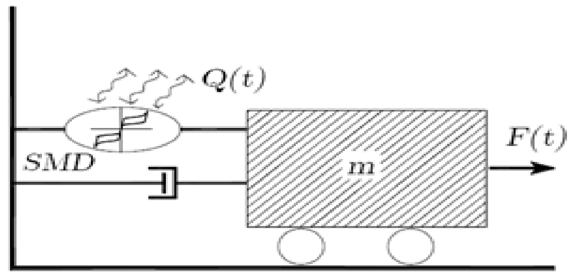


Figure 1: Schematic representation of a SMD model

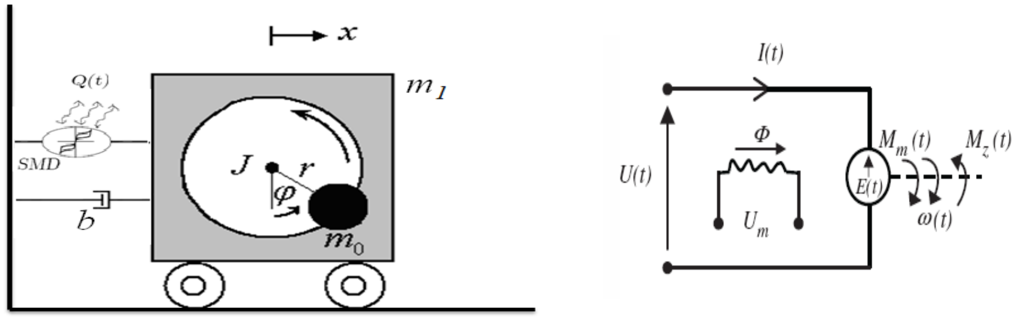
The SMOs are considered within a thermomechanical environment characterized by an harmonic forcing $F(t)=\gamma\cos(\alpha t)$ and a convective heat exchange $Q(\vartheta)=h(\vartheta_e-\vartheta)$ where γ and α are the excitation amplitude and frequency, ϑ_e the environment temperature and h the coefficient of convective heat exchange [1, 8]. At each time t the state of the SMO is described by displacement $x(t)$, velocity $v(t)$, an internal variable $\xi(t)\in[0,1]$ that models the internal state of the SMD and by the temperature $\vartheta(t)$. To model the complex hysteretic behavior of SMM the state depends not only on the actual value $\xi(t)$ but also on the value $\xi_0(t)$ of ξ at the beginning of the last process of change of ξ occurred before time *t*. As discussed in [1] the time evolution of the state takes place according to a system of 5 differential adimensional equations:

$$
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= -x+(\operatorname{sgn}(x)\lambda)\xi-\zeta v+F \\
\dot{\xi} &= Z\left[\operatorname{sgn}(x)v-JQ\right] \\
\dot{\vartheta} &= ZL\left[\frac{\Lambda}{J\lambda}+\vartheta\right]\left[\operatorname{sgn}(x)v-JQ\right]+Q
\end{aligned}
\tag{1}
$$

in which Z and Λ are constitutive functions that can take different expressions depending on suitable state-dependent criteria [1, 8]. The system reponse depends on 7 model parameters, J, λ, q_1, q_2, q_3, L, h whose physical meaning is discussed later (see also [8]) as well as on the damping coefficient ζ of the viscous damper (Fig. 1).

2.2 Non-ideal Excitation

Let us consider a vibrating system, which includes a direct current (DC) motor with limited power supply, operating on a structure (Figure 2). The excitation of the system is limited by a characteristic of the energy source (non-ideal energy source). Then, the coupling of the vibrating oscillator and the DC motor takes place. As the vibration of the mechanical system depends on the DC motor, also the motion of energy source depends on vibrations of the system. Hence, it is important to analyze what happens to the motor, as it influences the response of the system.



(a) (b)

Figure 2: (a) Mechanical model of the vibrating system and (b) the electrical schematic representation of the DC motor.

The vibrating system under consideration consists of a main structure of mass m_1, constrained by a linearly viscous element with damping coefficient b and a pseudoelastic SMD. The structure is also connected with a non-ideal DC motor, with driving rotor of moment of inertia J_M and an unbalanced mass m_0 with eccentricity *r*.

The electrical scheme of the DC motor representation is shown in Fig. 2(b). The corresponding equations that govern the motion of the DC motor are typically written in the form [9];

$$J_M \frac{d^2\varphi}{dt^2} = M_m(t) - M_z(t) - H(t) \tag{2}$$

$$U(t) = R_t I(t) + L_t \frac{dI(t)}{dt} + E(t) \tag{3}$$

where the time functions $U(t)$ and $I(t)$ are the voltage and the current in the armature, while R_t and L_t are resistance and inductance of the armature, $E(t)$ is the internally generated voltage, $M_z(t)$ is an external torque applied to the motor drive shaft, $H(t)$ is a frictional torque and $M_m(t)$ denotes the torque generated by the motor. The torque $M_m(t)$ and internal generated voltage $\mathrm{E}(\mathrm{t})$ can be expressed as:

$$M_m(t) = c_M \Phi I(t) \tag{4}$$

$$E(t) = c_E \Phi \omega(t) \tag{5}$$

where c_M, c_E are mechanical and electrical constants and Φ is the magnetic flux. Let us assume that external exciting current I_m and voltage U_m are constant and then the magnetic flux Φ is also constant in the considered model. Taking into account Eqs. (2) - (5), we can write the differential equations of the complete electro-mechanical system presented in Fig. 2, as follows:

$$Mx''+bx'+f-m_0r\left(\varphi'^2\sin\varphi-\varphi''\cos\varphi\right)=0 \quad (6)$$

$$\left(J_M+m_0r^2\right)\varphi''=c_M\Phi\tilde{I}(t)-\tilde{H}\left(\varphi'\right)-m_0rx''\cos\varphi \quad (7)$$

$$\frac{d\tilde{I}(t)}{dt}=-\frac{R_t}{L_t}\tilde{I}(t)-\frac{c_E\Phi}{L_t}\varphi'+\frac{\tilde{U}(t)}{L_t} \quad (8)$$

where a prime denotes a derivative with respect to dimensional time, $M=m_1+m_0$, and f is the SMD restoring force. It is convenient to work with dimensionless position and time, in such a way that Equations 6 - 8 are rewritten in the following form:

$$\ddot{u}+\zeta\dot{u}+u-\left(\operatorname{sgn}(u)\lambda\right)\xi-w_1\left(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi\right)=0 \quad (9)$$

$$\ddot{\varphi}=p_3I(\tau)-w_2\ddot{u}\cos\varphi-H\left(\varphi\right) \quad (10)$$

$$\dot{I}=U(\tau)-p_1I(\tau)-p_2\dot{\varphi} \quad (11)$$

$$\dot{\xi}=Z\left[\operatorname{sgn}(u)\dot{u}-JQ\right] \quad (12)$$

$$\dot{\vartheta}=ZL\left[\frac{\Lambda}{J\lambda}+\vartheta\right]\left[\operatorname{sgn}(u)\dot{u}-JQ\right]+Q \quad (13)$$

where

$$\omega_0^2=\frac{k}{M},\quad \mu=\frac{b}{2M\omega_0},\quad I=\frac{\tilde{I}}{I_r},\quad w_1=\frac{m_0r}{Mx_{st}},\quad w_2=\frac{m_0rx_{st}}{(J_M+m_0r^2)},\quad p_1=\frac{R_t}{L_t\omega_0},\quad p_2=\frac{c_E\Phi}{L_tI_r},$$

$$p_3=\frac{c_M\Phi I_r}{\left(J_M+m_0r^2\right)\omega_0^2},\quad U(\tau)=\frac{\tilde{U}(\tau)}{L_tI_r\omega_0},\quad H\left(\varphi\right)=\frac{\tilde{H}\left(\varphi'\right)}{\left(J_M+m_0r^2\right)\omega_0},\quad \tau=\omega_0t,\quad u=\frac{x}{x_{st}}$$

and x_{st} means a static displacement of the system, τ is the dimensionless time and I_r is a rated current in the armature and dots indicating differentiations with respect to dimensionless time, the function $H\left(\varphi\right)$ is the resistive torque applied to the motor which will be neglected henceforth.

3 NUMERICAL SIMULATIONS RESULTS

The evolution of a SMO depends on seven parameters , which may be grouped as follows: mechanical parameters: λ, q_1, q_2, q_3, that reflect the basic features of the device (type and arrangement of the material) and determine the basic shape of the pseudoelastic loop observed in isothermal conditions; thermal parameters: L, h that reflect the heat production, absorption and exchange with the environment and therefore determine the temperature variations of the device; a thermomechanical parameter J that determines the influence of the temperature variations on the transformation forces [1].

The numerical simulation of the vibrating system was carried out for parameters of the DC motor and mechanical parameters, which correspond to the values taken from [8].

Numerical dimensionless parameters are shown in table 1:

λ	q_1	q_2	q_3	L	h	J	w_1	w_2	p_1	p_2	p_3
8.125	0.98	1.2	1.017	0.12	0.08	3.174	0.2	0.3	0.3	3	0.15

Table 1: Systems Parameter used in simulation.

In non-ideal mechanical systems the oscillator cannot be driven by systems, whose amplitude and frequency are arbitrarily chosen, once the forcing source has a limited available energy supply. For this kind of oscillator, the excitation cannot be considered as given a priori, but it must be taken as a consequence of the dynamics of the whole system (oscillator and motor). Therefore, a non-ideal oscillator is, in fact, the combined dynamical system resulting from the coupling of a passive and an active oscillators, the latter serving as the driving source for the former. The resulting motion will be thus the outcome of the dynamics of the combined system. The dimensionless voltage applied across the armature U is the control parameter in non-ideal system. For each value of U, the non-ideal system presents one frequency and amplitude behavior. For example, the displacement time history and the angular velocity of the complete electro-mechanical model described by the full system of differential equations (9)-(13) for $U = 1$ are presented in figure 3. In this case, the motor frequency and amplitude response in the stationary state are $\varphi = 0.33$ and 0.0244, respectively. For this reason, the comparison with ideal system is complicated, because in this system, the amplitude and frequency do not change in time. However, in this work frequency input in the associated ideal system will be estimated as an average motor frequency and the corresponding amplitude excitation will be computed as the root mean square (RMS), also known as the quadratic mean, that is a statistical measure of the magnitude of a varying quantity. In order to understand the dynamical behavior of these systems and without loss of generality, here we will normalize the amplitude response in both cases.

Initially, in order to investigate the nonlinear dynamics of the non-ideal oscillator, we present the bifurcation diagram in Figure 4 where the control parameter is the dimensionless voltage applied across the armature U. As it will be shown, a large variety of periodic, quasi-periodic and chaotic responses is observed.

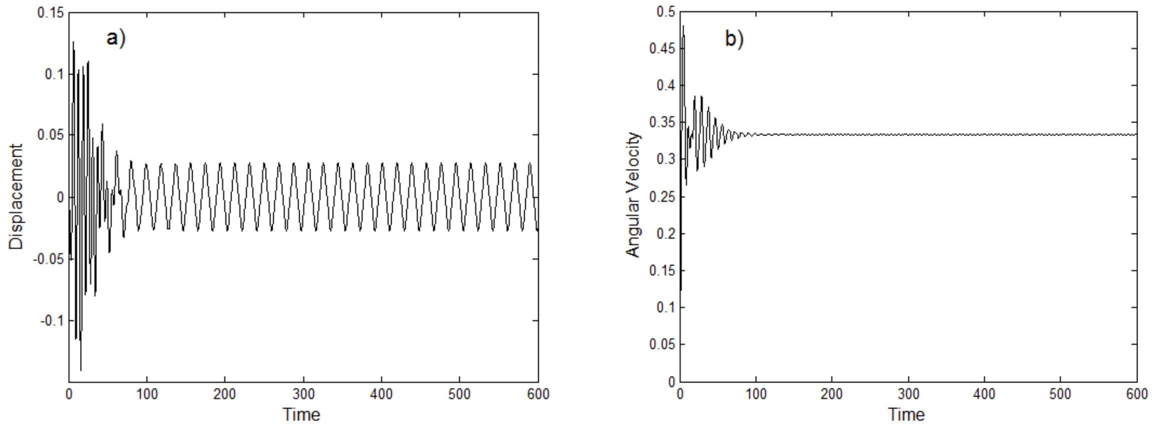


Figure 3: (a) Time history and (b) angular velocity for $U = 1$

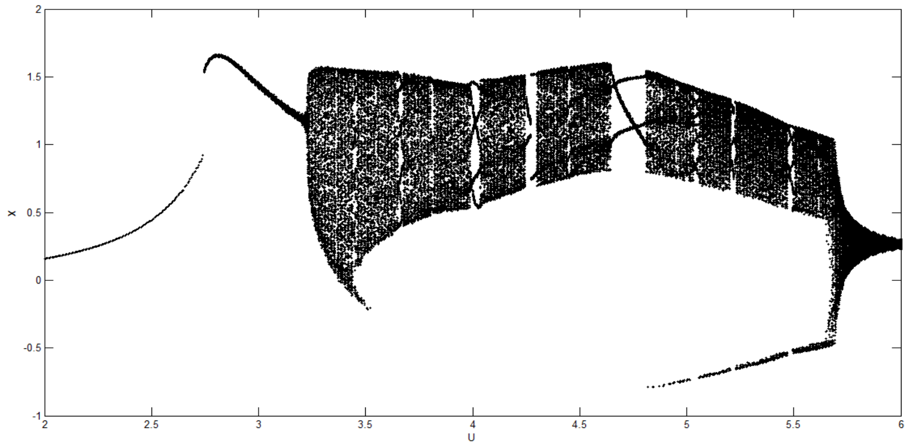


Figure 4: Bifurcation diagram showing displacement x as a function of U .

Figure 5 shows the dynamical response for $U = 3$, and for this value the motor frequency and amplitude excitation both increase. The motor frequency in the stationary state is $\varphi = 0.91$ and we will use this value as ideal frequency, i.e., $\alpha = 0.91$. The ideal amplitude system obtained by RMS is $\gamma = 1.01$. Note that, for the sake of comparison between the two systems, responses have been normalized to the same maximum amplitude value; as a consequence, the stiffness of the force-displacement relationship, which is the same in the two real responses, appears to be different. The periodic motion is observed in both cases, but now the SMD exhibits stress-induced phase transformations because the vibrations amplitude of the oscillator is bigger than previous one.

Figure 6 shows the dynamical response for $U = 4$. The motor frequency in the stationary state is $\varphi = 1.30$ and we use this value as ideal frequency, i.e., $\alpha = 1.30$. The amplitude of ideal system is obtained by RMS and in this case is $\gamma = 0.69$. Under these conditions, the non-ideal system presents a quasi-periodic behavior while the ideal system presents a regular motion. The SMD, in the ideal case, does not undergoes phase transformations and, therefore, the force and amplitudes are not enough to give rise to an hysteresis loop.

Figure 7 shows the dynamical response for $U = 4.5$. The motor frequency in the stationary state is $\varphi = 1.47$ and we use this value as ideal frequency, i.e., $\alpha = 1.47$. The amplitude of the ideal system is obtained by RMS and turns out to be $\gamma = 0.83$. Under these conditions, the nonideal system presents an irregular behavior while the ideal system presents a regular motion. The SMD, in the ideal case, does not present phase transformations and, therefore, again, there is no hysteresis loop.

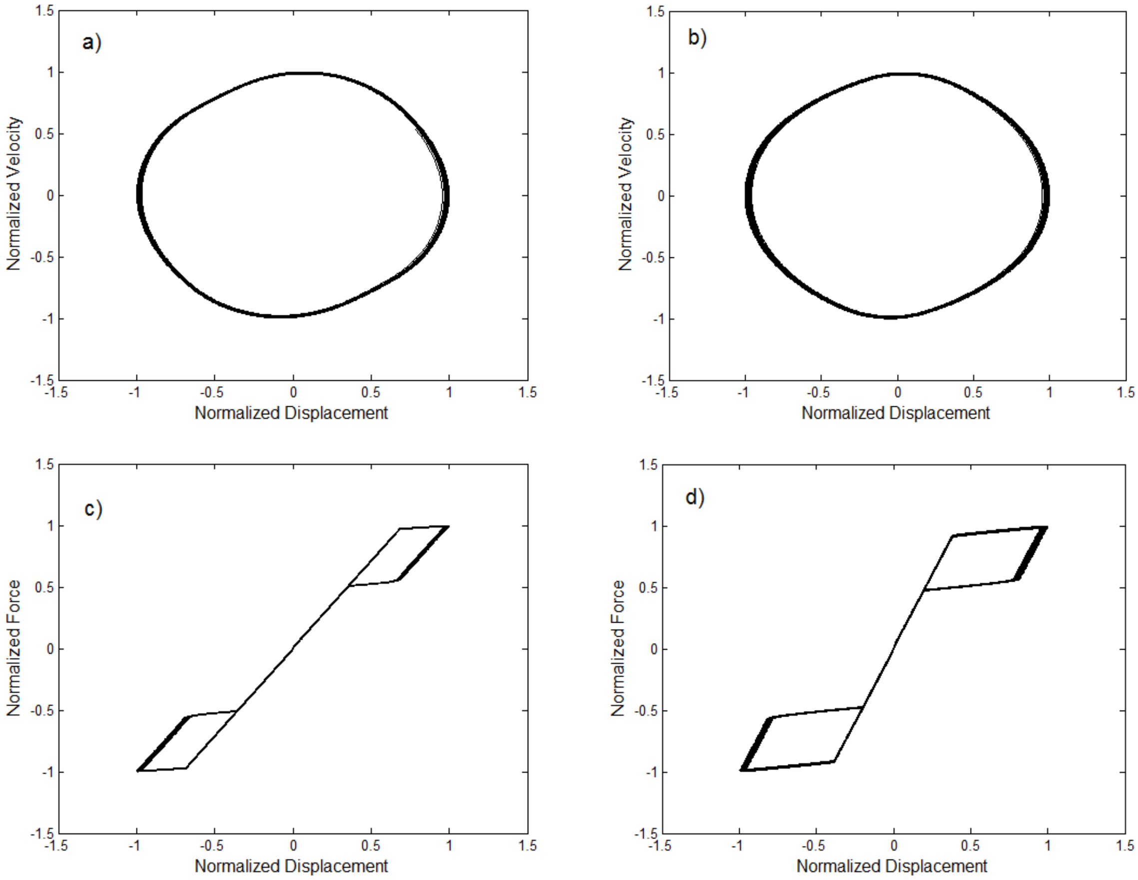
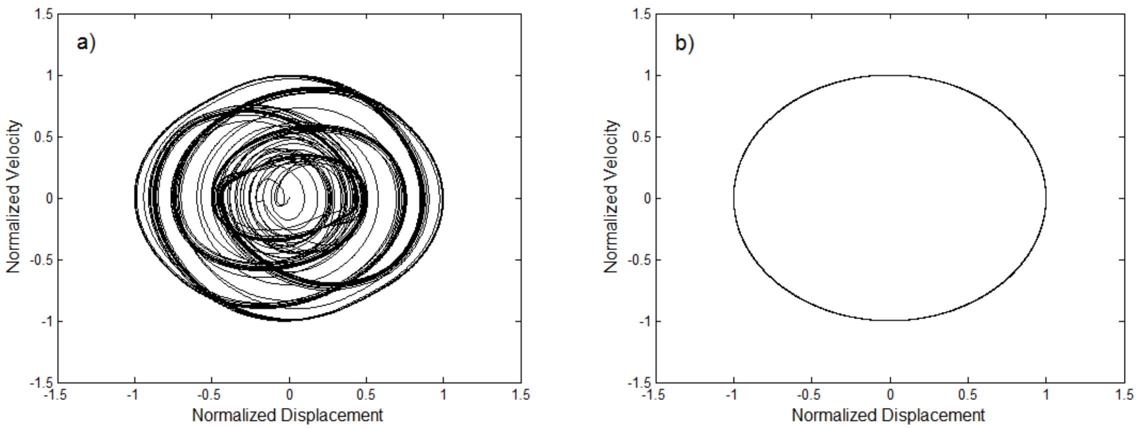


Figure 5: Phase portraits and force-displacement response and for $U = 3$: a) and c) nonideal model, b) and d) ideal model.



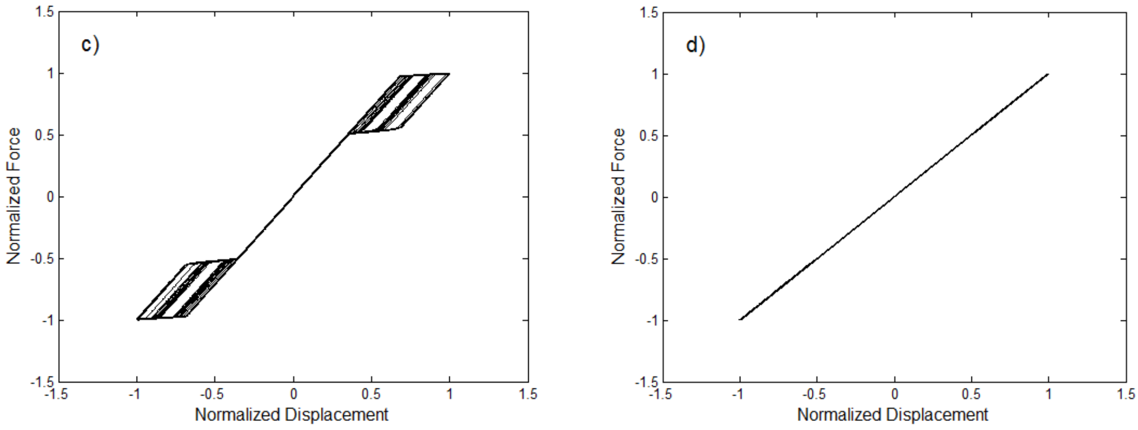


Figure 6: Phase portraits and force-displacement response for $U = 4$: a) and c) nonideal model, b) and d) ideal model.

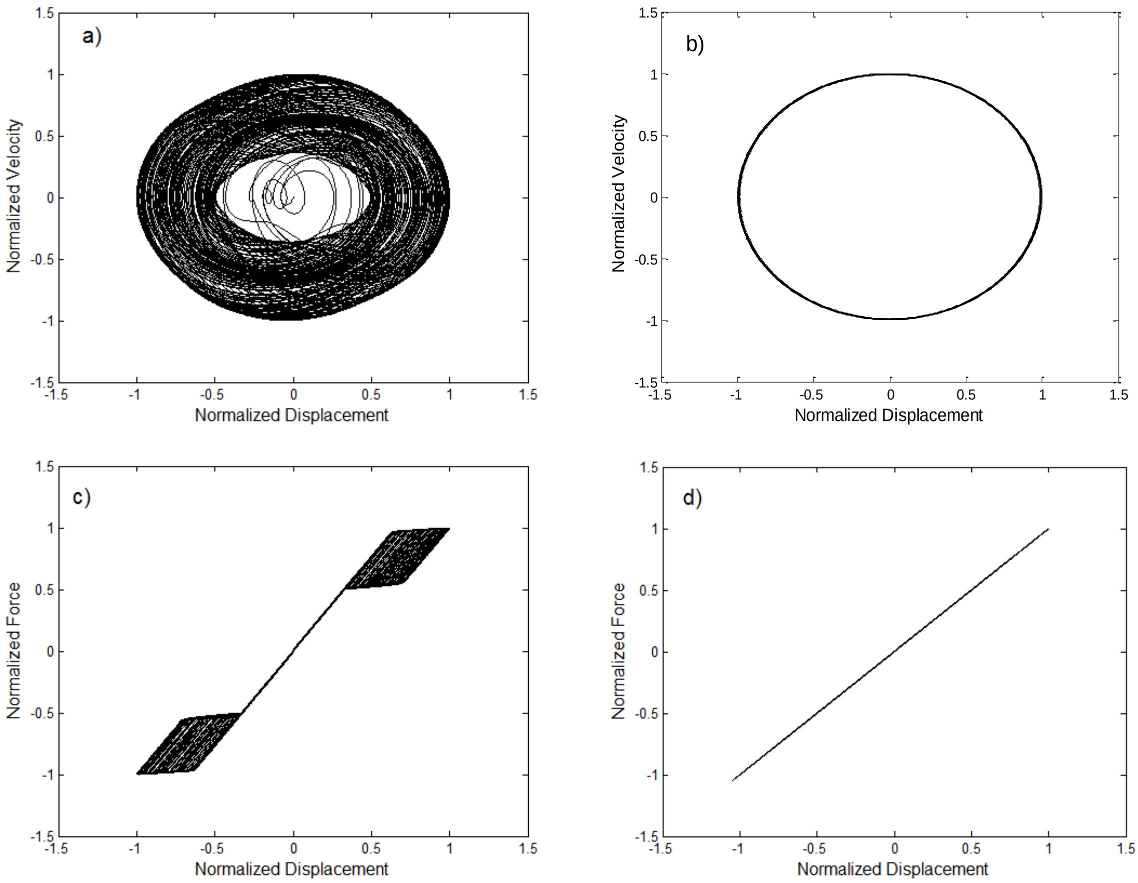


Figure 7: Phase portraits and force-displacement response for $U = 4.5$: a) and c) non-ideal model, b) and d) ideal model.

4 CONCLUSIONS

In this paper the influence of the non-ideal energy sources on the dynamics of a system composed by a pseudoelastic oscillator excited by a DC-motor driving an unbalanced rotating mass has been studied. For this system, the response is influenced by the electrical properties of the DC motor and the power supply.

The analysis carried out in this paper, emphasizes the different responses of ideal and non-ideal systems. Due to the richness of the mathematical model for the pseudoelastic force exerted by the SMD, various modifications of the force-displacement loop, can be obtained by different combinations of the parameters.

For a fixed set of model parameters, different phase portraits lead to different loop shapes that are characterized by stress-induced phase transformations. The dynamical response under varying frequency has been investigated. The analysis of non-ideal systems has shown that a significant vibration behavior, either the regular or non-regular, can be achieved for changes of the dimensionless voltage applied across the armature. On the other hand, the equivalent ideal system obtained from non-ideal system parameters, shows that the relation between frequency and amplitude, in most cases, does not feature the stress-induced phase transformations.

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