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ON THE EFFECT OF IMPULSIVE PARAMETRIC EXCITATION TO THE MODAL ENERGY CONTENT OF HAMILTONIAN SYSTEMS

T. Pumhoessel*[1], P. Hehenberger[1], K. Zeman[1]

[1]Institute of Mechatronic Design and Production, Johannes Kepler University Linz, Austria
thomas.pumhoessel@jku.at, peter.hehenberger@jku.at, klaus.zeman@jku.at

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Abstract. *Mechanical systems with time varying stiffness parameters have been investigated intensively in the past, where most attention was paid on systems with time-periodic coefficients. Besides the well known parametric resonances, the so-called parametric antiresonances, discovered by A. Tondl, have attracted significant interest in recent years. Parametric stiffness excitation at these frequencies results in a coupling of modes, and initiates an energy transfer between the participating modes, finally leading to a faster decay of vibration amplitudes compared to the case where no parametric excitation is present. In this contribution, parametric stiffness excitation of impulsive type is investigated. Based on the works of C.S. Hsu, the effect of parametric impulses to the modal energy content of mechanical systems is analyzed theoretically. For this purpose, a simple mechanical model consisting of two masses connected with springs, where the coupling stiffness to the inertial frame consists of a constant part and a part of impulsive type, is investigated. It is assumed that no damping is present in the system (Hamiltonian system). After a modal decomposition of the systems equations, it is shown analytically that using parametric stiffness excitation of impulsive type allows to extract energy from the mechanical system leading to a decay of the vibration amplitudes although the system is undamped. Therewith, parametric impulses can be applied in the sense of a control law, finally leading to a sequence of impulses which can then be replaced by a step shaped stiffness function. Furthermore, it is shown that impulsive parametric excitation enables to transfer energy from one mode to another in a targeted manner. One application is the transfer of energy from a weakly damped mode to a mode with higher damping, resulting finally in a faster decay of vibrations. With a simple numerical example, it is demonstrated that with this approach energy can be extracted efficiently from mechanical systems, and moreover, can be transferred in a systematic way from one mode to another.*

1 INTRODUCTION

Engineering systems with time-varying parameters, denoted as *parametrically excited systems* represent a special class of dynamical systems, and there exists abundant literature on this topic, see [1, 2], for example. Especially the phenomenon of parametric resonances of mechanical systems with time-periodic parameters has attracted much interest in the past, as they may lead to a breakdown of a mechanical structure, and therefore, they have to be avoided to guarantee a safe mode of operation. Besides the parametric resonances, the so-called parametric antiresonances, discovered by A. Tondl in 1989, see [3], have attracted significant interest in recent years. He showed that self-excited vibrations can be suppressed if the frequency of time-periodic stiffness parameters is equal or near certain parametric combination resonances, referred to as non-resonant parametric resonances or parametric antiresonances. Comprehensive investigations about this topic can be found in [4, 5], for example. Parametric stiffness excitation at the antiresonance frequency results in a coupling of vibrational modes, see [6]. The underlying physical mechanism was investigated first in [7], where it was shown that excitation at the parametric antiresonance frequency initiates an energy transfer from lower to higher modes as well as in the reverse direction, resulting finally in a faster decay of vibration amplitudes as higher modes usually possess higher damping ratios. In [8, 9] it was demonstrated using model-reduction techniques applied to multi-degree of freedom systems, that the parametric antiresonance-effect results in a concentration of the vibrational energy in only a few participating modes. Extensive investigations on targeted energy transfer can be found in [10], and references therein, where nonlinear elements are used successfully to transfer vibration energy in mechanical systems.
In this contribution, parametric stiffness excitation of impulsive type is investigated. Based on the results presented in [11, 12], the effect of parametric stiffness impulses of Dirac-delta type to the modal energy content of mechanical systems is first analyzed theoretically. It is shown that certain vibrational states allow to extract energy from the mechanical system, leading to a decrease of the vibration amplitudes although the system is undamped (Hamiltonian system). Moreover, it is shown that in undamped systems energy can be shifted in a targeted manner from lower to higher modes. At least, the analytical investigations are verified with a simple two-degree of freedom system, where the Dirac-delta impulses are replaced by rectangular ones to assist a more practical view of the presented findings.

2 ANALYTICAL INVESTIGATIONS

The equations of motion of a mechanical system with n degrees of freedom may have the following form

$$\mathbf{M}\mathbf{x}'' + \mathbf{K}\mathbf{x} + \left\{ \sum_{k=1}^{K} \varepsilon_k \mathbf{G}\delta(\tau - \tau_k) \right\} \mathbf{x} = \mathbf{0}, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ represent the constant symmetric mass and stiffness matrix. Furthermore it is assumed that $\mathbf{M}$ is nonsingular. Parametric stiffness excitation is introduced via a series of K Dirac delta functions $\delta(\tau-\tau_k)$, the symmetric matrix $\mathbf{G}$ and the scalar ε_k which is may different for each impulse. Using $\mathbf{x} = \mathbf{\Phi}\mathbf{y}$, where $\mathbf{\Phi}$ represents the modal matrix comprising the natural modeshapes of the corresponding system without parametric excitation, Eq. (1) can be written in the modal form

$$\mathbf{M}^*\mathbf{y}'' + \mathbf{K}^*\mathbf{y} = -\left\{ \sum_{k=1}^{K} \varepsilon_k \mathbf{G}^*\delta(\tau - \tau_k) \right\} \mathbf{y}, \tag{2}$$

with $\mathbf{M}^* = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathrm{diag}(\mu_1, \mu_2, \ldots \mu_n)$, $\mathbf{K}^* = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} = \mathrm{diag}(\nu_1, \nu_2, \ldots \nu_n)$ and $\mathbf{G}^* = \mathbf{\Phi}^T\mathbf{G}\mathbf{\Phi} = [\gamma_{ij}], \quad i, j = 1 \ldots n$. In the following, the effect of a single Dirac delta impulse, applied at an instant of time τ_k, to the modal energy content is investigated.
Using the Einstein summation, with the free index i and summation over the repeated index j, Eq. (2) yields to

$$\mu_i y_i'' + \nu_i y_i = -\varepsilon_k \delta(\tau - \tau_k) G_{ij}^* y_j. \tag{3}$$

For $\tau \neq \tau_k$, Eq. (3) is of autonomous type, and the total modal energy E_i of mode i is given by

$$E_i(\tau) = T_i(\tau) + U_i(\tau) = \mu_i y_i'^2/2 + \nu_i y_i^2/2, \tag{4}$$

where T_i and U_i represent the modal kinetic and the modal potential energy. Denoting the instant of time just before the application of the Dirac impulse with $\tau = \tau_{k-}$, and just after the impulse with $\tau = \tau_{k+}$, the variation of the total modal energy E_i caused by the Dirac impulse can be written in the form

$$\Delta E_{i,k} = \Delta T_{i,k} + \Delta U_{i,k} = W_{i,k}, \tag{5}$$

where, $\Delta T_{i,k} = T_{i,k+} - T_{i,k-}$ and $\Delta U_{i,k} = U_{i,k+} - U_{i,k-}$ represents the difference of the modal kinetic and the modal potential energy, and $W_{i,k}$ the work of the corresponding impulsive elastic force on the right side of Eq. (3).
According to [11], the modal displacements $\mathbf{y_{k+}}$ and modal velocities $\mathbf{y_{k+}}'$ just after a Dirac impulse are given by

$$\begin{bmatrix} \mathbf{y_{k+}} \\ \mathbf{y_{k+}}' \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\varepsilon_k \mathbf{M}^{*^{-1}} \mathbf{G}^* & \mathbf{I} \end{bmatrix}}_{\mathbf{J}_k} \begin{bmatrix} \mathbf{y_{k-}} \\ \mathbf{y_{k-}}' \end{bmatrix} \tag{6}$$

where $\mathbf{J}_k$ represents the so-called *jump matrix*, and $\mathbf{y_{k-}}$ and $\mathbf{y_{k-}}'$ the modal displacements and velocities just before the impulse. From Eq. (6) it can easily be seen that the modal velocity $\mathbf{y}_{k+}'$ is different from $\mathbf{y}_{k-}'$, whereas for the modal displacements $\mathbf{y}_{k+} = \mathbf{y}_{k-}$ holds, i.e. the potential energy remains unchanged across a Dirac delta impulse, whereas the kinetic energy is subject of variation. Hence, Eq. (5) simplifies to

$$\Delta E_{i,k} = \Delta T_{i,k} = W_{i,k}. \tag{7}$$

Summarizing over all modes, the energy balance equation of the overall system is given by

$$\Delta E_k = \Delta T_k = W_k. \tag{8}$$

The total work W_k of the impulsive elastic forces is equal to the negative value of the variation of the potential U_k of the Dirac-type spring. The switching behaviour of the stiffness of the Dirac-type spring allows to extract energy from, or feed energy to the mechanical system, i.e. the Dirac-type spring acts as a kind of controlled energy buffer (CEB).
Henceforth, all analytical investigations are restricted to systems with two degrees of freedom, representing the simplest model to describe modal energy transfer effects. In this case, it is easy to verify that the following relations hold

$$y_{1,k+} = y_{1,k-}, \quad y_{2,k+} = y_{2,k-}, \tag{9}$$

$$y_{1,k+}' = y_{1,k-}' - \frac{1}{\mu_1}\varepsilon_k \beta_{1,k-}, \quad y_{2,k+}' = y_{2,k-}' - \frac{1}{\mu_2}\varepsilon_k \beta_{2,k-}, \tag{10}$$

with the abbreviations

$$\beta_{1,k-} = \gamma_{11} y_{1,k-} + \gamma_{12} y_{2,k-}, \;\; \beta_{2,k-} = \gamma_{21} y_{1,k-} + \gamma_{22} y_{2,k-}. \tag{11}$$

The variation of the kinetic energy of the first mode, caused by a Dirac delta impulse, is of the form

$$\Delta T_{1,k} = T_{1,k+} - T_{1,k-} = -\varepsilon_k \beta_{1,k-} \left[y'_{1,k-} - \frac{1}{2\mu_1} \varepsilon_k \beta_{1,k-} \right], \tag{12}$$

and of the second mode

$$\Delta T_{2,k} = T_{2,k+} - T_{2,k-} = -\varepsilon_k \beta_{2,k-} \left[y'_{2,k-} - \frac{1}{2\mu_2} \varepsilon_k \beta_{2,k-} \right]. \tag{13}$$

From Eqs. (12) and (13) it can be seen that the effect of a Dirac delta impulse to the modal kinetic energy depends on ε_k and on the values of the state variables $y_{i,k-}$ and $y'_{i,k-}$, $i = 1, 2$, just before the application of the impulse. Furthermore, $\Delta T_{1,k}$ is a quadratic function in ε_k with zeroes at $\varepsilon_k = \varepsilon_{k,0} = 0$ and

$$\varepsilon_k = \varepsilon_{k,A} = 2\mu_1 \frac{y'_{1,k-}}{\beta_{1,k-}(y_{1,k-}, y_{2,k-})}. \tag{14}$$

Using $\varepsilon_k = \vartheta \varepsilon_{k,A}$, Eq. (12) yields to

$$\Delta T_{1,k} = 2\mu_1 y'^2_{1,k-} (\vartheta^2 - \vartheta), \tag{15}$$

which is negative within the interval $\vartheta \in]0, 1[$, respectively if $\varepsilon_k \in]0, \varepsilon_{k,A}[$, i.e., the kinetic energy of the first mode is reduced by the Dirac delta impulse. In the same way, it can be seen that $\Delta T_{2,k}$ has zeroes at $\varepsilon_k = \varepsilon_{k,0} = 0$ and at

$$\varepsilon_{k,B} = 2\mu_2 \frac{y'_{2,k-}}{\beta_{2,k-}(y_{1,k-}, y_{2,k-})}, \tag{16}$$

and $\Delta T_{1,k} + \Delta T_{2,k}$ at $\varepsilon_k = \varepsilon_{k,0} = 0$ and at

$$\varepsilon_k = \varepsilon_{k,S} = \frac{\beta_{1,k-} y'_{1,k-} + \beta_{2,k-} y'_{2,k-}}{\frac{1}{2} \left(\frac{\beta^2_{1,k-}}{\mu 1} + \frac{\beta^2_{2,k-}}{\mu 2} \right)}. \tag{17}$$

Figure (1) (left) provides a sketch of $\Delta T_{1,k}$, $\Delta T_{2,k}$ and $W_k = \Delta T_{1,k} + \Delta T_{2,k}$ for the exemplary case where $\varepsilon_{k,B} < \varepsilon_{k,S} < \varepsilon_{k,A}$. Depending on ε_k, the modal energy content of both modes can be affected by a stiffness impulse of Dirac delta type. For both modes, ranges of ε_k can be identified allowing a reduction or an increase of the corresponding modal energy content.
To extract energy from the mechanical system, it is required that $W_k < 0$, i.e. ε_k has to be within the interval $]0, \varepsilon_{k,S}[$. It is obvious to see that the maximum energy is extracted if $\varepsilon_k = \varepsilon_{k,S}/2$.
A special case is represented by $\varepsilon_k = \varepsilon_{k,S}$, where $\Delta T_{2,k} = -\Delta T_{1,k}$ and hence, the amount of energy that is extracted from one mode is feeded to the other mode, where no additional (external) energy is required, ($W_k = 0$). It has to be pointed out that $\varepsilon_k = \varepsilon_{k,S}$ does not specify a direction of energy transfer. The necessary condition to transfer energy from the first to the second mode is that concurrently $\Delta T_{1,k} < 0$, which means that $\varepsilon_k = \varepsilon_{k,S}$ has to be within the interval $]0, \varepsilon_{k,A}[$, see Fig. (1) (left).

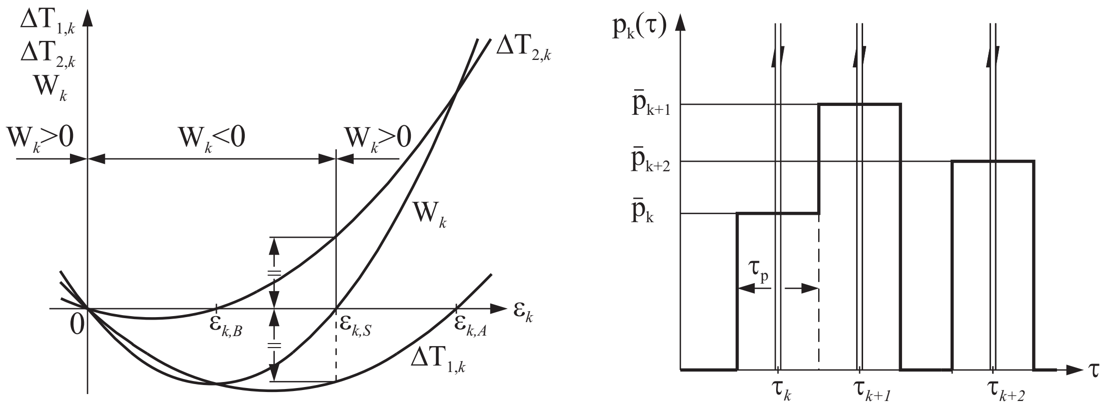

Figure 1: Effect of a Dirac delta stiffness impulse to the modal energy content (left), replacing Dirac delta impulses by rectangular ones (right).

The Dirac delta functions $\delta(\tau - \tau_k)$ in the equations of motion Eq. (2) are now replaced by rectangular impulses $p_k(\tau)$ according to

$$p_k(\tau) = \begin{cases} \bar{p}_k & \text{if } \tau_k - \tau_p/2 \leq \tau \leq \tau_k + \tau_p/2, \text{ and} \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

The strength $\varepsilon_k \mathbf{G}$ of the Dirac impulse is related to $p_k(\tau)$ by $\varepsilon_k \mathbf{G} = \bar{p}_k \mathbf{G} \tau_p$, i.e. $\bar{p}_k = \varepsilon_k/\tau_p$. Therewith, the modally transformed equations of motion can be written in the form

$$\mathbf{M}^* \mathbf{y}'' + \mathbf{K}^* \mathbf{y} = -\sum_{k=1}^{K} p_k \mathbf{G}^* \mathbf{y}, \tag{19}$$

approximating the dynamics of the original system Eq. (2). If $p_k = 0$, the equations of motion Eq. (19) are of autonomous type, and at an arbitrary instant of time τ, the state vector at $\tau + \tau_p$ is given by the simple equation

$$\mathbf{y}(\tau + \tau_p) = \mathbf{T}_0 \mathbf{y}(\tau), \tag{20}$$

where $\mathbf{T}_0 = \mathbf{T}_0(\mathbf{M}^*, \mathbf{K}^*, \tau_p)$. If a Dirac delta impulse at $\tau + \tau_p$ exists that allows to extract or transfer energy, the Dirac impulse is replaced by a rectangular one, see Fig. (1) (right). At the instant of time τ_k, the state-vector $\mathbf{y}(\tau_k + \tau_p)$ is given by

$$\mathbf{y}(\tau_k + \tau_p) = \mathbf{T}_{k,0} \mathbf{T}_k \mathbf{y}(\tau_k), \tag{21}$$

with $\mathbf{T}_k = \mathbf{T}_k(\mathbf{M}^*, \mathbf{K}^* + \bar{p}_k \mathbf{G}^*, \tau_p/2)$ and $\mathbf{T}_{k,0} = \mathbf{T}_{k,0}(\mathbf{M}^*, \mathbf{K}^*, \tau_p/2)$. If there exists again a Dirac delta impulse at $\tau_k + \tau_p$ allowing to extract or transfer energy, a rectangular impulse with center at $\tau_{k+1} = \tau_k + \tau_p$ and the amplitude $\bar{p}_{k+1} \mathbf{G}^*$ is applied, otherwise $\bar{p}_{k+1}$ is set to zero. Finally, a step-shaped stiffness modulation function is achieved, as exemplary depicted in Fig. (1) (right).

3 NUMERICAL EXAMPLE

Figure (2) shows a sketch of the investigated mechanical system of Hamiltonian type with two degrees of freedom $\mathbf{x} = [x_1, x_2]^T$, pinned on one end. The masses m_i are coupled via

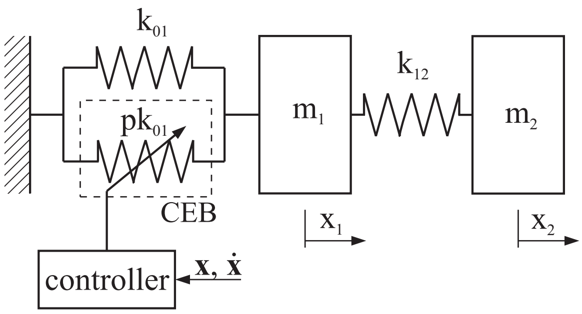

Figure 2: Sketch of the investigated mechanical system.

springs with stiffness parameters $k_{i-1,i}$, $i = 1, 2$. The coupling stiffness $k(t)$ of m_1 to the inertial system consists of a constant and a time-varying part according to

$$k(t) = k_{01}(1+p), \text{ where } p = \sum_{k=1}^{K} p_k(t). \tag{22}$$

Introducing a time-transformation $\tau = \Omega_R t$, where $\Omega_R = \sqrt{k_R/m_R}$ represents a reference frequency, and non-dimensional system parameters $M_i = m_i/m_R$ and $K_{i-1,i} = k_{i-1,i}/k_R$, the non-dimensional equations of motion can be written in the form

$$\mathbf{M}\mathbf{x}'' + \mathbf{K}\mathbf{x} = -\sum_{k=1}^{K} p_k \mathbf{G}\mathbf{x}, \tag{23}$$

with the matrices

$$\mathbf{M} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}, \; \mathbf{K} = \begin{bmatrix} K_{01}+K_{12} & -K_{12} \\ -K_{12} & K_{12} \end{bmatrix}, \; \mathbf{G} = \begin{bmatrix} K_{01} & 0 \\ 0 & 0 \end{bmatrix}. \tag{24}$$

Applying a modal transformation $\mathbf{x} = \mathbf{\Phi}\mathbf{y}$ to Eq. (23), where $\mathbf{\Phi}$ comprises the natural mode-shapes of the system without parametric excitation, the equations of motion are of the form of Eq. (19). The non-dimensional system parameters

$$M_1 = 1.5, \; M_2 = 1, \; K_{01} = 2, \; K_{12} = 1, \; \tau_p = 0.5, \tag{25}$$

are used for the numerical investigations. To extract energy from the mechanical system, ε_k has to be within the interval $]0, \varepsilon_{k,S}[$ as it was shown in Section 2. The simplest approach to define ε_k for the numerical calculations is the linear form $\varepsilon_k = \alpha\varepsilon_{k,S}$, where $\alpha = 1/2$ is used here. Moreover, ε_k is limited to $\varepsilon_{k,max} = 1/8$.

Figure (3) (left) depicts the results for the modal displacements y_1 and y_2, the stiffness modulation function p and the total energy E_1 and E_2 of the first and the second vibrational mode, as well as the total energy $E_1 + E_2$ (top to bottom). As initial condition, a superposition of a first and a second mode deflection according to $\mathbf{y}_0 = [1, \; 0.1]^T$ and $\mathbf{y}_0' = [0, \; 0]^T$, is used. Up to $\tau = 50$, the mechanical system shows vibrations with constant amplitude as the parametric excitation is switched off ($p = 0$). The energies E_1 and E_2 are constant and equal their initial value. Switching on the parametric excitation results in a, more or less, "bang-bang" behavior of p. It has to be pointed out that each of the depicted parametric impulses consists of a series of

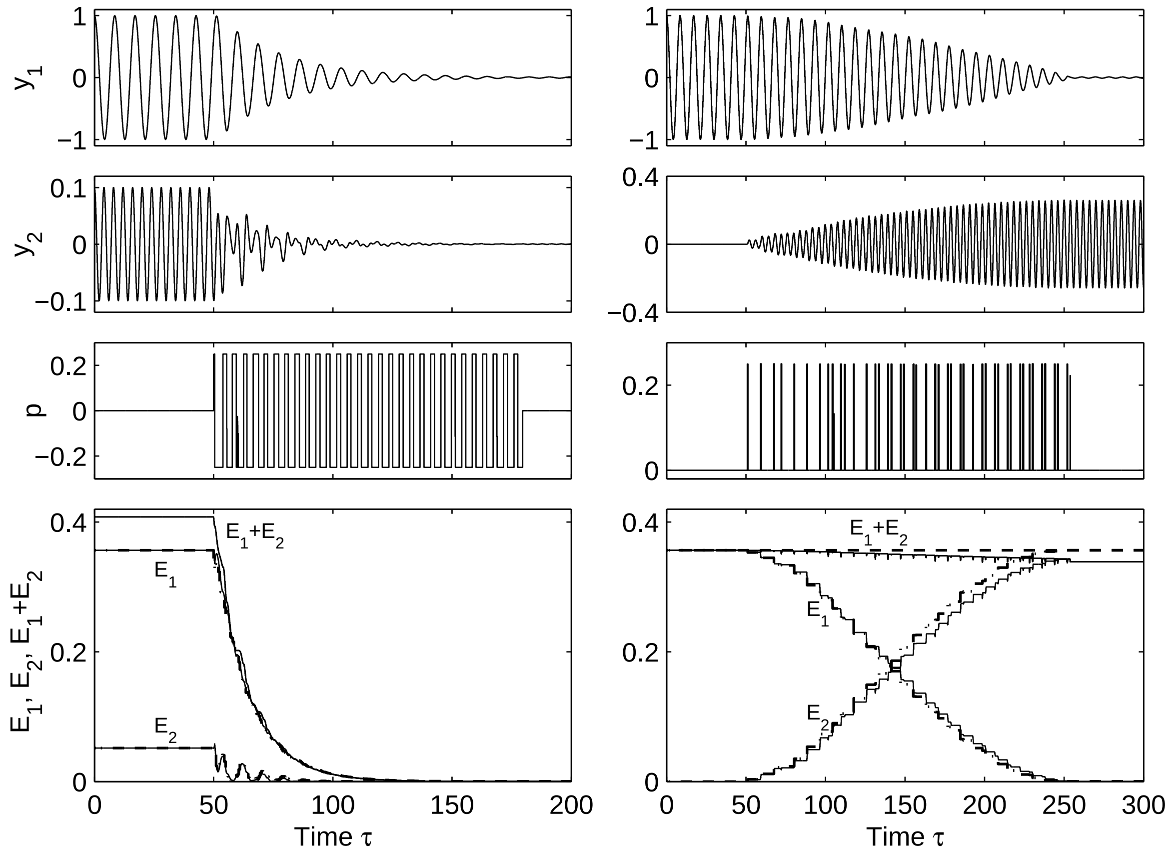

Figure 3: Results for the case where energy is extracted from the mechanical system (left column), and the case where energy is transferred from first to second mode (right column).

impulses of duration τ_p. The first impulses ($\tau > 50$), show an irregular, not-periodic behaviour, becoming a periodic sequence with increasing time. The amplitudes of the modal vibration signals y_1 and y_2 decrease very fast, and therewith the corresponding total energies E_1 and E_2, see bottom plot of Fig. (3) (left). This means that energy is extracted very efficiently from the mechanical system. The parametric excitation is switched off when $(E_1 + E_2) < 1e - 4$ holds. It is interesting that E_2, although showing an overall decreasing behavior, is decreasing and increasing repeatedly, until it approaches zero. This is founded in the fact that the condition $W_k < 0$ does not impose any restriction to the sign of $\Delta T_{1,k}$ and $\Delta T_{2,k}$. It is ensured solely that W_k is negative, i.e. energy is extracted. The dashed lines represent the total modal energies if Dirac impulses are used instead of rectangular ones. The differences between the exact solution using Dirac impulses and the approximate solution using rectangular impulses are very small, which is of course mainly affected by the impulse duration τ_p. The corresponding physical coordinates x_1 and x_2 are shown in Fig. (4) (left). Although the system is undamped, impulsive parametric excitation allows to efficiently extract energy from the mechanical system, leading to a fast decay of the vibration signals.

Another interesting property of impulsive parametric excitation is the possibility to transfer energy from one mode to another. Figure (3) (right) shows the results where a first mode deflection $\mathbf{y}_0 = [1,\ 0]^T$ and $\mathbf{y}'_0 = [0,\ 0]^T$ is used as initial condition. As in the previous case, the parametric excitation is switched on at $\tau = 50$, and ε_k is limited to $\varepsilon_{k,max} = 1/8$. Moreover, ε_k is limited to positive values. Switching on the parametric excitation $\tau = 50$ is leading to a decrease of the

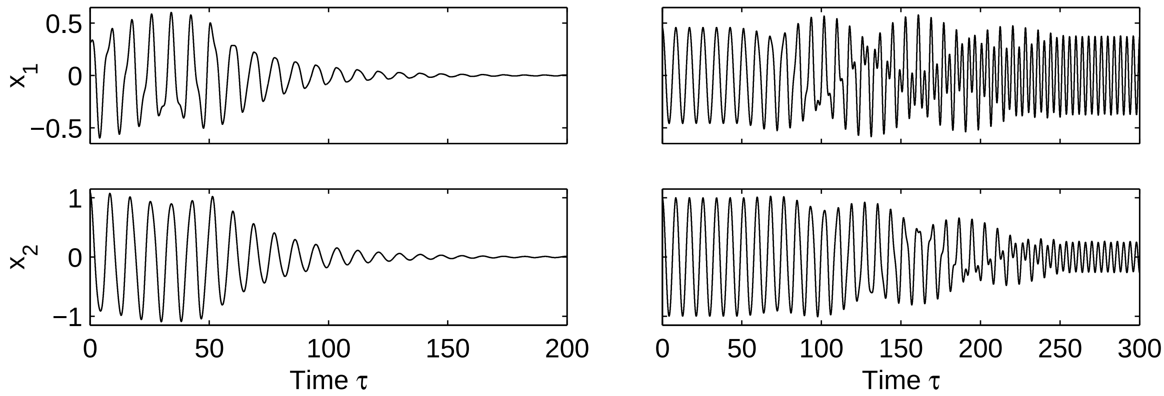

Figure 4: Timeseries of the physical coordinates for extracting energy from the mechanical system (left column), and transferring energy from first to second mode (right column).

total modal energy E_1 of the first mode, accompanied by an almost symmetric increase of the total energy E_2. The total energy $E_1 + E_2$ decreases slightly as the impulsive parametric excitation is of the non-ideal rectangular form. The dashed lines represent the solution using Dirac delta impulses, where $E_1 + E_2$ remains constant. At $\tau = 250$, first mode vibrations have almost vanished, and the system exhibits second mode vibrations only, hence, the vibration energy of the first mode was shifted to the second one. The timeseries of the physical coordinates x_1 and x_2 are presented in Fig. (4) (right), showing the transition from first to second mode vibrations. Figure (5) explains the effect of impulsive parametric excitation to the modal energy content in detail. In the left column the results for the case where energy is extracted from the mechanical system are depicted. Within a very short timespan p switches from zero at point 1 to 0.25 at point 2, see Fig. (5) (left). As the displacement $x_1 \approx 0$, the potential energy

$$U_P = pK_{01}x_1^2/2 \tag{26}$$

of the CEB is also equal to zero. From 2 to 3, the work W_P of the elastic force $pK_{01}x_1$,

$$W_{P,23} = -\int_{\tau_2}^{\tau_3} pK_{01}x_1x_1' d\tau \tag{27}$$

is negative, i.e. energy is extracted from the mechanical system ($E_1 + E_2$ is decreasing) and fed to the CEB, increasing its potential energy U_P. From 3 to 4, p is switched off, and therefore, the accumulated energy U_P is set to zero, i.e. is extracted from the CEB. At point 5, $U_P < 0$ as p is negative. The work W_P is still negative from 5 to 6, i.e. energy is extracted from the system and feeded to the CEB until U_P reaches zero at point 6. Switching p to zero from 6 to 7 prepares for a new cycle. Within the two timespans from 1 to 4 and 4 to 7, energy is extracted from the mechanical system, $(E_1 + E_2)$ is decreasing, where the total energy E_1 of the first mode is continuously decreasing, and E_2 shows an increasing behaviour.

Figure (5) (right), shows the results for modal energy transfer. From point 1 to point 2, p switches from zero to 0.25. As $x_1 \neq 0$, the potential energy of the CEB increases, i.e. external energy is feeded to the CEB. With ongoing time, W_P is decreasing first, achieves a minimum, and is then increasing to almost the same level as in point 2. Accordingly, the potential energy U_P and the total energy $E_1 + E_2$ at point 3 are quite the same as in point 2. The total energy of first and second mode, E_1 and E_2, show a totally different behaviour. As E_1 is decreasing from

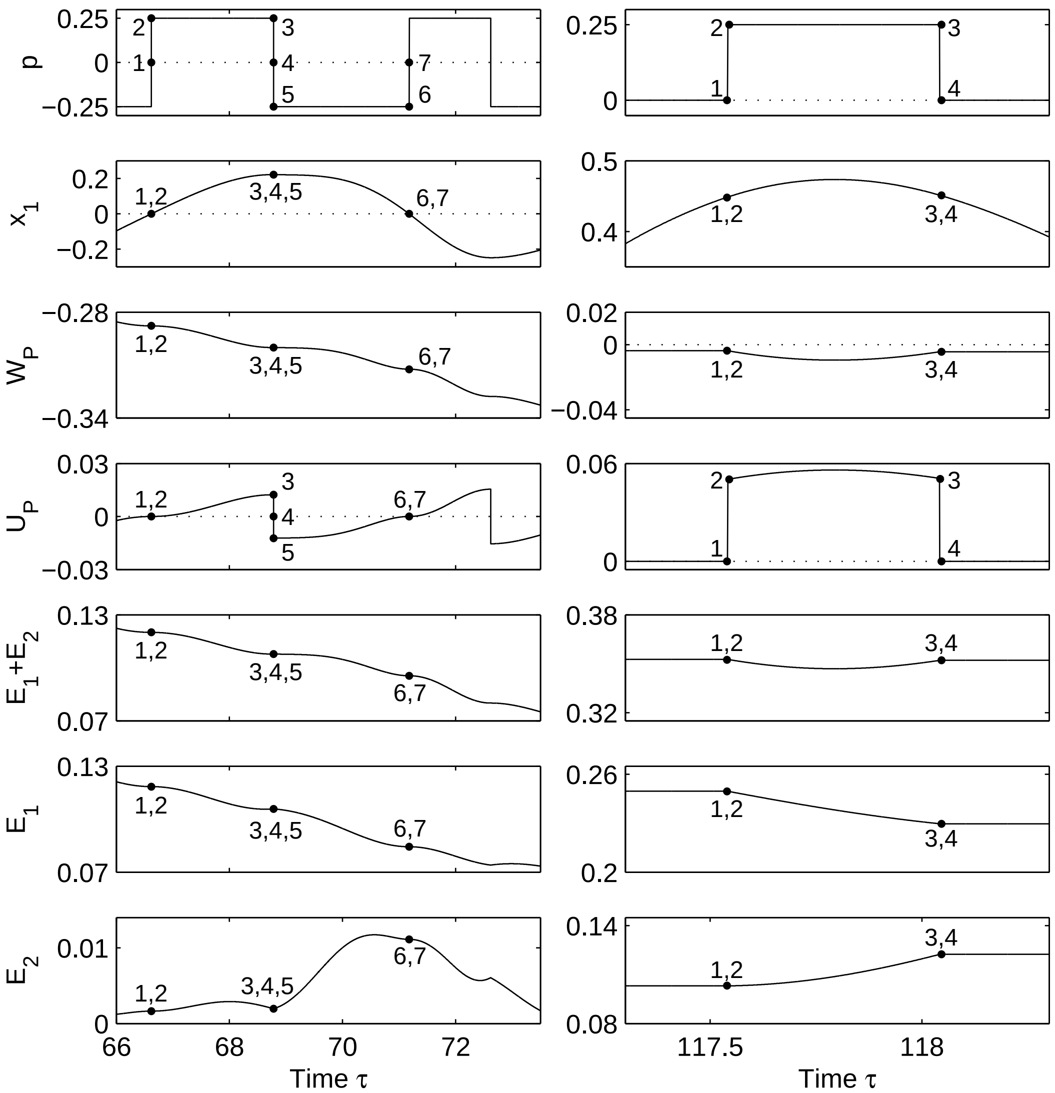

Figure 5: Principles of extracting energy (left column), and transferring energy (right column).

2 to 3, E_2 increases almost symmetrically, i.e. energy extracted from the first mode is fed to the second mode. From 3 to 4, p is switched off. Setting U_P to zero means that energy is extracted from the CEB which is approximately equal to the energy feeded to the CEB from 1 to 2, i.e. almost zero energy is necessary to transfer energy from the first to the second vibrational mode. The energy removed from the CEB (i.e. from the entire system) from 3 to 4 has to be stored to feed it to the system with the next impulse.

4 CONCLUSIONS

In this contribution, it was shown that impulsive parametric stiffness excitation allows to extract energy from a mechanical system and to transfer energy from one mode to another. The case of modal energy transfer reveals its advantages in damped systems as higher modes usually possess higher damping ratios leading to a faster decay of vibrational amplitudes compared to the case without impulsive parametric excitation.

5 ACKNOWLEDGEMENT

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