

## A METHODOLOGY TO SUPPRESS THE RESIDUAL VIBRATIONS OF EXCITED COMPLIANT SYSTEMS

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**Abstract.** *In its initial phase, the paper deals with defining the problems concerning the generation of spurious residual vibrations of non-periodic step functions. Step motion functions kinematically excite a mechanical system with compliant links. Attention is focused on non-periodic displacement laws with a rest interval in which it is usually carried out a required technological operation and in which those residual vibrations are undesirable. The kinematic excitation of the system is implemented by a conventional or electronic cam. The special way of compensating the residual vibrations is demonstrated on a dynamic stand of electronic cam with one compliant link in the driven part of the kinematic chain.*

## 1 INTRODUCTION

Conventional cam mechanisms and electronic cams have virtually the same, i.e., the drive of a working link of a manufacturing machine mechanism. As conventional cams we mean compound cam mechanisms with any basic cam mechanisms as described in References [1][5]. Those mechanisms are well known with their pros and cons. Positive properties are for example their high dynamics, relatively low cost and variability of structures; their negatives are their single purpose, the influence of backlashes (clearances), compliances in the input and output kinematic chain, wear (needed as spare parts).

Electronic cams [3] are mechanisms that consist of control (controller) and drive (inverter, servomotor). We talk often about the system of electronic cam because the application of electronic cam unites recently separate disciplines (mechanics, software, electronics, control, etc.) in it and it is a classic case of a rapidly developing field called mechatronics. The positives of electronic cams are their programmability and usability in production systems as elements of flexible automation, low maintenance and reliability. Their negatives are for example: their lower dynamics, higher acquisition costs, high qualification requirements for the preparation and design of an application.

The paper deals with the use of conventional and electronic cams in rigid and flexible automation, which is the implementation of non-periodic (step) movements with structural elements such as are various step mechanisms [5] and turntables in this case. In the one direction, the subject of interest in this paper is the study of residual oscillation (vibration) in the rest areas of motion functions and finding ways of its minimizing without feedback intervention, i.e., to determine the residual spectra of selected displacement laws and based on them, to set operating speed, angle of displacement or moment of inertia of workload in such a way so that this oscillation (vibration) is minimal. Then, in the other direction, we study the effect of superposition of the basic displacement law with harmonic compensation (correction) pulse, which minimizes this spurious oscillation [2][3].

## 2 THE APPLICATION RESEARCH OF CONVENTIONAL AND ELECTRONIC CAMS

For the purposes of experiments, dynamic stands for conventional cam mechanisms and electronic cams were built. In Figure 1, there is a 3kW system of Yaskawa's electronic cam (controller, inverter, and servomotor) with a pliable driven part with working mass of inertia.

In the following paragraphs, it is then created a numerical model of electronic cam mechanism shown in Figure 1. The model was validated by an independent measurement. The measurement results are not listed due to the limited range of this paper.



Figure 1: Electronic cam stand.

### 3 THE DISCRETE MODEL OF CONVENTIONAL CAM MECHANISM WITH FLEXIBLE CONSTRAINTS

Conventional cam mechanisms with flexible constraints in the driven and driving mechanism parts are characterized by the fact that the combination of two discrete mass system parts with rotational or translational motion, which are part of the same mechanism link (in terms of kinematic labeling of mechanism links), is implemented by a flexible constraint. The mechanisms of this group have always 1° of degree of freedom at rigid constraints. By introducing flexible constraints, the number of degrees of freedom of the mechanism system that is dependent on the number and arrangements of constraints increases.

Real mechanisms, used for the construction of machines and equipment, are relatively complicated mechanical systems. The starting point for a specific dynamic solution is to determine a suitable computational model. In the class of computational models of mechanisms with flexible constraints, we will include those models of mechanisms by which the appropriate flexible links are considered to be immaterial. The cam mechanism is characterized by a discrete distribution of masses of the individual links of a system that are assumed to be perfectly rigid. The mentioned system has the finite number of degrees of freedom, depending on the number and arrangement of flexible constraints. Deriving the equations of motion is discussed in [3] in more detail.

### 4 EQUATIONS OF MOTION OF THE ELECTRONIC CAM DISCRETE MODEL

We will derive the equations of motion in the same manner as it is in the case of a conventional cam mechanism. Assigning values to variables in the reality of electronic cam will be depicted below. Due to the analogy with the conventional mechanism, also the links with zero moments of inertia are plotted, which, however, have their mechanical equivalent in the form of a conventional mechanism with two output compliances.

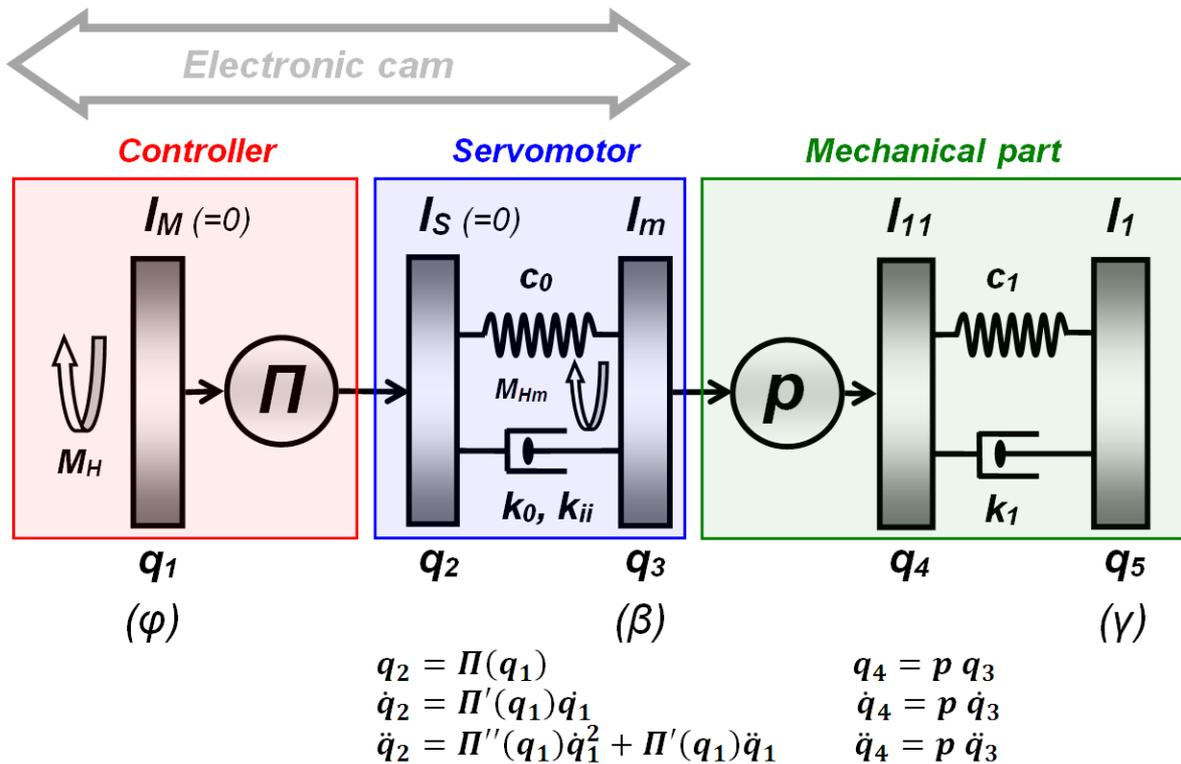


Figure 2: Electronic cam discrete model.

The equations of motion of the discrete model of an electronic cam in the form of

$$M_H = \{c_0[\Pi(\varphi) - \beta] + k_0[\Pi'(\varphi)\dot{\varphi} - \dot{\beta}]\} \Pi'(\varphi) \quad (1)$$

$$(I_m + I_{11}p^2)\ddot{\beta} = c_0[\Pi(\varphi) - \beta] - c_1p(p\beta - \gamma) + k_0[\Pi'(\varphi)\dot{\varphi} - \dot{\beta}] - k_1p(p\dot{\beta} - \dot{\gamma}) \quad (2)$$

$$I_1\ddot{\gamma} = c_1(p\beta - \gamma) + k_1(p\dot{\beta} - \dot{\gamma}) \quad (3)$$

where generally  $\dot{\varphi} \neq konst.$  Equations (2) and (3) describe the behavior of the electronic cam in the case when coordinate  $\varphi$  is the dependent variable (slave) of the virtual shaft (master). For example, the equations then describe the behavior when starting-up revolutions along a predefined ramp, as it is common with electronic cams. Practically, however, stable operation is solved.

At this point, we will provide a short note to a difference from the numerical solution of a conventional cam mechanism model. Here, we will not describe the principles of controlling servo drives; we will only note that most servo drives have a cascade control structure with torque, speed and position feedback. Controllers are generally proportional (P) and proportionally integral (PI). Thus, we will try to intervene in the numerical solution of equations of motion in such a way so that a characteristic quantity, which is a servo motor positional deviation (PERR: a difference of the actual position on the servo shaft from the theoretical one), can correspond to the reality of P/PI modes at most. PERR is a criterion of the accuracy of the given model and its course was compared with two independent sources. The one is the measurement and the other is the virtual model of a controlled mechanical system created in the MSC.ADAMS and MSC.EASY5 program systems. Due to the limited extent of this paper, the results of the comparison are not presented.

## 5 RESIDUAL SPECTRA OF THE DISPLACEMENT LAWS OF CONVENTIONAL AND ELECTRONIC CAMS

The accuracy of the end position in the rest interval of function of motion is assessed according to the extreme acceleration value of a working link because the links of a cam mechanism are considered as pliable in dynamic models. The criterion of positional accuracy is the residual spectrum of the second derivative ( $a_R$ ) of the response to the kinematic excitation of a pliable system with a displacement law [1]. The residual spectrum, specific for the given displacement law, will be used for determining the parameters (speed, angle of displacement or moment of inertia) at which the oscillation is minimized. The presented results in the paper are the outcome of a purely numerical solution based on the data file of a displacement law (0<sup>th</sup>, 1<sup>st</sup> and 2<sup>nd</sup> derivative) and the parameters of models with pliable links. The numerical solution is simple. In the cycle for/next of relative natural frequency  $\nu$  (in Figure 5 and 6 as „ny1“), a numerical solution of equations (1) up to (3) proceeds whose each cycle run is the resulting maximum acceleration in the rest area of the equation of motion. The graphical expression of those values depending on  $\nu$  is the searched residual spectrum. Relative natural frequency  $\nu$  represents the number of oscillations performed in the time of one displacement or the implemented angle of displacement (time period minus rest time). Relationship  $\nu$  [1] with speed  $n[\text{min}^{-1}]$ , angle of displacement  $\phi[\text{deg}]$  and moment of inertia of load  $I_1[\text{kg}\cdot\text{m}^2]$  is

$$\nu = \phi f / 6n, \quad f = (1/2\pi) \sqrt{c_1/I_1}. \quad (4)$$

With regard to the extent of the paper, there are given only illustrative results for electronic cams. The following figure shows the residual spectra for the polynomial displacement law of the 5<sup>th</sup> degree, harmonic and parabolic with a displacement of 68[deg] and an angle of displacement of 90[deg] at a load of 0.1[kgm<sup>2</sup>].

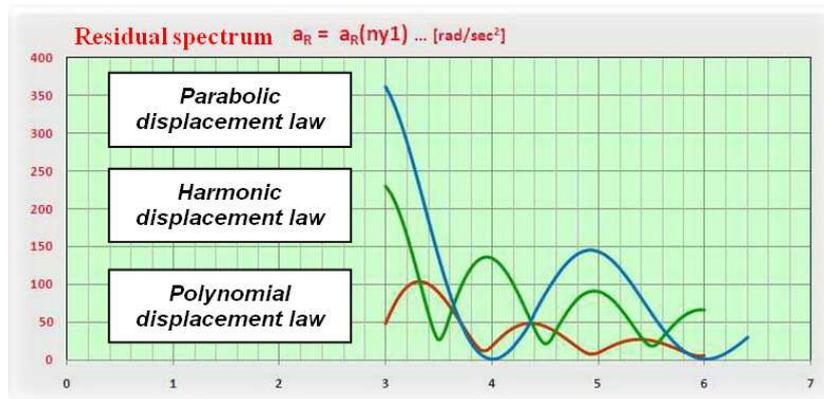


Figure 3: Residual spectra of the electronic cam model.

## 6 COMPENSATION OF RESIDUAL OSCILLATIONS BY SUPERPOSING THE DISPLACEMENT LAW WITH HARMONIC PULSE

The stand model according to Figure 1 has one natural frequency due to compliance in the driven part of the mechanism. At the end of the displacement working part, it will then excite harmonics of the natural frequency. According to [2][3], then, for damping the oscillation, it will be sufficient to place the pulse of the same frequency, the same amplitude and the opposite phase at the end of the displacement. The following part of the article will deal with the extension of this idea, resulting in practical verification.

Extension of this idea means that it is not necessary to use exactly the given pulse, for whose determination it is necessary to know the natural frequency of the modeled system. It is just sufficient to use any pulse that will generate oscillations of the same amplitude but opposite phase with regard to the oscillations generated by the original displacement law at the end of the working part of the original displacement law. By superposing this pulse with the displacement law, we will then get the signal that may well excite oscillations in the working part, but before going into the rest part, their absorption (damping) will occur. It is just this rest area which is crucial for the most manufacturing processes.

The following Figure 4 graphically demonstrates the creating of a modified displacement law. To the parabolic displacement law (characterized by a jump course of the 2<sup>nd</sup> derivative of the displacement or acceleration), it is added a pulse in a length of 50° in the point of 120°. The pulse has an amplitude size of 1.46° so that it is only slightly noticeable in the displacement course in an amplitude of 30°, but in the acceleration course, it can be very well observed.

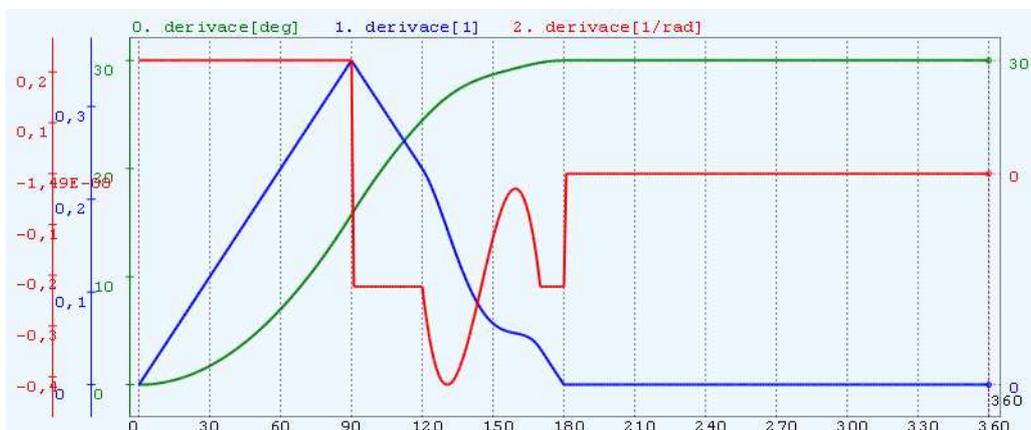


Figure 4: An example of creating a modified displacement law.

The mere superposition of the initial displacement law with compensation pulse would lead to a change of the resulting displacement, therefore, it is necessary to multiply the initial displacement law by an amplitude of the compensation pulse, see the below relations. Displacement law is defined by means of unified unit displacement law and a parameter defining the size of the displacement.

$$dis\_law = displacement * unit\_dis\_law \quad (5)$$

Similarly, also the compensation pulse can be defined. The resulting relationship can be then written as follows, where  $A$  represents the amplitude of the compensation pulse.

$$mod\_dis\_law = (displacement + A) * unit\_dis\_law - A * unit\_puls \quad (6)$$

The following Figure shows the effect of shaft compliance in a practical example. The green dashed line represents the desired displacement law. This is a parabolic displacement law with a displacement of  $30^\circ$  during the half-turn ( $180^\circ$ ) of the output. The response to this signal at a speed of 130 rev/min is shown in the red curve. The damped oscillations are evident throughout the whole rest interval. The blue curve then shows the response of the system to the displacement law modified by the compensation pulse. The residual oscillations (vibrations) in the rest part are completely attenuated. There is also seen a slightly larger deformation of the response during the displacement caused by a change of the scale of the original (initial) displacement law. Compensation pulse is inserted just before the end of the displacement (approximately into a point of  $130^\circ$ ); therefore, it itself does not bring any deformation. Usually, there are more positions where the pulse can be placed, which will be explained in the next Chapter.

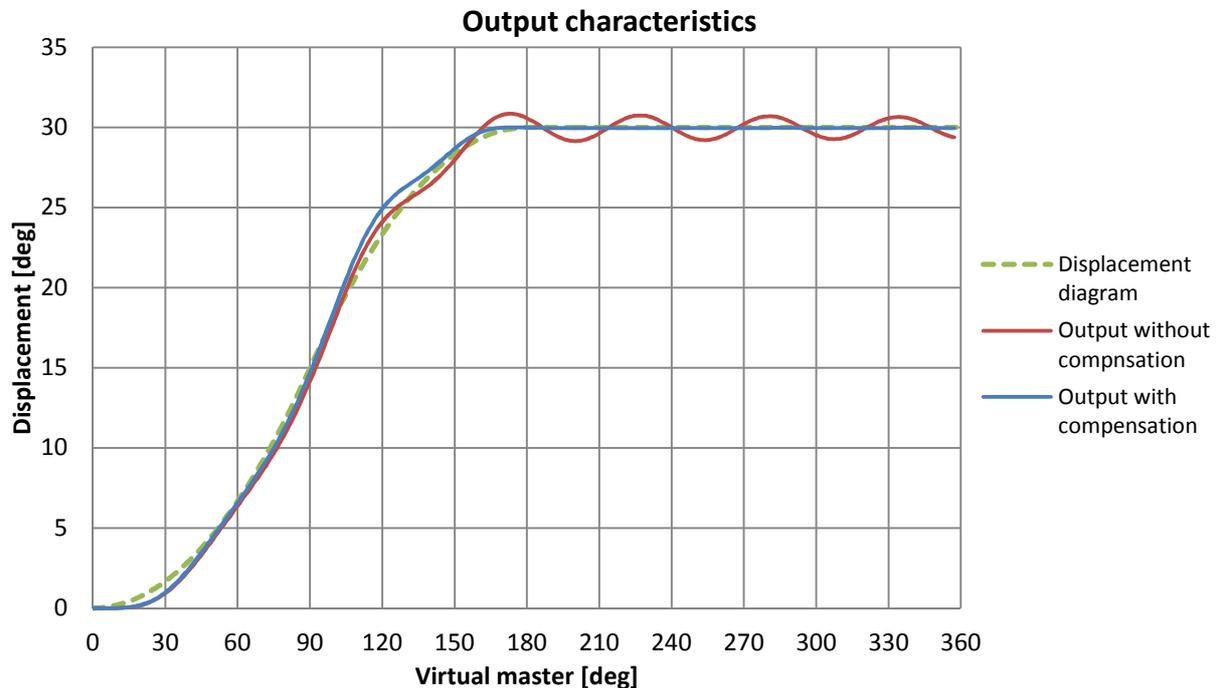


Figure 5: Output characteristics measured on the test stand.

## 7 THE EFFECT OF THE PARAMETERS OF THE COMPENSATION PULSE

The compensation pulse is basically only a scaled unit displacement law placed in the correct position. At first, the function or type of displacement law, determining its shape, will be established. In all the graphs in this paper, it is a polynomial of the 5<sup>th</sup> degree. Also other

functions, such a parabola or cycloidal function were tested. In general, it can be said that similar results could be achieved with them. Furthermore, to define the pulse accurately, only three parameters: length, amplitude and position will be required. Length and position are in degrees with regard to the rotation of the virtual master. Due to the fact that the speed of the virtual master is always assumed to be constant, it would be possible to replace angular units with time ones.

By fixing the pulse length (duration) at a constant value, the influence of the other two parameters on the size of residual oscillations can be graphically illustrated, as shown in the following Figures 6 and 7. In both cases, the pulse lengths were chosen randomly owing to the natural frequency, which is 14.5 Hz (a pulse period corresponding to the natural frequency is  $26.9^\circ$ ). The initial pulse position from  $0^\circ$  up to  $160^\circ$  is plotted in the horizontal axis. Of course, the usability of the upper limit is influenced by the pulse length in such a way so that the pulse does not intervene (extend) in the rest area of the displacement law. It is evident that with regard to the horizontal axis, there are just several minima and their distance is defined by their natural frequency. In the vertical axis, there is plotted a pulse size from  $-5^\circ$  up to  $5^\circ$ . It is obvious, that minima can be found for both positive and negative pulse sizes. The graphs also show a slight shift of the minima to the horizontal axis with an increasing pulse position. This phenomenon is dependent on the size of damping the flexible shaft in such a way that it increases with the size of damping. The earlier the pulse is placed, the larger must be its amplitude so that it can generate oscillations of the desired size at the end of the displacement working part. When comparing both graphs, it is evident that the pulse length has a direct influence on the pulse size. This is logical because when increasing the pulse length, the amplitude of oscillations generated by this pulse will decrease. The pulse cannot be extended indefinitely in such a way because its large amplitude would too deform the initial displacement law. From the viewpoint of the servomotor, also the maximum size of acceleration of the modified displacement law is important. In terms of the compensation pulse length it is not good to choose a too large or too small value. Methodology for determining this value depending on other parameters will be the subject of further research.

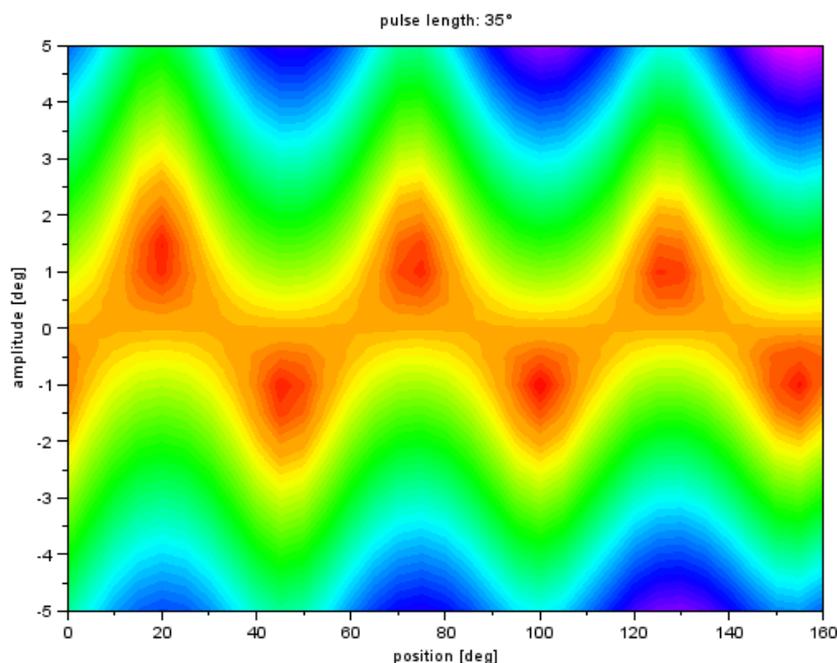


Figure 6: Effect of the position and pulse size on the size of oscillations (red minimum, violet/purple maximum) at a pulse length of  $35^\circ$ .

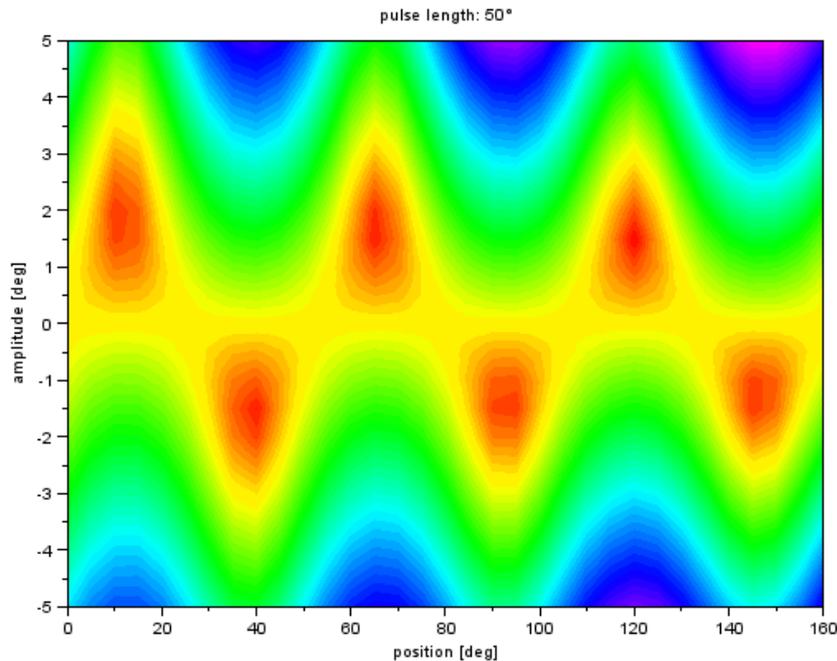


Figure 7: Effect of the position and pulse size on the size of oscillations (red minimum, violet/purple maximum) at a pulse length of  $50^\circ$ .

## 8 DETERMINING THE PULSE PARAMETERS

For practical application of this method it is assumed that a mathematical model of the system is not known. Thus, to find the ideal parameters of the pulse, an iterative method was used. At first, a method of halving (bisection) the interval was used (separately for the one axis and then for the second axis). This way of finding minima was very sensitive to an appropriate setting of the initial conditions. It was also adversely influenced by the errors arising from the measurement of the size of residual oscillations. Therefore, it was gradually modified and ultimately entirely replaced by the Nelder–Mead simplex method [4]. The method is characterized by a relatively rapid convergence, it does not require knowledge of the derivative of the minimized function and at the same time it is also robust with regard to possible errors arising from the measurement. In general, it is multi-dimensional, so the eventual expansion of the minimized function with the pulse length parameter would not present any major problems.

The first step of the method is the choice of the initial simplex, which is an object consisting of  $n+1$  vertices, where  $n$  is the size of the minimized function. As shown in Figures 6 and 7, there are usually several minima. Thus, the choice of the initial simplex directly decides to which particular local minimum the function will converge. It is usually best to find the latest minimum before the end of the displacement because the pulse here will have the smallest amplitude or smallest acceleration.

The next step is then the iterative transformation of the simplex according to the rules of the Nelder–Mead method. In our case, there are used transformations: reflection, expansion, contraction and reduction.

## 9 TEST STAND

The experimental model is based on Yaskawa's components. Specifically, there are the PLC MP2300, the SGDA-A3-AE servopack and the SGMGH-30D servomotor with an output of 2.9kW. The measurement of output data is ensured by an external encoder with a resolution

of  $0.015^\circ$ , connected via the LIO-02 communication card. The PLC is programmed with a „ladder diagram“ and it incorporates the electronic cam control program and auxiliary subroutines for measuring the output data. To facilitate development, experimentation and interpretation of the results, other functionality was moved to the superior PC application. Communication between the PC and the PLC is via an Ethernet interface using Yaskawa API libraries developed by our company in Visual Basic [6]. The application itself is developed in C# language and in addition to control functions (start, stop, speed adjustment, etc.) it mainly detects the iterative mechanism for finding the ideal position of the compensation pulse (loading the measured data, calculation of a new modified displacement law and sending it to the PLC).

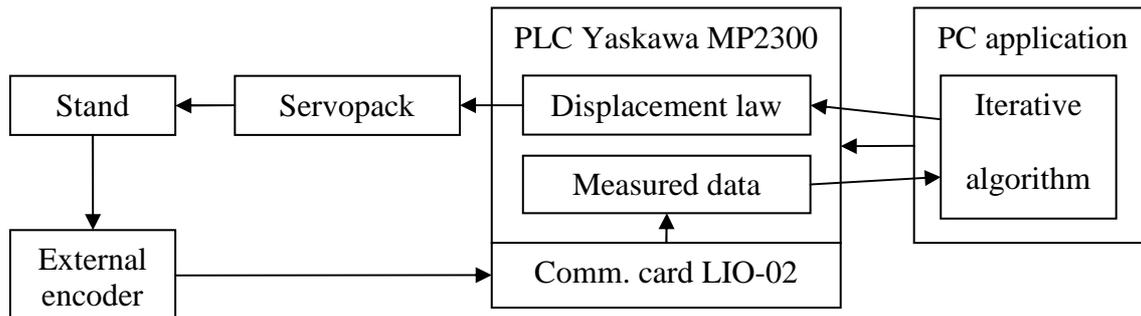


Figure 8: Diagram of data flows of the test stand.

## 10 CONCLUSIONS

Based on the residual spectra of displacement laws, which are the result of a numerical solution of models, it is possible to determine the speed, angle of displacement or moment of inertia in such a way so that residual oscillation (vibration) in the rest area of motion functions is minimal. These conclusions were verified on the dynamic stand. One of the interesting results is that the parabolic displacement law preferably compensates residual oscillations, even though the response of the system to this displacement law is the strongest because of its discontinuous course of the 2<sup>nd</sup> derivative.

The method of compensating oscillations using a superposition with harmonic pulse does not seek to replace existing methods; it only fills the empty space. By its nature, this is a feedback method, although it does not necessarily require the actual information on the exact position of the output shaft. It is sufficient for it when it receives the information about the „size“ of the oscillations in the current cycle at the end of the cycle. For experimental purposes, the crucial part of the algorithm was carried out in the PC application, but for practical use it should not be a problem to pass the algorithm directly to the PLC (especially if the PLC supports programming by means of structured text according to the now common standard IEC EN 61131-3). The algorithm can then function as a subroutine of „auto-tuning“ that the machine operator will start when booting the machine operation or changing some machine parameters (speed, changing the work mass, etc.), or the subroutine may be constantly active and when detecting deterioration in the quality of control, it will run itself the correction (compensation) process.

Unlike other feedback methods, this method does not intervene into any control structures of the servo drive. These interventions are usually quite expensive (only more expensive PLC with specialized SW modules enable them). A disadvantage of this method is the partial deformation of the working part of the displacement law, but on the contrary, it causes no delay. The process of finding the ideal position of compensation pulse lasts for several cycles and at the poorly chosen initial conditions, the first steps of the iterative algorithm may even deterio-

rate the quality of control. For its function, it does not require knowledge of the model of the controlled system. However, if the model is available, depending on its accuracy, the process of iteration can be speeded up (to choose the initial simplex more closely to the ideal state).

There are a large number of sophisticated methods for suppressing residual oscillations, but their practical use is still relatively rare. The implementation of this method is inexpensive and its use is easy for the operators; therefore, we are confident that it will find its place.

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