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A CO-SIMULATION METHOD FOR COUPLING COMPLEX FOUNDATION MODELS AND WIND TURBINE MODELS IN AERO-ELASTIC SIMULATIONS

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Abstract. *In order to achieve the goal of 20% renewable energy in 2020, as set by the European Union, large offshore wind farms are either under construction or in development throughout Europe. As many of the shallow sites, that are close to the shore, are already developed or under development, the current trend is to go further offshore into deeper waters, with ever larger wind turbines. Therefore, the traditional solution of using monopile foundations is no longer sufficient, and one needs to resort to more complex types of offshore foundations, such as tripods or jackets. Appropriate models for these more complex foundations are often not available in aero-elastic codes. Therefore, different methodologies for including approximate models or superelement models of the foundation in aero-elastic simulations have been developed, even tough their accuracy has never been evaluated with full scale simulations. In this paper, a different methodology will be presented for including complex foundation models in aero-elastic simulations. In this method the (aero-elastic) simulations for the wind turbine and the foundation are separated over two separate programs and coupled through their interfaces. This means that both components have their "own" Newmark time integrations, which are coupled by means of the interface forces and displacements. By taking the equivalent interface stiffness of the neighboring components into account, one iterates on the global problem in the local iterations performed on component level. The method has several benefits. First and foremost one is able to use the full foundation model in aero-elastic simulations; hence one is able to verify the accuracy of the earlier mentioned methods that use approximated or superelement models. In addition to this, one only needs to modify the aero-elastic code once and is then free to couple the simulations to any other code for the foundation, such that the coupled responses are obtained.*

1 Introduction

In order to ramp up the production of renewable energy, we have seen an exponential increase in the number of wind turbines installed, both onshore and offshore. As many of the favorable onshore locations are usually also densely populated and therefore not suited for large wind farms, many of these farms will have to be built offshore. In addition to this, offshore locations offer a number of other advantages, such as a higher and more uniform wind and, in theory, there are no size constraints for offshore wind turbines. In order to bring down the levelized cost of offshore wind energy, larger turbines are being developed and installed. Installing these bigger turbines in deeper waters requires more complex types of marine foundations, such as jacket, tripods or even floating foundations[2], as the design criteria can no longer be met using the traditional monopile foundations.

However, combining these more complex structures with the aero-elastic wind turbine models leads to some challenges, as the models of these offshore structures generally consist of significantly more degrees of freedom (DoF) and thus lead to a huge increase in computational effort. In addition, these complex aero-elastic codes where initially develop for simulating the responses of onshore wind turbines and by the time the first offshore wind turbines where designed, these codes where extended to allow for including monopile and gravity based marine foundation models. As these foundation types are relatively simple, this imposed only limited challenges for extending the software. However, integrating more complex foundation models will require a significant amount of changes to the aero-elastic code. An easier option for integrating the marine foundations in the aero-elastic simulations, is by including the (linearized) models as reduced superelements[6]. However, in this case one neglects the nonlinear contributions resulting from the hydro-elastics, soil-structure interaction and possible ice-structure coupling.

A second option is to model the foundation in an external program and to couple the time simulations of the wind turbine and foundation substructures. This can be done by establishing a line of communication between the two programs over which one sends the interface forces and displacements, such that one can use these to enforce compatibility and equilibrium at the interface between the two models. In this paper a substructured time integration method is presented that allows for such a coupled time integration. As the method in fact only reorganizes the equations of the associated non-substructured model, the accuracy and stability properties of the original time integration scheme, the generalized-α , are maintained.

Starting in section 2, the concepts of dual assembly and time integration are briefly introduced. In section 3 it is shown how the substructured simulations are coupled using the interface displacements, forces and the equivalent effective interface stiffness of the foundation. Some of the first results of a verification study are shown in section 4 and finally in section 5 a brief summary and outlook are given.

2 Coupling of numerical models for time simulations using dual assembly

This section contains the mathematical idea and concepts on which the substructuring approach proposed here is based. As a start, the equations of motion and concept of *dual assembly* are discussed. The section is organized such that in section 2.1 the initially uncoupled equations of motion of the wind turbine and foundation are coupled using the concept of dual assembly. Section 2.2 introduces the time integration method and solution strategy for solving the coupled set of equations as a single structure.

2.1 Coupling of the equations of motion

Firstly, the theory behind the proposed approach is briefly introduced, starting with an unconstraint model of the turbine as shown in figure 1. On a high level, the model of the wind

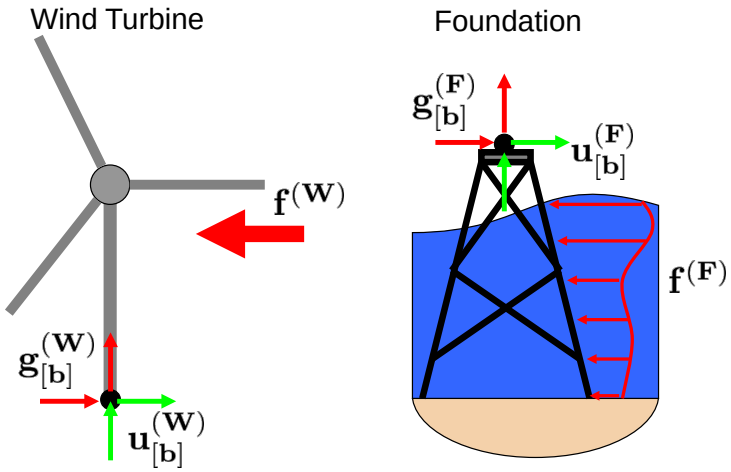

Figure 1: Wind turbine and Foundation model, with annotations for forces and displacements

turbine can be described as a nonlinear flexible multibody model which is excited by (non-linear) aerodynamic forces ($\boldsymbol{f}^{(W)}$), as is shown in Eq. (1).

$$\boldsymbol{M}^{(W)}\left(\boldsymbol{u}^{(W)}\right)\ddot{\boldsymbol{u}}^{(W)} + \boldsymbol{p}^{(W)}\left(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}\right) = \boldsymbol{f}^{(W)}(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}) + \boldsymbol{g}^{(W)}, \tag{1}$$

where, $\boldsymbol{M}^{(W)}\left(\boldsymbol{u}^{(W)}\right)$ is the configuration dependent mass matrix, $\boldsymbol{p}^{(W)}\left(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}\right)$ is the vector of internal non-linear elastic and damping forces, $\boldsymbol{f}^{(W)}(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)})$ being the (non-linear) external load (mainly) due to the wind and $\boldsymbol{g}^{(W)}$ are the forces from the neighboring substructure (the foundation) and are referred to as the *interface forces*. By now regarding the lowest tower node as being the boundary of the wind turbine model (denoted by subscribt $[b]$) and the rest of the model as being internal (denoted by subscribt $[i]$), the DoF can be partitioned according to

$$\boldsymbol{u}^{(W)} = \begin{bmatrix} \boldsymbol{u}^{(W)^T}_{[i]} & \boldsymbol{u}^{(W)^T}_{[b]} \end{bmatrix}. \tag{2}$$

Note that in a similar fashion one can partition the structural matrices and force vectors accordingly. The model of the offshore foundation (figure 1) is usually also a linear or non-linear FE model, hence its equations of motion are (on a global level) similar to those of the wind turbine.

$$\boldsymbol{M}^{(F)}\ddot{\boldsymbol{u}}^{(F)} + \boldsymbol{p}^{(F)}\left(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}\right) = \boldsymbol{f}^{(F)}(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}) + \boldsymbol{g}^{(F)} \tag{3}$$

For practical applications the mass matrix of the foundation is usually constant. The method proposed in this work is however derived for the more general case of configuration dependent mass. The normal approach for coupling models would now be to use *primal* assembly[7] to couple the two models in the same way as one would couple individual finite elements to build a full finite element model. The primal approach would result in a single large model. As this would then require that the aero-elastic code is modified to include more DoF, wave loading, etc, this is not an approach we want to take. A more logical approach would be to use *dual*

assembly[7], where compatibility and equilibrium between the substructures is enforced using Lagrange multipliers. Thereby resulting in the set of coupled equations of motion in Eq. (4).

$$\begin{cases} \boldsymbol{M}^{(W)}\left(\boldsymbol{u}^{(W)}\right)\ddot{\boldsymbol{u}}^{(W)} + \boldsymbol{p}^{(W)}\left(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}\right) + \boldsymbol{B}^{(W)^T}\boldsymbol{\lambda} = \boldsymbol{f}^{(W)}(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}) \\ \boldsymbol{M}^{(F)}\ddot{\boldsymbol{u}}^{(F)} + \boldsymbol{p}^{(F)}\left(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}\right) + \boldsymbol{B}^{(F)^T}\boldsymbol{\lambda} = \boldsymbol{f}^{(F)}(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}) \\ \boldsymbol{B}\boldsymbol{u} = \boldsymbol{0} \end{cases} \tag{4}$$

Here $\boldsymbol{B}^{(\star)}$ is a signed Boolean matrix, that locates the boundary DoF within the full set of DoF, such that:

$$\boldsymbol{u}_{[b]}^{(W)} - \boldsymbol{u}_{[b]}^{(F)} = \boldsymbol{B}^{(W)}\boldsymbol{u}^{(W)} + \boldsymbol{B}^{(F)}\boldsymbol{u}^{(F)} = \boldsymbol{0}, \tag{5}$$

where one can see that this relation thus enforces compatibility between the two subsystems. Equilibrium at the interface is satisfied by introducing Lagrange multipliers, $\boldsymbol{\lambda}$, that represent the interface force intensities.

$$\begin{bmatrix} \boldsymbol{g}^{(W)} \\ \boldsymbol{g}^{(F)} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{B}^{(W)^T} \\ \boldsymbol{B}^{(F)^T} \end{bmatrix}\boldsymbol{\lambda}, \tag{6}$$

Finally, Eq. (4) is written in a compact block-matrix format.

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{p} + \boldsymbol{B}^T\boldsymbol{\lambda} = \boldsymbol{f} \\ \boldsymbol{B}\boldsymbol{u} = \boldsymbol{0} \end{cases}, \tag{7}$$

where,

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}^{(W)}\left(\boldsymbol{u}^{(W)}\right) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}^{(F)}\left(\boldsymbol{u}^{(F)}\right) \end{bmatrix},\ \boldsymbol{p} = \begin{bmatrix} \boldsymbol{p}^{(W)}\left(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}\right) \\ \boldsymbol{p}^{(F)}\left(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}\right) \end{bmatrix},\ \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}^{(W)} \\ \boldsymbol{u}^{(F)} \end{bmatrix},$$
$$\boldsymbol{f} = \begin{bmatrix} \boldsymbol{f}^{(W)}\left(\dot{\boldsymbol{u}}^{(W)}, \boldsymbol{u}^{(W)}\right) \\ \boldsymbol{f}^{(F)}\left(\dot{\boldsymbol{u}}^{(F)}, \boldsymbol{u}^{(F)}\right) \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}^{(W)} & \boldsymbol{B}^{(F)} \end{bmatrix}$$

The compact form given in Eq. (7) will be used throughout this paper.

2.2 Performing simulations using the dual assembled system of equations

In order to solve the dually assembled set of equations of motion, as is given in Eq. (7), one needs to first discretize the equations in time, as will be shown in section 2.2.1. The next step is to solve the discretized nonlinear equations, which is done by means of a Newton-Raphson process, as is shown in section 2.2.2. Using the Newton-Raphson process, one iterates within a time step to obtain equilibrium between all the internal inertia, damping and elastic forces and the external forces.

2.2.1 Time discretization using the generalized-α method

As the Newmark time discretization[4], in which a Taylor expansion is used to deduce the velocities and displacements of the next time station, is the most common for computing the time response of a (nonlinear) finite element model, it will also be applied for the current method. The relation between the accelerations, velocities and displacements between the current and previous time step is given,

$$\begin{aligned} \dot{\boldsymbol{u}}_n &= \dot{\boldsymbol{u}}_{n-1} + (1-\gamma)h\ddot{\boldsymbol{u}}_{n-1} + \gamma h\ddot{\boldsymbol{u}}_n = \dot{\hat{\boldsymbol{u}}}_n + \gamma h\ddot{\boldsymbol{u}}_n \\ \boldsymbol{u}_n &= \boldsymbol{u}_{n-1} + h\dot{\boldsymbol{u}}_{n-1} + h^2(\frac{1}{2}-\beta)\ddot{\boldsymbol{u}}_{n-1} + h^2\beta\ddot{\boldsymbol{u}}_n = \hat{\boldsymbol{u}}_n + h^2\beta\ddot{\boldsymbol{u}}_n, \end{aligned} \tag{8}$$

where β and γ are the Newmark integration parameters, h the time step size and $\hat{\dot{\boldsymbol{u}}}_n$ and $\hat{\boldsymbol{u}}_n$ are referred to as the *predictors* and are dependent on the displacements, velocities and accelerations of the previous time step. The generalized-α scheme[1] is a generalization of the Newmark scheme according to:

$$(1-\alpha_m)\boldsymbol{f}_n^{\text{iner}}+\alpha_m\boldsymbol{f}_{n-1}^{\text{iner}}+(1-\alpha_f)\left(\boldsymbol{f}_n^{\text{damp}}+\boldsymbol{f}_n^{\text{elas}}-\boldsymbol{f}_n^{\text{ext}}\right)+\alpha_f\left(\boldsymbol{f}_{n-1}^{\text{damp}}+\boldsymbol{f}_{n-1}^{\text{elas}}-\boldsymbol{f}_{n-1}^{\text{ext}}\right)=\boldsymbol{0} \quad (9)$$

where, α_f controls the weight for averaging the elastic, damping and external forces and α_m controls the weight for averaging the inertial forces, such that these parameters control the amount of numerical damping of the time integration scheme. If the parameters are chosen such that $\alpha_m \leq \alpha_f \leq \frac{1}{2}$, $\beta = \frac{1}{4}(1-\alpha_m+\alpha_f)^2$ and $\gamma = \frac{1}{2}-\alpha_m+\alpha_f$ the result is an unconditionally stable second-order scheme that introduces numerical damping for the high frequencies while introducing only very small errors in the lowest frequency of interest.. Clearly, if $\alpha_m = 0$, on finds the HHT-α method[3] and if also $\alpha_f = 0$ the method reduces to Newmark's constant average acceleration scheme. As the generalized-α method can be seen as a further generalization of the original Newmark method, this time integration schem will be used throughout this section. It should be noted however, that by changing the α_m, α_f, β and γ parameters used, the method can be used with any of the Newmark time integrations schemes as well.

2.2.2 Time integration of the time discretized equations of motion

The time discretized version of Eq. (7) can be written for a certain time step $t(n) = t_n$.

$$\begin{cases} \boldsymbol{M}_n\ddot{\boldsymbol{u}}_n + \boldsymbol{p}_n + \boldsymbol{B}^T\boldsymbol{\lambda}_n = \boldsymbol{f}_n \\ \boldsymbol{B}\boldsymbol{u}_n = \boldsymbol{0} \end{cases}, \quad (10)$$

Note that in the time discretized equations of motion given here, $\ddot{\boldsymbol{u}}_n, \dot{\boldsymbol{u}}_n, \boldsymbol{u}_n$ are related to each other through the time discretization applied in the Newmark scheme, given in Eq. (8). In order to solve this set of nonlinear equations of motions, Newton-Raphson iterations will be used. The first iterations is then started from an initial guess, that is obtained from substituting the *predictors* in the equations of motion. These predictors are obtained from setting the accelerations to zero for the first iteration ($\ddot{\boldsymbol{u}}_n = \boldsymbol{0}$) and substituting this in Eq. (8), such that the predictors for time t_n are obtained.

$$\begin{aligned} \hat{\dot{\boldsymbol{u}}}_n &= \dot{\boldsymbol{u}}_{n-1} + (1-\gamma)h\ddot{\boldsymbol{u}}_{n-1} \\ \hat{\boldsymbol{u}}_n &= \boldsymbol{u}_{n-1} + h\dot{\boldsymbol{u}}_{n-1} + h^2(\frac{1}{2}-\beta)\ddot{\boldsymbol{u}}_{n-1} \end{aligned} \quad (11)$$

As the set of equations (10) we are trying to solve is nonlinear, a direct solution can not be found. Hence, Eq. (10) needs to be written in a residual form that is consistent with the generalized-α method (see Eq. (9)).

$$\begin{cases} \boldsymbol{r}_n = \boldsymbol{M}_{\alpha_m}\ddot{\boldsymbol{u}}_{\alpha_m} + \boldsymbol{p}_{\alpha_f} + \boldsymbol{B}^T\boldsymbol{\lambda}_{\alpha_f} - \boldsymbol{f}_{\alpha_f} \\ \boldsymbol{B}\boldsymbol{u}_n = \boldsymbol{0} \end{cases}, \quad (12)$$

where,

$$\begin{cases} \boldsymbol{M}_{\alpha_m}\ddot{\boldsymbol{u}}_{\alpha_m} = (1-\alpha_m)\boldsymbol{M}_n\ddot{\boldsymbol{u}}_n + \alpha_m\boldsymbol{M}_{n-1}\ddot{\boldsymbol{u}}_{n-1} \\ \boldsymbol{p}_{\alpha_f} = (1-\alpha_f)\boldsymbol{p}_n + \alpha_f\boldsymbol{p}_{n-1} \\ \boldsymbol{f}_{\alpha_f} = (1-\alpha_f)\boldsymbol{f}_n + \alpha_f\boldsymbol{f}_{n-1} \\ \boldsymbol{\lambda}_{\alpha_f} = (1-\alpha_f)\boldsymbol{\lambda}_n + \alpha_f\boldsymbol{\lambda}_{n-1} \end{cases}$$

In order to solve Eq. (12), the Newton-Raphson scheme is used to minimize the residual. In the Newton-Raphson scheme, one uses the Jacobian ($\boldsymbol{S}$) of a nonlinear function for determining a correction. Or in other words, one uses a sequence of linearized problems to find a solution for the associated nonlinear function. As the interface forces are also unknowns in Eq. (12), one also needs to solve for these, resulting in the Newton-Raphson step shown in Eq. (13).

$$\begin{bmatrix} \boldsymbol{S}^{(W)} & \boldsymbol{0} & \boldsymbol{B}^{(W)^T} \\ \boldsymbol{0} & \boldsymbol{S}^{(F)} & \boldsymbol{B}^{(F)^T} \\ \boldsymbol{B}^{(W)} & \boldsymbol{B}^{(F)} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{u}_n^{(W)} \\ \Delta \boldsymbol{u}_n^{(F)} \\ (1-\alpha_f)\Delta \boldsymbol{\lambda}_n \end{bmatrix} = \begin{bmatrix} -\boldsymbol{r}_n^{(W)} \\ -\boldsymbol{r}_n^{(F)} \\ \boldsymbol{0} \end{bmatrix}, \tag{13}$$

where,

$$\boldsymbol{S}^{(s)} = (1-\alpha_f)\left(\boldsymbol{K}_t^{(s)} + \boldsymbol{C}_t^{(s)}\frac{\gamma}{h\beta}\right) + (1-\alpha_m)\left(\boldsymbol{M}^{(s)}\frac{1}{h^2\beta} + \frac{\partial \boldsymbol{M}^{(s)}}{\partial \boldsymbol{u}^{(s)}}\ddot{\boldsymbol{u}}^{(s)}\right), \tag{14}$$

and

$$\boldsymbol{K}_t = \frac{\partial \boldsymbol{p}^{(s)}}{\partial \boldsymbol{u}^{(s)}} - \frac{\partial \boldsymbol{f}^{(s)}}{\partial \boldsymbol{u}^{(s)}}, \quad \boldsymbol{C}_t = \frac{\partial \boldsymbol{p}^{(s)}}{\partial \dot{\boldsymbol{u}}^{(s)}} - \frac{\partial \boldsymbol{f}^{(s)}}{\partial \dot{\boldsymbol{u}}^{(s)}}, \quad \text{for } s = W, F \ ,$$

which are the tangent stiffness ($\boldsymbol{K}_t$) and tangent damping ($\boldsymbol{C}_t$) matrices. Using the computed updates, an updated set of displacements, velocities and accelerations for both substructures can be computed. In addition to this, the interface forces are also updated by $\Delta\boldsymbol{\lambda}$.

$$\begin{aligned} \boldsymbol{u}_n &= \boldsymbol{u}_n + \Delta \boldsymbol{u}_n \\ \dot{\boldsymbol{u}}_n &= \dot{\boldsymbol{u}}_n + \frac{\gamma}{\beta h}\Delta \boldsymbol{u}_n \\ \ddot{\boldsymbol{u}}_n &= \ddot{\boldsymbol{u}}_n + \frac{1}{\beta^2 h}\Delta \boldsymbol{u}_n \\ \boldsymbol{\lambda}_n &= \boldsymbol{\lambda}_n + \Delta \boldsymbol{\lambda} \end{aligned} \tag{15}$$

By substituting the new set of accelerations, velocities and displacement into the residual equation given in Eq. (12), the updated residual is found. One continues this process until the residual is smaller than a pre-set tolerance, after which one starts the computations for the next time step.

3 Substructuring the time integration in the aero-elastic code

Most of the aero-elastic codes are specialized pieces of code, which have been developed over the last two decades. As the goal of the method presented in this paper is to separate the time integration schemes for the wind turbine and foundation substructures within an existing aero-elastic code, the desire is to minimize the changes to the current implementation of the time integration scheme in the aero-elastic code. The approach is on a high level visualized in figure 2, where it is clear that both substructures are modeled and time integrated in different programs, but are coupled through the Newton-Raphson iterations.

The equations to solve in case of a global Newton-Raphson iteration are given in Eq. (13). Instead of solving this block of equations directly, one can also solve it by a number of intermediate steps. By using the effective stiffness matrix and the residual of the foundation model, the "uncoupled" part of the foundation's response is computed as the first step.

$$\Delta \tilde{\boldsymbol{u}}_n^{(F)} = -\boldsymbol{S}^{(F)^{-1}}\boldsymbol{r}_n^{(F)}, \tag{16}$$

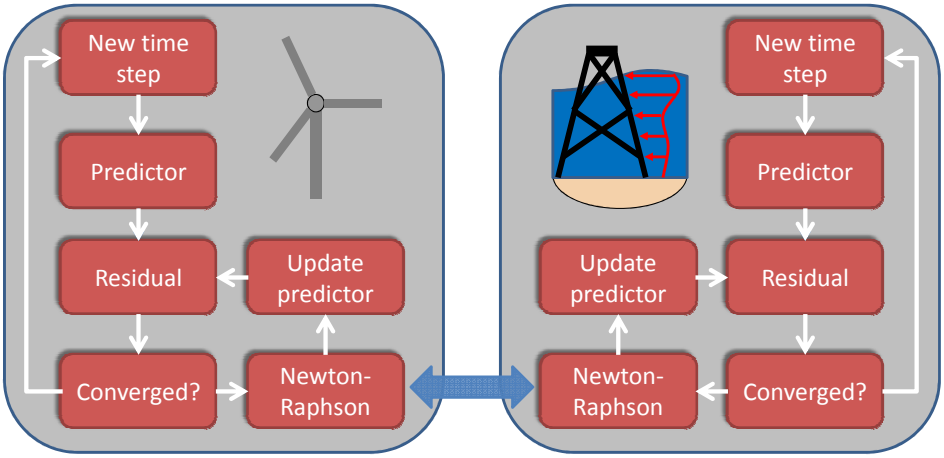

Figure 2: Coupling the time integration schemes

At this first iteration no update on the displacements has been computed for the wind turbine yet, therefore $\Delta \boldsymbol{u}_n^{(B)} = \boldsymbol{0}$. By substituting the "uncoupled" response of the foundation ($\Delta \tilde{\boldsymbol{u}}_n^{(F)}$) and $\Delta \boldsymbol{u}_n^{(B)} = \boldsymbol{0}$ into the compatibility equation on the last line in Eq. (13), one obtains a first estimate of the interface force.

$$(1-\alpha_f)\Delta\tilde{\boldsymbol{\lambda}}_n = \left(\boldsymbol{B}^{(F)}\boldsymbol{S}^{(F)^{-1}}\boldsymbol{B}^{(F)^T}\right)^{-1}\left(\boldsymbol{B}^{(F)}\Delta\tilde{\boldsymbol{u}}_n^{(F)}\right) \tag{17}$$

This force can be interpreted as a the interface force required to set the interface displacements of the foundation substructure to zero, thus effectively being a constraint force. By updating the residual of the wind turbine substructure with this constraint force

$$\boldsymbol{r}_n^{(W)+} = \boldsymbol{r}_n^{(W)} + (1-\alpha_f)\boldsymbol{B}^{(W)^T}\Delta\boldsymbol{\lambda}_n, \tag{18}$$

one transfers the effect of the residual of the foundation structure, to the wind turbine model. By assembling the equivalent effective interface stiffness of the foundation into the effective stiffness matrix of the foundation, one finds the update on the displacements for the turbine model.

$$\left(\boldsymbol{S}^{(W)} + \boldsymbol{B}^{(W)^T}\left(\boldsymbol{B}^{(F)}\boldsymbol{S}^{(F)^{-1}}\boldsymbol{B}^{(F)^T}\right)^{-1}\boldsymbol{B}^{(W)}\right)\Delta\boldsymbol{u}_n^{(W)} = -\boldsymbol{r}_n^{(W)+} \tag{19}$$

Now, by substituting $\Delta\tilde{\boldsymbol{u}}_n^{(F)}$ and $\Delta\boldsymbol{u}_n^{(W)}$ into the compatibility equation of Eq. (13), one finds the correct update on the interface forces.

$$(1-\alpha_f)\Delta\boldsymbol{\lambda}_n = \left(\boldsymbol{B}^{(F)}\boldsymbol{S}^{(F)^{-1}}\boldsymbol{B}^{(F)^T}\right)^{-1}\left(\boldsymbol{B}^{(W)}\Delta\boldsymbol{u}_n^{(W)} + \boldsymbol{B}^{(F)}\Delta\tilde{\boldsymbol{u}}_n^{(F)}\right) \tag{20}$$

The update for the interface force is now used to find the coupled response of the foundation model.

$$\Delta\boldsymbol{u}_n^{(F)} = \Delta\tilde{\boldsymbol{u}}_n^{(F)} - \boldsymbol{S}^{(F)^{-1}}\boldsymbol{B}^{(F)^T}\Delta\boldsymbol{\lambda}_n \tag{21}$$

After which the displacements, velocities and accelerations for both substructures, as well as the interface forces are updated, as is shown in Eq. (15). The different steps that have been presented in the equations of this section, have been visualized in figure 3. Here it can be seen that for each iteration, the interface displacements of the wind turbine are transferred to the simulation of the foundation, the equivalent effective interface stiffness of the foundation

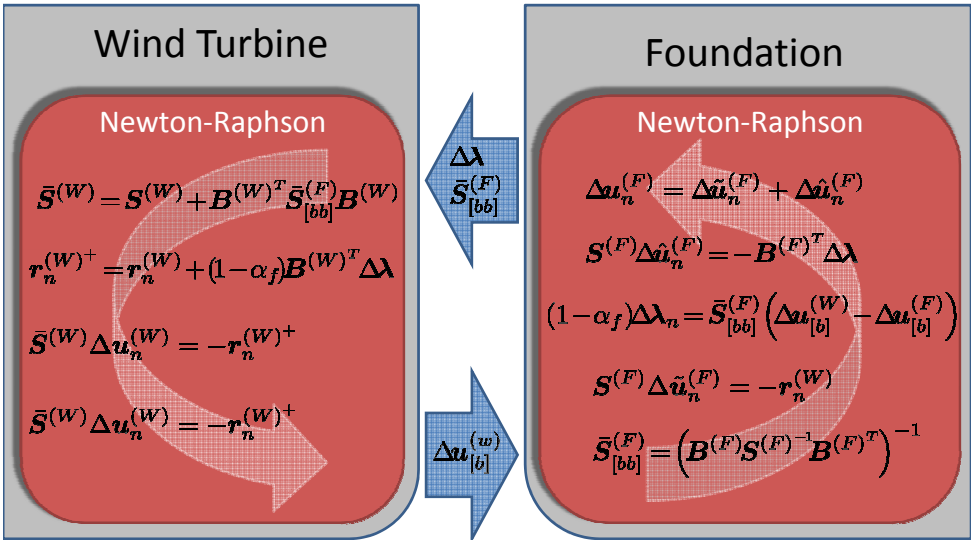

Figure 3: Data exchange between the two programs

and the resulting interface forces are transferred back to the wind turbine simulation, thereby allowing one to find the coupled response.

It should be noted that, as one in fact only reorganizes the linearized problem posed in Eq. (13) over two separate Newton-Raphson schemes, the result found will be (within numerical precision) exactly identical. This also means that the substructured time integration scheme has the same accuracy and stability as one would obtain from solving the associated unsubstructured time integration. A clear disadvantage of the proposed approach is the fact the the method is a sequential method; i.e. the substructures have to "wait" for each other, thereby limiting the efficiency of the method.

4 First results of a verification study

The method is implemented into the aero-elastic code BHawC[5] (Bonus Energy Horizontal axis wind turbine Code), developed at Bonus Energy A/S and Siemens Wind Power A/S. In the current verification study, the model of a SWT-3.6-120 with a 87.5 meter tower is installed on the simplest possible foundation model; a single Euler beam element with a length of 20 m, a diameter of 6 m and a wall thickness of 50mm. The wind turbine part is time integrated in BHawC, the foundation element is modeled and integrated in Matlab and both time integrations are coupled through the Matlab COM Engine using the method presented in section 3.

A single time simulation is performed with a mean wind speed at hub height of 10 m/s, including turbulence and shear effects. In this test case, no wave forces are applied at the single beam element foundation structure. The simulation is started from a quasi-static equilibrium, followed by a 20 second dynamic initialization, which is performed to damp out any transients due to the initialization. After initialization, the simulations are performed for 600 seconds real time, with a step size of 0.02 seconds.

As currently, the results are not yet verified through an independent second model, only the results of the substructured model are shown. Still, a number of results provide valuable information. Firstly, the compatibility between the two substructures can be checked, as no gap between the models is allowed. In figure 4 the interface displacements in y-direction (parallel to the wind direction) of both substructures is shown. As the largest relative error on the compatibility that is found is in the same order as the displacement tolerance in the Newton-Raphson

iteration (10^{-5}), it can be regarded as numerically exact. Another important observation here is that, as the wind turbine substructure is unconstraint, it would be floating in space if it was not coupled to the foundation substructure, which is clearly not the case. In addition to this,

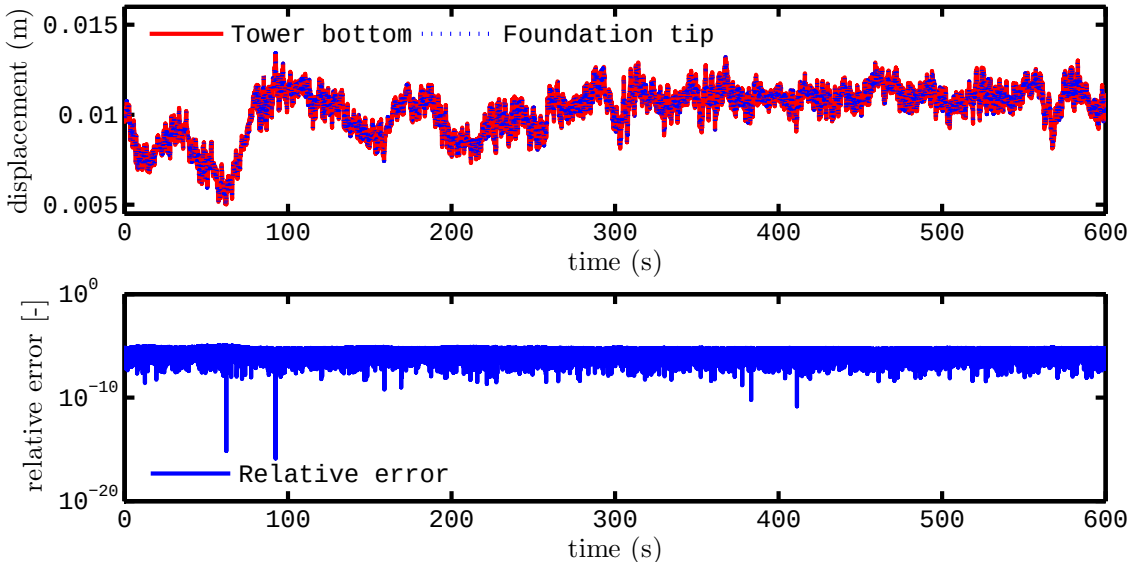

Figure 4: a. Interface displacement in y-direction on both sides of the interface; b. Relative error on the compatibility condition

the number of iterations per time step can be evaluated, as a large number of iterations could indicate that there are issues with the method. From figure 5 it can be seen that the number of required iterations per time step is relatively small and constant. These results give great

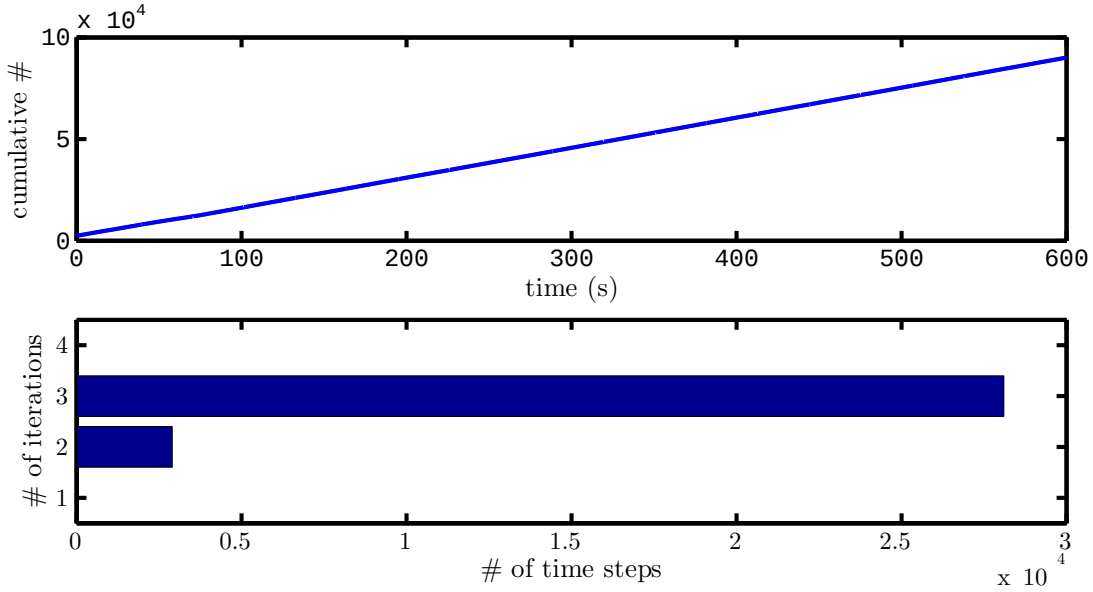

Figure 5: a. Cumulative number of iterations; b. Histogram of the number of iteration per time step

confidence in the method and its implementation in the aero-elastic code. Obviously the results and accuracy will need to be verified by a second independent set of results obtained using a

traditional, un-substructured, approach. The work on this verification has recently started and it is expected to yield results in the near future.

5 Summary and future work

In this paper a method was presented for coupling two separate simulations to obtain the coupled responses of an offshore wind turbine and its foundation. Only small changes in the different steps of the time integration scheme are required, thereby leading to an easy implementation in existing simulation software. In addition to this, as long as the coupling between the different substructures is only in a limited number of DoF, very limited amounts of data need to be exchanged between the different programs. From the theory it is shown that the method has no effect on the stability and the accuracy of the underlying Newmark time integration methods. The biggest disadvantage however, is that the method is a sequential (or staggered) scheme, thereby limiting the computational efficiency.

The first results presented in this paper indicate that the method has been applied successfully. These results will need to be verified by a second set of results that is obtained from an un-substructure time integration approach in order to give absolute confidence in the method and its implementation.

Important next steps for this work will be to generalize the method in order to be able to couple more then two subsystems and in addition to this, to extend it into a parallel method. If both steps are taken, it is expected that this will generate a efficient framework for coupling different modeling and simulation tools.

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