11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

ANALYTICAL SOLUTION FOR WAVE PROPAGATION IN A VISCOELASTIC WALL STRIP UNDER TRANSVERSE IMPACT LOADING

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Keywords: Wave Propagation, Analytical Solution, Viscoelastic Strip, Transverse Impact, Numerical Inverse Laplace Transform.

Abstract. *This work concerns the investigation of non-stationary waves propagated in a viscoelastic strip of infinite length. The strip is subjected to a transverse pressure load the amplitude of which is described by cosine function in spatial domain and by Heaviside function in time. Using the exact 2D theory, the equations of motion are derived for the standard model of linear viscoelastic solid. The system of two integro-differential equations is then solved by means of integral transforms and the exact formulae for the Laplace transforms of relevant displacement components are derived. The inverse Laplace transform is then performed by means of a numerical algorithm based on FFT and Wynn's epsilon accelerator. To prevent the precision and the quality of the solution derived, multi-precision implementation of this algorithm was performed in Maple, so the results obtained can be considered of analytical accuracy. Finally, these results are compared to numerical ones obtained by the help of finite element software MSC.Marc/Mentat. It is clear from presented results that very good agreement between both solutions was achieved, both close and far from the area of excitation. Presented analytical solution can be utilised in problems of optimal design of structures with viscoelastic layers subjected to impact loading.*

1 INTRODUCTION

The elastodynamic problems of strips of finite or infinite length under different boundary conditions have been studied by many authors since the mid of the 20th century. Existing works can be divided into two groups in general. The first one contains studies concerning strips with cracks or cavities. Different methods are used for the investigation of stationary or transient stress fields in such cases. Itou in [1] used the combination of integral transform method with Schmidt's method to solve the scattering of shock waves by a cylindrical cavity in an infinite elastic strip. Ghosh and Mandal in [2] dealt with a diffraction problem of P-wave by a specific type of crack in a long elastic strip. By using the Fourier transform and the numerical solution of a Fredholm integral equation, stress intensity factors are calculated in this work.

In the second case, the authors usually investigate the attenuation behaviour of elastic layered strips in view of stationary and non-stationary waves propagated. In [3] the analytical solution to the problem of an infinite two-layered elastic strip subjected to a transient loading is presented. Using the formulae derived by means of the method of characteristics, the designs that minimise the stress amplitude are proposed for different layered structures. The problem of optimal design of a layered elastic strip is solved also in [4]. For several specific layered strips with free-fixed boundaries, the authors derived a stress transfer function for the case of an impulsive loading. The material properties and the thickness of layers are then discussed in relation to the attenuation of stress waves propagated.

As in all papers mentioned above, this work concerns the solution to a problem of stress waves propagated in an infinite strip by means of an analytical approach. But contrary to those, a viscoelastic homogeneous isotropic strip will be investigated. Using the method of integral transform, the transient response of the strip to a specific type of pressure loading is solved. As in [5], where the same problem for an elastic strip is solved, the Laplace and Fourier transforms are used for this purpose. When the formulae for the transforms of displacement components are derived, the numerical inverse Laplace transform is utilised as in [1], but multi-precision computations in Maple were used to avoid the loss of the solution accuracy. Finally, obtained results are compared with those of numerical simulation performed in finite element code MSC.Marc/Mentat.

2 PROBLEM DESCRIPTION

The problem solved is schematically depicted in Figure 1. We will assume an infinite strip of the total height $2d$ made of an isotropic homogeneous linear viscoelastic material the properties of which will be described by the discrete model of standard solid in Zener's configuration [6]. Let us denote the parameters of elastic elements of the model in conventional way, i.e. E_i, G_i and μ_i, where the subscripts $i = 1, 2$ correspond to the alone standing spring and to the spring in series with the dashpot in the model, respectively. The viscous material properties will be described by the coefficients of normal λ and shear η viscosities and by the Poisson's ratio ν. Hereinafter we will assume that $\nu = \mu_2$ for simplicity.

As in [5], the upper edge of the strip is subjected to a pressure load which is non-zero only in an area of the length $2h$ (see Figure 1). We will suppose that the amplitude of applied load is constant through the strip thickness and is given by the cosine function in the strip longitudinal direction. This cosine variation approximately corresponds to the contact pressure distribution in the case of a strip impacted by a cylindrical impactor with a spherical nose. Further, let the Heaviside function describes the variation of the pressure in time. With respect to the coordinate system depicted in Figure 1 and with respect to mentioned assumptions, the boundary condition

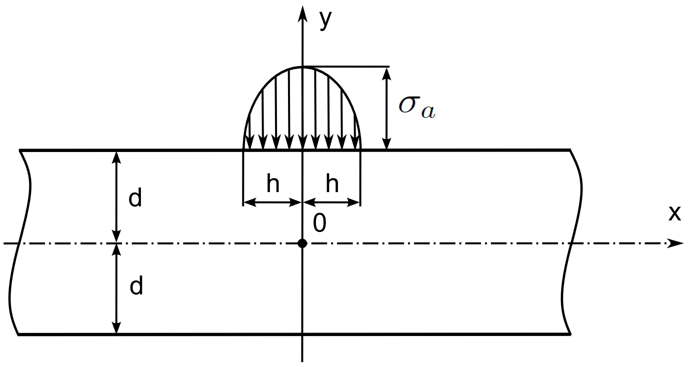

Figure 1: The scheme of the problem.

for the shear stress τ_{xy} and for the normal stress σ_y can be then expressed in the form

$$\tau_{xy}(x,d,t) = 0\,, \quad \tau_{xy}(x,-d,t) = 0\,, \quad \sigma_y(x,-d,t) = 0\,, \tag{1}$$
$$\sigma_y(x,d,t) = \begin{cases} -\sigma_a \cos\!\left(\frac{\pi}{2}\frac{x}{h}\right) H(t) & \text{for } x \in \langle -h, h\rangle, \\ 0 & \text{otherwise}\,, \end{cases}$$

where σ_a denotes the pressure amplitude and $H(t)$ is the Heaviside function in time. Finally, initial conditions for displacement components and their time derivatives will be assumed zero for simplicity.

Considering the geometry and the boundary conditions specified, the plane stress problem in xy plane will be solved and the functions $u(x,y,t)$ and $v(x,y,t)$ describing the displacement in x and y directions, respectively, will be looked for.

3 ANALYTICAL SOLUTION OF THE PROBLEM

3.1 Governing equations

Let us introduce the governing equations needed for the problem formulation. First, the kinematic equations for the theory of small-deformation can be written as

$$\varepsilon_x = \frac{\partial u}{\partial x}\,, \quad \varepsilon_y = \frac{\partial v}{\partial y}\,, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\,, \tag{2}$$

where the functions $\varepsilon_x(x,y,t)$, $\varepsilon_y(x,y,t)$ and $\gamma_{xy}(x,y,t)$ are the normal and shear strain components, respectively.

The equations of motion of two-dimensional continuum have the well known form [7]

$$\rho\,\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\,, \quad \rho\,\frac{\partial^2 v}{\partial t^2} = \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\,, \tag{3}$$

while the normal and shear stress components are denoted by the functions $\sigma_x(x,y,t)$, $\sigma_y(x,y,t)$ and $\tau_{xy}(x,y,t)$. The constant ρ in Eqs. (3) represents the material density.

The constitutive equations for standard viscoelastic solid can be derived based on the superposition principle [6]. Superimposing the strain rates and the stresses for the elements in series and for the parallel branches of the Zener's model, respectively, one can express the non-zero stress components as

$$\sigma_x = \sum_{i=1}^{2} \frac{E_i}{1-{\mu_i}^2}\,(\varepsilon_x + \mu_i\varepsilon_y) - \frac{{E_2}^2}{\lambda\,(1-{\mu_2}^2)} \int_0^t (\varepsilon_x + \mu_2\varepsilon_y)\, e^{-\frac{E_2}{\lambda}(t-\tau)} d\tau\,, \tag{4}$$

$$\sigma_y = \sum_{i=1}^{2} \frac{E_i}{1-{\mu_i}^2}\,(\varepsilon_y + \mu_i\varepsilon_x) - \frac{{E_2}^2}{\lambda\,(1-{\mu_2}^2)} \int_0^t (\varepsilon_y + \mu_2\varepsilon_x)\, e^{-\frac{E_2}{\lambda}(t-\tau)} d\tau\,,$$

$$\tau_{xy} = (G_1 + G_2)\,\gamma_{x,y} - \frac{{G_2}^2}{\eta} \int_0^t \gamma_{xy} e^{-\frac{G_2}{\eta}(t-\tau)} d\tau\,.$$

The integrals in Eqs. (4) characterise the memory effect of the material and are related to its damping behaviour.

Next, it is useful to introduce the dilatation $s(x, y, t)$ and the rotation $r(x, y, t)$ for the plane stress problem as in [5]. There hold

$$s = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\,, \quad r = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\,. \tag{5}$$

3.2 Equations of motion

Substituting Eqs. (2) and (4) into Eqs. (3) and introducing Eqs. (5), the equations of motion can be expressed in the form

$$\frac{\partial^2 u}{\partial t^2} = \left(c_{21}^2 + c_{22}^2\right)\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x \partial y}\right) + \left(c_{31}^2 + c_{32}^2\right)\frac{\partial s}{\partial x} \tag{6}$$

$$-\alpha \int_0^t \left(c_{32}^2 \frac{\partial s}{\partial x} - 2c_{22}^2 \frac{\partial^2 v}{\partial x \partial y}\right) e^{-\alpha(t-\tau)} d\tau - \beta c_{22}^2 \int_0^t \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) e^{-\beta(t-\tau)} d\tau\,,$$

$$\frac{\partial^2 v}{\partial t^2} = \left(c_{21}^2 + c_{22}^2\right)\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right) + \left(c_{31}^2 + c_{32}^2\right)\frac{\partial s}{\partial y}$$

$$-\alpha \int_0^t \left(c_{32}^2 \frac{\partial s}{\partial y} - 2c_{22}^2 \frac{\partial^2 u}{\partial x \partial y}\right) e^{-\alpha(t-\tau)} d\tau - \beta c_{22}^2 \int_0^t \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) e^{-\beta(t-\tau)} d\tau\,,$$

where the constants α, β and c_{ij} ($i = 2, 3,\ j = 1, 2$) are defined by

$$\alpha = \frac{E_2}{\lambda}\,, \quad \beta = \frac{G_2}{\eta}\,, \quad c_{2j} = \sqrt{\frac{G_j}{\rho}} \quad \text{and} \quad c_{3j} = \sqrt{\frac{E_j}{\rho\left(1-\mu_j^2\right)}} \quad \text{for} \quad j = 1, 2. \tag{7}$$

These parameters represent the reciprocals of the normal and shear relaxation times and the constants analogous to the phase velocities of rotational (shear) waves in general continuum and of dilatational waves in two-dimensional continuum, in sequence.

Since there hold the relation between the viscosity coefficients λ and η analogous to those for the elastic constants E_2 and G_2, i.e. $\lambda/\eta = 2(1 + \mu_2)$, the parameters α and β are equal (see [6]). Using this equality and the relations Eqs. (5), the coupled system Eqs. (6) for the unknown functions $u(x, y, t)$ and $v(x, y, t)$ can be rewritten to an uncoupled system

$$\frac{\partial^2 s}{\partial t^2} = \left(c_{31}^2 + c_{32}^2\right)\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right) - \alpha c_{32}^2 \int_0^t \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right) e^{-\alpha(t-\tau)} d\tau\,, \tag{8}$$

$$\frac{\partial^2 r}{\partial t^2} = \left(c_{21}^2 + c_{22}^2\right)\left(\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2}\right) - \alpha c_{22}^2 \int_0^t \left(\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2}\right) e^{-\alpha(t-\tau)} d\tau\,,$$

in which the dilatation $s(x, y, t)$ and the rotation $r(x, y, t)$ are the required functions. Comparing Eqs. (8) with the appropriate equations for elastic problem [5], i.e.

$$\frac{\partial^2 s}{\partial t^2} = c_3^2\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right), \quad \frac{\partial^2 r}{\partial t^2} = c_2^2\left(\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2}\right), \tag{9}$$

where c_2 and c_3 are the velocities of shear waves and of dilatational waves in two-dimensional elastic medium, respectively, the analogy is obvious. Eqs. (8) together with conditions Eqs. (1) represent the final system of equations of motion which have to be solved.

3.3 Solution procedure and the Laplace transforms of displacement components

Several analytical methods to solve the boundary value problem formulated in the previous subsection can be found in literature. With respect to the previous works published by the authors (see e.g. [8]-[10]), the integral transform method was used to solve Eqs. (8) with constraints Eqs. (1). In particular, the Laplace transform in time and the Fourier transform in spatial domain were used. Here we will present only the basic ideas of the solution procedure.

First, the Laplace transform is applied to the system Eqs. (8). Taking into account the zero initial conditions and denoting the Laplace transforms of dilatation and rotation as $\bar{s}(x, y, p)$ and $\bar{r}(x, y, p)$, respectively, where p is a new complex variable, the transformed equations of motion can be written as

$$\bar{s} = \left(\frac{C_3}{p}\right)^2\left(\frac{\partial^2 \bar{s}}{\partial x^2} + \frac{\partial^2 \bar{s}}{\partial y^2}\right), \quad \bar{r} = \left(\frac{C_2}{p}\right)^2\left(\frac{\partial^2 \bar{r}}{\partial x^2} + \frac{\partial^2 \bar{r}}{\partial y^2}\right), \tag{10}$$

where the complex functions $C_2(p)$ and $C_3(p)$ are defined by relations

$$C_2(p) = \sqrt{\left(1 - \frac{\alpha}{p+\alpha}\right) c_{22}^2 + c_{21}^2} \quad \text{and} \quad C_3(p) = \sqrt{\left(1 - \frac{\alpha}{p+\alpha}\right) c_{32}^2 + c_{31}^2}\,. \tag{11}$$

Now let us assume that the solution of the uncoupled system of Eqs. (10) can be expressed in the form of Fourier integrals. Since the external pressure load is even function of the coordinate x, the displacements u and v will be described by odd and even functions of x, respectively. Then, using the relations Eqs. (5), it is easy to show that the dilatation $s(x, y, t)$ and the rotation $r(x, y, t)$, as well as their Laplace transforms, will be even and odd in x, respectively. Hence, one can write

$$\bar{s}(x, y, p) = \frac{1}{\pi}\int_0^\infty A(\omega, y, p)\cos(\omega x) d\omega\,, \quad \bar{r}(x, y, p) = \frac{1}{\pi}\int_0^\infty B(\omega, y, p)\sin(\omega x) d\omega\,. \tag{12}$$

Substituting Eqs. (12) into the transformed equations of motion Eqs. (10) we obtain a system of two uncoupled ODEs for the functions $A(\omega, y, p)$ and $B(\omega, y, p)$

$$\left(p^2 + \omega^2 C_3(p)^2\right) A(\omega, y, p) - C_3(p)^2 \frac{\partial^2}{\partial y^2} A(\omega, y, p) = 0\,, \tag{13}$$
$$\left(p^2 + \omega^2 C_2(p)^2\right) B(\omega, y, p) - C_2(p)^2 \frac{\partial^2}{\partial y^2} B(\omega, y, p) = 0\,,$$

the general solution of which can be written as

$$A(\omega, y, p) = A_1 \cosh(\Lambda_1(\omega, p)\, y) + A_2 \sinh(\Lambda_1(\omega, p)\, y)\,, \tag{14}$$
$$B(\omega, y, p) = B_1 \cosh(\Lambda_2(\omega, p)\, y) + B_2 \sinh(\Lambda_2(\omega, p)\, y)\,.$$

The complex functions $\Lambda_1(\omega, p)$ and $\Lambda_2(\omega, p)$ are defined as

$$\Lambda_1(\omega, p) = \sqrt{\omega^2 + \frac{p^2}{C_3(p)^2}}\,, \quad \Lambda_2(\omega, p) = \sqrt{\omega^2 + \frac{p^2}{C_2(p)^2}}\,, \tag{15}$$

and A_i, B_i ($i = 1, 2$) in Eqs. (14) are the constants of integration. Substituting Eqs. (14) into Eqs. (12) we obtain the formulae for the Laplace transforms of dilatation and rotation. The required functions of the Laplace transforms $\bar{u}$ and $\bar{v}$ can be then simply derived by using the relations Eqs. (5).

The last step in the solution procedure consists in the determination of A_i, B_i for $i = 1, 2$. The boundary conditions Eqs. (1) will be used for this purpose. First, using Eqs. (4) and Eqs. (2) the boundary conditions are rewritten in terms of displacement components u and v and then transformed into the Laplace domain. Introducing the general formulae for $\bar{u}$ and $\bar{v}$ into the transformed conditions, one obtains a system of four algebraic equations for the unknown functions A_i, B_i ($i = 1, 2$). The solution of this system can be written in the compact form

$$A_1 = -\frac{a(\omega)\, k_5 \sinh(d\, \Lambda_2(\omega, p))}{2pk_{10}}\,, \quad A_2 = \frac{a(\omega)\, k_5 \cosh(d\, \Lambda_2(\omega, p))}{2pk_{11}}\,, \tag{16}$$
$$B_1 = -\frac{a(\omega)\, k_4 \cosh(d\, \Lambda_1(\omega, p))}{2pk_{11}}\,, \quad B_2 = \frac{a(\omega)\, k_4 \sinh(d\, \Lambda_1(\omega, p))}{2pk_{10}}\,,$$

where $a(\omega)$ is the cosine Fourier integral of applied external loading and it is easy to show that

$$a(\omega) = \frac{4\pi h \sigma_a \cos(\omega h)}{\pi^2 - 4\omega^2 h^2}\,. \tag{17}$$

The other functions involved in Eqs. (16) were introduced by formulae

$$k_1(\omega, p) = E_v(p) - \frac{2G_v(p)\, C_3(p)^2 \omega^2}{p^2}\,, \quad k_2(\omega, p) = E_v(p) + 2\left(1 + \frac{C_3(p)^2 \omega^2}{p^2}\right) G_v(p)\,, \tag{18}$$
$$k_3(\omega, p) = \frac{4G_v(p)\, C_2(p)^2 \Lambda_2(\omega, p)\, \omega}{p^2}\,, \quad k_4(\omega, p) = \frac{2G_v(p)\, C_3(p)^2 \Lambda_1(\omega, p)\, \omega}{p^2}\,,$$
$$k_5(\omega, p) = 2\left(1 + \frac{2C_2(p)^2 \omega^2}{p^2}\right) G_v(p)\,, \quad k_6(\omega, p) = \frac{C_3(p)^2 \omega}{p^2}\,,$$

$$k_7(\omega,p)=\frac{2C_2(p)^2\omega}{p^2}\,,\quad k_8(\omega,p)=\frac{C_3(p)^2\Lambda_1(\omega,p)}{p^2}\,,\quad k_9(\omega,p)=\frac{2C_2(p)^2\Lambda_2(\omega,p)}{p^2}\,,$$

$$k_{10}(\omega,p)=-k_2k_5\cosh(d\,\Lambda_1(\omega,p))\sinh(d\,\Lambda_2(\omega,p))+k_3k_4\cosh(d\,\Lambda_2(\omega,p))\sinh(d\,\Lambda_1(\omega,p))\,,$$

$$k_{11}(\omega,p)=k_2k_5\cosh(d\,\Lambda_2(\omega,p))\sinh(d\,\Lambda_1(\omega,p))-k_3k_4\cosh(d\,\Lambda_1(\omega,p))\sinh(d\,\Lambda_2(\omega,p))\,,$$

where the complex functions

$$E_v(p)=\frac{E_1\,\mu_1}{1-\mu_1^2}+\frac{E_2\,\mu_2}{1-\mu_2^2}\left(1-\frac{\alpha}{p+\alpha}\right)\quad\text{and}\quad G_v(p)=G_1+G_2\left(1-\frac{\alpha}{p+\alpha}\right)\tag{19}$$

characterise the material properties of the strip.

The final formulae for the Laplace transforms of required displacement components u and v can be then expressed based on the presented results as

$$\bar{u}(x,y,p)=-\frac{1}{\pi}\int\limits_0^\infty\Big[\Big(A_1\cosh(\Lambda_1(\omega,p)y)+A_2\sinh(\Lambda_1(\omega,p)y)\Big)k_6 \tag{20}$$

$$+\Big(B_1\sinh(\Lambda_2(\omega,p)y)+B_2\cosh(\Lambda_2(\omega,p)y)\Big)k_9\Big]\sin(\omega x)d\omega\,,$$

$$\bar{v}(x,y,p)=\frac{1}{\pi}\int\limits_0^\infty\Big[\Big(A_1\sinh(\Lambda_1(\omega,p)y)+A_2\cosh(\Lambda_1(\omega,p)y)\Big)k_8 \tag{21}$$

$$+\Big(B_1\cosh(\Lambda_2(\omega,p)y)+B_2\sinh(\Lambda_2(\omega,p)y)\Big)k_7\Big]\cos(\omega x)d\omega\,,$$

so that the solution to the problem can be formally written in the form

$$u(x,y,t)=\frac{1}{2\pi i}\int\limits_{Br_1}\bar{u}(x,y,p)\,e^{pt}dp\,,\quad v(x,y,t)=\frac{1}{2\pi i}\int\limits_{Br_1}\bar{v}(x,y,p)\,e^{pt}dp\,. \tag{22}$$

The functions describing the spatio-temporal distribution of other mechanical quantities, e.g. stress or strain components, can be then simply derived by using Eqs. (20) and (22) and relevant relations previously stated.

The evaluation of the Bromwich-Wagner integrals in Eqs. (22) can be done exactly by means of the theorem of residue. This approach involves the calculation of dispersion curves (see [5]). As verified, this is very time-consuming procedure. Therefore, the numerical inverse Laplace transform (NILT) was used in this work. To avoid the loss of the quality of analytical solution, a suitable NILT algorithm was implemented in Maple environment using multi-precision computations [11]. In particular, the FFT based algorithm combined with the Wynn's epsilon accelerator was used. The accuracy and the efficiency of this approach were previously verified by authors for instance in [10]. The analytical results obtained for specific material and geometric parameters are presented in the following section.

4 NUMERICAL SOLUTION AND RESULTS COMPARISON

Numerical results were obtained using the finite element simulation performed in the commercial code MSC.Marc/Mentat. These results helped to check the correctness of the process of the analytical solution derivation and its evaluation, and conversely, the analytical solution was used to verify the capabilities of the finite element model.

The model of the problem was created based on the Figure 1 and Eqs. (1). A strip of the total height $2d = 40\,\mathrm{mm}$ was loaded by a pressure of the amplitude $\sigma_a = 10^6\,\mathrm{Pa}$ in the region

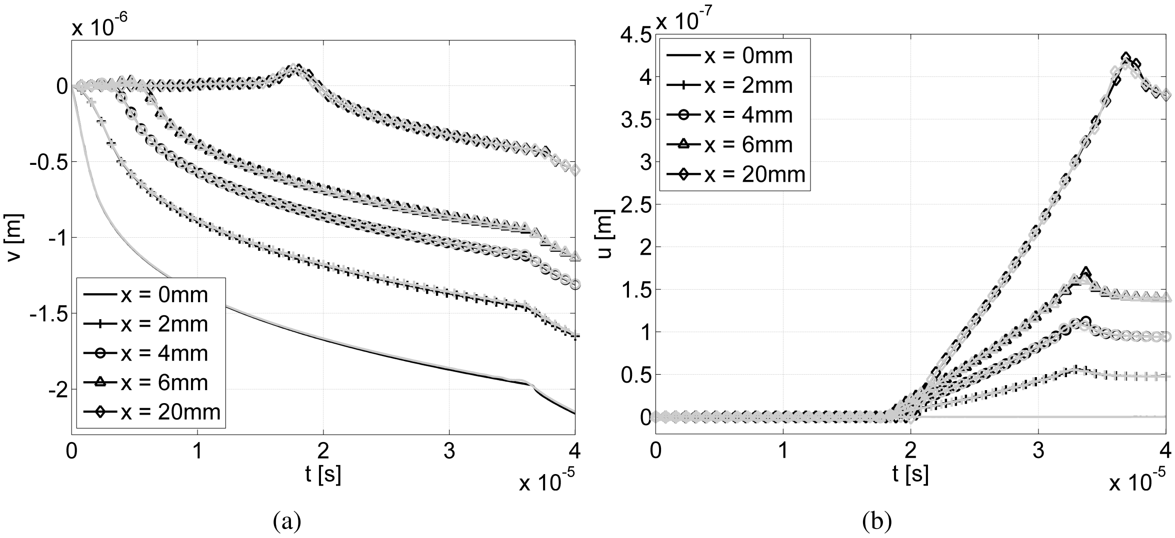

Figure 2: Comparison of analytical (*black lines*) and numerical (*gray lines*) results for different values of x: (a) the vertical displacement v for $y = 20\,\mathrm{mm}$, (b) the horizontal displacement u for $y = -20\,\mathrm{mm}$.

of length $2h = 4\,\mathrm{mm}$. With respect to the problem symmetry, only one half of the strip was modelled using the four-node isoparametric elements and a plane stress problem was solved. The mesh with elements of basic size $0.4 \times 0.4\,\mathrm{mm}$ was once refined near the area of excitation. The material parameters used for the representing of strip properties were taken from [10]: $\rho = 1100\,\mathrm{kg\,m^{-3}}$, $\nu = \mu_1 = \mu_2 = 0.4$, $\eta = 10432.9\,\mathrm{Pa\,s^{-1}}$, $\lambda = 29212.0\,\mathrm{Pa\,s^{-1}}$, $E_1 = 3.603{\times}10^9\,\mathrm{Pa}$ and $E_2 = 7.918{\times}10^8\,\mathrm{Pa}$. The implicit Newmark algorithm with a step $4{\times}10^{-8}\,\mathrm{s}$ was used for integration in time. The value of the integration step was determined based on the mesh size and on the maximal velocity of waves propagated in the strip, analogously as in [12].

The comparison of analytical and numerical results was made in selected points lying at the upper and at the bottom edge of the strip. In particular, the points with $x = \{0, 2, 4, 6, 20\}\,\mathrm{mm}$ were chosen to compare the results both in the vicinity of the external load and relatively far from it. The time histories of u and v were compared only up to $40\,\mu\mathrm{s}$ to avoid the interaction with the waves reflected from the strip end for a strip of finite length was modelled using FEM. It can be said that a great agreement of analytical and numerical results was achieved in all points monitored. This is demonstrated in Figure 2 where the values of the vertical displacement v and the horizontal displacement u for $y = 20\,\mathrm{mm}$ and $y = -20\,\mathrm{mm}$ are compared, respectively. The detailed analysis of results presented in Figure 2(b) has shown that the size of finite elements used is too large to represent relatively steep change of u at time about $t = 35\,\mu\mathrm{s}$, so the mesh of the model should be at least once refined.

5 CONCLUSIONS

The presented work dealt with the investigation of the transient response of an infinite viscoelastic strip under a specific type of pressure load. The system of equations of motion was derived for a strip the viscoelastic properties of which were represented by standard linear viscoelastic solid. Using the classical method of integral transforms, the exact formulae for the Laplace transforms of displacement components were obtained. The inverse Laplace transform back to time domain was carried out by means of FFT-based algorithm with Wynn's epsilon accelerator using multi-precision computations in Maple. Obtained results were then compared with those from numerical simulation performed in the finite element code MSC.Marc/Mentat.

The comparison has shown a great agreement between analytical and numerical results, verifies the correctness of the analytical solution derivation and points out the limitations of the numerical model used.

Presented solution can be used for example as a benchmark for numerical methods and for analysing of the influence of viscoelastic properties on non-stationary waves propagated in the solids of strip-like geometry. Modifying the boundary conditions, the presented solution procedure can be advantageously used in optimal design of impact absorbers, as well.

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