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FREE AND FORCED OSCILLATIONS OF THIN BARS BY THE GENERAL MICROPOLAR THEORY OF ELASTICITY WITH INDEPENDENT FIELDS OF DISPLACEMENTS AND ROTATIONS

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Abstract. *In this paper the hypothesis method based on asymptotical properties of the solution of the initial-boundary problem of the flat micropolar theory of elasticity with independent fields of displacements and rotations in thin rectangle is developed. As a result, the applied dynamic model of micropolar elastic thin bars with independent fields of displacements and rotations with the full account of transverse shift deformations is constructed. The applied dynamic model of micropolar elastic thin bars without the account of transverse shifts is also constructed. On the basis of the constructed models problems on natural and forced oscillations of micropolar elastic bars with the simply supported ends are considered. In the result, the frequencies, forms of natural oscillations, the amplitude of the forced oscillations and resonance conditions for micropolar bars are defined in formulas. The results of numerical calculations demonstrate the specific features of natural oscillations of thin bars of the micropolar elastic material. It is shown that in thin bars of micropolar material there exists proper frequency of acoustic resonance, which is practically independent of the sizes of thin bars, and depends only on the physical and inertial properties of micropolar material. It is also shown that in the choice of micropolar material there is certain possibility to regulate values of low frequencies of bar's oscillations. The latter is essential for the resonance phenomenon. While studying natural oscillations of micropolar elastic thin bars, on the basis of the numerical analysis the constructed models (with and without the account of transverse shifts) are compared with each other and with the classical model. Finally certain conclusions are drawn at and several recommendations are formulated.*

1 INTRODUCTION

Presently, the construction of mathematical models of micropolar elastic thin bars, plates and shells is considered to be one of the most actual ones from the viewpoint of structural mechanics of solid deformable bodies [1]. The review of works in this direction is carried out in work [2].

In works [3-6] the hypothesis method for constructing the static and dynamic deformation models of thin plates and shells is developed, which is based on the mathematical (asymptotical) properties of the solutions of boundary or initial-boundary problems of the micropolar theory of elasticity in thin areas. In the micropolar theories of plates and shells, constructed in works [3-6], the transverse shift and related thereto deformations are completely taken into account. In the given work the mentioned approach is developed and the applied dynamic bend theory of micropolar elastic thin bars with independent fields of displacements and rotations is constructed.

2 PROBLEM STATEMENT

Let us consider an isotropic micropolar elastic parallelepiped of the constant height $2h$, length a and thickness $2h_1=1$. We will place the coordinate plane x_1x_3 in the median plane of the parallelepiped. Axis x_3 is directed along the height, and axis x_1 along the length of parallelepiped which divides height $2h$ into two halves. We will assume that in a parallelepiped a flat intense condition along the direction of axis x_2 is carried out.

The basic equations of the flat dynamic problem (in the area of rectangle $0\le x_1\le a,\ -h\le x_3\le h$) of micropolar theory of elasticity with independent fields of displacements and rotations (or otherwise, by general Cosserat continuum) look as [7]:

The motion equations

$$\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{31}}{\partial x_3}=\rho\frac{\partial^2V_1}{\partial t^2},\quad \frac{\partial\sigma_{13}}{\partial x_1}+\frac{\partial\sigma_{33}}{\partial x_3}=\rho\frac{\partial^2V_3}{\partial t^2},\quad \frac{\partial\mu_{12}}{\partial x_1}+\frac{\partial\mu_{32}}{\partial x_3}-(\sigma_{13}-\sigma_{31})=J\frac{\partial^2\omega_2}{\partial t^2} \tag{1}$$

Physical geometrical relations

$$\begin{gathered}\gamma_{11}=\frac{\partial V_1}{\partial x_1}=\frac{1}{E}(\sigma_{11}-\nu\sigma_{33}),\quad \gamma_{13}=\frac{\partial V_3}{\partial x_1}+\omega_2=\frac{\mu+\alpha}{4\mu\alpha}\sigma_{13}-\frac{\mu-\alpha}{4\mu\alpha}\sigma_{31}\\ \gamma_{33}=\frac{\partial V_3}{\partial x_3}=\frac{1}{E}(\sigma_{33}-\nu\sigma_{11}),\quad \gamma_{31}=\frac{\partial V_1}{\partial x_3}-\omega_2=\frac{\mu+\alpha}{4\mu\alpha}\sigma_{31}-\frac{\mu-\alpha}{4\mu\alpha}\sigma_{13}\\ \chi_{32}=\frac{\partial\omega_2}{\partial x_3}=\frac{1}{B}\mu_{32},\quad \chi_{12}=\frac{\partial\omega_2}{\partial x_1}=\frac{1}{B}\mu_{12}\end{gathered} \tag{2}$$

Here $\sigma_{11},\sigma_{13},\sigma_{31}$ and σ_{33} are force stresses; μ_{12} and μ_{32} are momentum stresses; $\gamma_{11},\gamma_{33},\gamma_{13},\gamma_{31}$ are deformations; χ_{32} and χ_{12} are flexure-torsions; V_1 and V_3 are linear displacements; ω_2 is independent rotation of round rectangle's points over the axis x_2; $E,\ \mu,\alpha$ and B are elastic constants of micropolar body material (E and μ are accordingly the classical elasticity module and the shift module, α and B are the new elastic constants connected with the properties of the micropolar material), J is the measure of inertia in the rotation process of the particles.

On obverse lines $x_3=\pm h$ of the rectangle force and momentum boundary conditions are considered to be set (we will consider the bend problem):

$$\sigma_{31} = p_1, \quad \sigma_{33} = \pm p_3, \quad \mu_{32} = \pm m_2 \tag{3}$$

On the rectangle edges $(x_1 = 0, x_1 = a)$ we will accept the below-mentioned basic variants of boundary conditions of flat problem of the micropolar theory of elasticity:

$$1)\ \sigma_{11} = p_1^*(x_3,t),\ \sigma_{13} = p_3^*(x_3,t),\ \mu_{12} = m_2^*(x_3,t) \tag{4}$$

$$2)\ \sigma_{11} = p_1^*(x_3,t),\ V_3 = V_3^*(x_3,t),\ \mu_{12} = m_2^*(x_3,t) \tag{5}$$

$$3)\ V_1 = V_1^*(x_3,t),\ V_3 = V_3^*(x_3,t),\ \omega_2 = \omega_2^*(x_3,t) \tag{6}$$

On the basis of initial conditions at $t = 0$, values of the components of the displacements V_1, V_3, independent rotation ω_2, linear and rotary speeds vectors components of the body points are set.

It is supposed that the rectangle height is small in comparison with its length $(2h << a)$.

Let us switch to the construction of dynamic bend deformation model of a micropolar bar on the basis of the hypotheses method.

3 THE BASIC HYPOTHESES. MATHEMATICAL DYNAMIC BEND MODELS OF MICROPOLAR ELASTIC THIN BARS

The qualitative results of initial approach of the asymptotic integration method of initial-boundary problem given by (1) - (7) in thin rectangular area [8], allows to formulate the following rather general assumptions (hypotheses) in the basis of the construction of one-dimensional dynamic model of micropolar elastic thin bars with independent fields of displacements and rotations:

1. A normal element, initially perpendicular to the symmetry axis x_1 of a rectangle stays rectilinear after deformation, but this time not perpendicular to the deformed axis, rotating freely at a certain angle without changing its length. Besides, free rotation ω_2 is considered as a constant function over the rectangle height. So we have the linear law for the displacements V_1, V_3 and free rotation ω_2 over the rectangle thickness

$$V_3 = w(x_1,t), \quad V_1 = x_3\psi_1(x_1,t) \tag{7}$$

$$\omega_2 = \Omega_2(x_1,t) \tag{8}$$

Here w is the deflection of the bar, Ω_2 is the angle of the free rotation, ψ_1 is the full angle of the rotation of a normal element.

Note that for the components of displacements vector this hypothesis represents classical hypothesis Timoshenko for elastic bars [9]. On this basis, as in works [3-6], let us call the hypothesis given by the equations (7), (8) the Timoshenko kinematic hypothesis in micropolar bar theory.

We will add kinematic hypothesis (7), (8) to the following static hypotheses:

2. While determining the deformations, flexures-torsions, and strength and momentum stress, for the strength stress σ_{31} we will first assume that

$$\sigma_{31} = \overset{0}{\sigma}_{31}(x_1,t) \tag{9}$$

After definition of the indicated values, we finally determine the value of σ_{3i} as the sum of the value (9) and result obtained by integrating the first motion equation from (1). We require that its value averaged over the plate thickness should be equal to zero.

3. In the generalized Hooke's law (2) we can ignore the force stress σ_{33} in relation to the strength stress σ_{ii}

According to the accepted kinematic hypothesis (7),(8) we will switch to calculating deformations and flexures-torsions.

Considering formulas (2) and taking into account (7),(8) (it is necessary to consider the geometrical equations), for deformations γ_{13}, γ_{31} we will receive:

$$\gamma_{13} = \Gamma_{13}(x_1, t), \ \gamma_{31} = \Gamma_{31}(x_1, t) \tag{10}$$

Here

$$\Gamma_{13} = \frac{\partial w}{\partial x_1} + \Omega_2, \quad \Gamma_{31} = \psi_1 - \Omega_2 \tag{11}$$

Based on equations (2) (considering the geometrical equations), with the account of formulas (7),(8) for deformation γ_{11} we will receive

$$\gamma_{11} = x_3 K_{11}(x_1, t) \tag{12}$$

Here

$$K_{11} = \frac{\partial \psi_1}{\partial x_1} \tag{13}$$

If we consider the same formula (2), but now in view of elasticity laws, and accept formulas (12) and use hypothesis 3), force stress σ_{11} will be expressed as

$$\sigma_{11} = x_3 \hat{\sigma}_{11}(x_1, t), \ \hat{\sigma}_{11} = E K_{11} \tag{14}$$

With the help of equation for deformation (2) (considering the geometrical equations) and on the basis of the formula (7) we will receive

$$\gamma_{33} = 0 \tag{15}$$

Based on equations (2) (considering the physical equations), taking into account the assumption 2), for force stresses σ_{13} and σ_{31}^0 we will have:

$$\sigma_{13} = (\mu + \alpha)\Gamma_{13} + (\mu - \alpha)\Gamma_{31}, \ \sigma_{31}^0 = (\mu + \alpha)\Gamma_{31} + (\mu - \alpha)\Gamma_{13} \tag{16}$$

If we consider the second motion equation from the eq. (1), and take into account assumption 2) and formulas (16), with the account of boundary condition from eq. (3), for force stress σ_{33} we will receive:

$$\sigma_{33} = -x_3 \left(\frac{\partial \sigma_{13}}{\partial x_1} - \rho \frac{\partial^2 w}{\partial t^2} \right) = x_3 \frac{\tilde{p}_3}{h} \tag{17}$$

Considering geometrical equations from (2) and taking into account formula from (8) for the deformation χ_{12} we will receive

$$\chi_{12} = \kappa_{12}(x_1, t) = \frac{\partial \Omega_2}{\partial x_1} \tag{18}$$

Now we will consider the same equation, but this time we will consider the corresponding physical relation. For the momentum stress μ_{12} we will have

$$\mu_{12} = \mu_{12}(x_1, t) = B\kappa_{12} \tag{19}$$

Accepting the geometrical equation from (2) and with the account of relations (8) for χ_{32} we will have

$$\chi_{32} = 0 \tag{20}$$

Let us consider the third motion equation from eq. (1). If we consider eq. (8) and formulas (19) (and also boundary condition from (3)), for momentum stress μ_{32} we will have

$$\mu_{32} = -x_3\left[\frac{\partial \mu_{12}}{\partial x_1} + (\sigma_{31} - \sigma_{13}) - J\,\frac{\partial^2 \Omega_2}{\partial t^2}\right] = x_3\,\frac{\tilde{m}_2}{h} \tag{21}$$

Now, we will pass to the performance of the second part of assumption 3), when all quantities are defined. Adhering to hypothesis 3) for the force tangents stress pressure σ_{31} we will finally have:

$$\sigma_{31} = \sigma_{31}^0(x_1,t) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial \hat{\sigma}_{11}(x_1,t)}{\partial x_1} - \rho\frac{\partial^2 \psi_1}{\partial t^2}\right) \tag{22}$$

Formula (22) provides distribution of force stress σ_{31} over the rectangle height (the square law).

With the view to reducing two-dimensional problem (1), (2) to the applied one-dimensional (which is already executed for deformations, flexure-torsions, displacements, rotations, force and momentum stresses), instead of the force and momentum stress tensor components, we will introduce statically thereto equivalent integral characteristics over the rectangle thickness, that is efforts (N_{13}, N_{31}) and moments (M_{11}, L_{12})

$$N_{13} = \int_{-h}^{h}\sigma_{13}dx_3,\ N_{31} = \int_{-h}^{h}\sigma_{31}dx_3,\ M_{11} = \int_{-h}^{h}\sigma_{11}\alpha_3 dx_3,\ L_{12} = \int_{-h}^{h}\mu_{12}dx_3 \tag{23}$$

As a result, the basic system of the dynamic equations of the bend deformation of micropolar elastic thin bars with independent fields of displacements and rotations is expressed as follows

The movement equations

$$\frac{\partial N_{13}}{\partial x_1} = 2\rho h\frac{\partial^2 w}{\partial t^2} - 2\tilde{p}_3,\quad N_{31} - \frac{\partial M_{11}}{\partial x_1} + \frac{2\rho h^3}{3}\frac{\partial^2 \psi_1}{\partial t^2} = 2h\tilde{p}_1$$
$$\frac{\partial L_{12}}{\partial x_1} + N_{31} - N_{13} = 2Jh\frac{\partial^2 \Omega_2}{\partial t^2} - 2\tilde{m}_2 \tag{24}$$

Elasticity relations

$$N_{13} = 2h[(\mu+\alpha)\Gamma_{13} + (\mu-\alpha)\Gamma_{31}],\quad N_{31} = 2h[(\mu+\alpha)\Gamma_{31} + (\mu-\alpha)\Gamma_{13}]$$
$$M_{11} = \frac{2Eh^3}{3}K_{11},\quad L_{12} = 2Bh\kappa_{12} \tag{25}$$

Geometrical relations

$$\Gamma_{13} = \frac{\partial w}{\partial x_1} + \Omega_2,\quad \Gamma_{31} = \psi_1 - \Omega_2,\quad K_{11} = \frac{\partial \psi_1}{\partial x_1},\quad \kappa_{12} = \frac{\partial \Omega_2}{\partial x_1} \tag{26}$$

The system of the dynamic equations of bend deformations of micropolar elastic thin bars (24) - (26) represents the system of equations of the sixth order, of the hyperbolic type. It is a system with 11 equations in relation to the 11 unknown functions $w, \psi_1, \Omega_2, \Gamma_{13}, \Gamma_{31}, K_{11}, \kappa_{12}, N_{13}, N_{31}, M_{11}, L_{12}$.

The "softened" boundary conditions at the bar edge (for example, on $x_1 = 0$) look as:

$$M_{11} = M_{11}^* \text{ either}, \psi_1 = \psi_1^* \text{ or}, N_{13} = N_{13}^*\ w = w^* \text{ or } L_{12} = L_{12}^* (\Omega_2 = \Omega_2^* 27)$$

It is necessary to insert initial conditions at $t = 0$ for the quantities $w, \psi_1, \Omega_2, \frac{\partial w}{\partial t}, \frac{\partial \psi_1}{\partial t}, \frac{\partial \Omega_2}{\partial t}$.

Transverse shifts and related thereto deformations are completely taken into account in the micropolar bar model (24) - (26) with independent fields of displacements and rotations.

It is easy to bring the system of equations (24) - (26) to the system of equations in relation to w, ψ_1 and Ω_2

$$(\mu-\alpha)\frac{\partial\psi_1}{\partial x_1}+(\mu+\alpha)\frac{\partial^2 w}{\partial x_1^2}+2\alpha\frac{\partial\Omega_2}{\partial x_1}=\rho\frac{\partial^2 w}{\partial t^2}-\frac{\tilde{p}_3}{h}$$

$$(\mu+\alpha)\psi_1+(\mu-\alpha)\frac{\partial w}{\partial x_1}-2\alpha\Omega_2-\frac{Eh^2}{3}\frac{\partial^2\psi_1}{\partial x_1^2}+\frac{\rho h^2}{3}\frac{\partial^2\psi_1}{\partial t^2}=\tilde{p}_1 \tag{28}$$

$$B\frac{\partial^2\Omega_2}{\partial x_1^2}+2\alpha\psi_1-2\alpha\frac{\partial w}{\partial x_1}-4\alpha\Omega_2=J\frac{\partial^2\Omega_2}{\partial t^2}-\frac{\tilde{m}_2}{h}$$

If in the model of micropolar bars (24) - (26) it is conditionally accepted that $\alpha=0$, as a result, the model of the classical Timoshenko theory of elastic thin bars [9] (with some insignificant difference, connected with a static hypothesis 2)) is separated.

If in model of micropolar bars (24) - (26) we neglect transverse shifts i.e. if we will accept that

$$\psi_1=-\frac{\partial w}{\partial x_1} \tag{29}$$

we will receive the model of micropolar bars, in case of which the normal element rotates remaining perpendicular to the bar's deformed axis.
Formula (29) provides that by neglecting transverse shifts, instead of the generalized kinematic Timoshenko hypothesis we will be guided by generalized Bernoulli hypothesis for micropolar bars (i.e. along with (29) we will consider formulas (8) to be right).

Now we can come to the solution of specific problems, i.e. we will study free and forced bend oscillations of micropolar bar on the basis of the constructed dynamic bend models of micropolar elastic thin bars.

4 FREE OSCILLATIONS OF MICROPOLAR ELASTIC THIN BARS

Let us consider the problem on bend free oscillations of simply supported micropolar elastic thin bars on the basis of the general model of micropolar bars with independent fields of displacements and rotations (24) - (26).

The resolving equations of the model (24) - (26) in relation to the displacement w and rotations Ω_2 and ψ_1 look as. (28) (in which it is necessary to accept $\tilde{p}_1=\tilde{p}_3=\tilde{m}_2=0$). To these equations we will attach boundary conditions of hinged edge

$$w=0,\ M_{11}=0,\ L_{12}=0 \text{ at } x_1=0,a \tag{30}$$

For the solution of the boundary problem (28), we will present (30) in the form of:

$$w=A_m^1 e^{ip_m t}\sin\frac{m\pi}{a}x_1,\ \ \Omega_2=A_0^2 e^{ip_0 t}+A_m^2 e^{ip_m t}\cos\frac{m\pi}{a}x_1 \tag{31}$$

$$\psi_1=A_0^3 e^{ip_0 t}+A_m^3 e^{ip_m t}\cos\frac{m\pi}{a}x_1 \tag{32}$$

Here $A_m^1, A_0^2, A_m^2, A_0^3, A_m^3$ are constants, p_0, p_m are frequencies of natural oscillations of micropolar bar with free rotation. Solution (31), (32) completely satisfies boundary conditions (30). Satisfying the differential equations (28), as a result we will receive the homogeneous algebraic equations in relation to A_0^2, A_0^3 and A_m^1, A_m^2, A_m^3. We will demand that the determinants of matrixes corresponding to these algebraic systems of equations are equal to zero.

Then for the definition of natural frequencies p_0 p_m we will receive the following algebraic equations of the fourth and sixth degrees accordingly:

$$p_0^4 - \left[(\mu+\alpha)\frac{3}{\rho h^2} + \frac{4\alpha}{J} \right] p_0^2 + \frac{4\mu\alpha}{J}\frac{3}{\rho h^2} = 0 \tag{36}$$

$$C_{m1} p_m^6 - C_{m2} p_m^4 + C_{m3} p_m^2 - C_{m4} = 0 \tag{37}$$

Here

$$C_{m1} = J\frac{\rho^2 h^2}{3}, C_{m2} = \frac{\rho^2 h^2}{3}\left[B\left(\frac{m\pi}{a}\right)^2 + 4\alpha \right] + J\rho\left[\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 + \mu + \alpha + (\mu+\alpha)\frac{h^2}{3}\left(\frac{m\pi}{a}\right)^2 \right]$$

$$\begin{aligned} C_{m3} &= J\left(\frac{m\pi}{a}\right)^2 \left[4\mu\alpha + (\mu+\alpha)\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 \right] + B\rho\left(\frac{m\pi}{a}\right)^2 \left[\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 + \right. \\ &\left. + (\mu+\alpha) + (\mu+\alpha)\frac{h^2}{3}\left(\frac{m\pi}{a}\right)^2 \right] + 4\alpha\rho\left[\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 + \mu + \mu\frac{h^2}{3}\left(\frac{m\pi}{a}\right)^2 \right] \\ C_{m4} &= B\left(\frac{m\pi}{a}\right)^4 \left[4\mu\alpha + (\mu+\alpha)\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 \right] + 4\mu\alpha\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^4 \end{aligned} \tag{38}$$

Let us also consider the problem on natural oscillations of thin bars on the basis of the model in case of which transverse shifts were neglected (it is necessary to accept $\tilde{p}_1 = \tilde{p}_3 = \tilde{m}_2 = 0$). Boundary conditions of the hinged edge look as (30). Solution of the mentioned boundary problem we will present in the form (31) (for w and Ω_2). For the definition of natural frequencies p_0 and p_m we will receive the following formulas:

$$p_0 = \sqrt{\frac{4\alpha}{J}} \tag{39}$$

$$p_m = \sqrt{\frac{C_{m2} \mp \sqrt{C_{m2}^2 - 4C_{m1}C_{m3}}}{2C_{m1}}} \tag{40}$$

Here

$$\begin{aligned} C_{m1} &= \rho J\left(1 + \frac{h^2}{3}\left(\frac{m\pi}{a}\right)^2\right), C_{m3} = \left(\frac{m\pi}{a}\right)^4 \left[B\frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2 + 4\alpha\left(B + \frac{Eh^2}{3}\right) \right] \\ C_{m2} &= \rho\left(1 + \frac{h^2}{3}\left(\frac{m\pi}{a}\right)^2\right)\left(4\alpha + B\left(\frac{m\pi}{a}\right)^2\right) + J\left(\frac{m\pi}{a}\right)^2\left(4\alpha + \frac{Eh^2}{3}\left(\frac{m\pi}{a}\right)^2\right) \end{aligned} \tag{41}$$

Let us now pass to presenting the results of numerical calculations. It should be noted that, in contrast to classical model, an additional spectrum of natural frequencies is obtained according to the general micropolar model with independent fields of displacements and rotations. This is caused by the free rotation of bar points generating new degrees of freedom.

Let us consider low frequencies on the basis of afore-mentioed micropolar models and the classical model (see Table 1). As far as micropolar model (24) - (26) (with the full account of transverse shift deformations) is considered to be a relatively exact model, therefore we will make the comparison in relation to this model. Low frequency of bar oscillations on the basis of micropolar model (24) - (26) and classical model very strongly differ for values up to $a = 0.3\,m$. At enough massive sizes (from $a = 0{,}5m$ to $a = 0.8м$ and above) results of micropolar and classical model are rather close to each other. Low frequency on the basis of the model in which transverse shift deformations were neglected, at the small sizes ($a \le 0{,}015m$) strongly differs from low frequency on the basis of model (24) - (26) and at the large sizes of a the results are rather close.

Table 1. Frequencies of own oscillations of thin micropolar bars

Physical parameters of the bar's material: α1,6 МПа, μ2 МПа, λ3 МПа, B = 300 H;
Material density: ρ =1114 kg/m^3; an inertia measure at rotation: J=5,31·10^{-6} kg/m; $\delta = h/a = 1/40$

Bar sizes		Classical model of Timoshenko type			Micropolar model without the account of shift			Micropolar model with the account of shifts				
		m=0		m=1	m=0	m=1		m=0		m=1		
a, sm	h, sm	P_0^1, Hz	P_1^1, Hz	P_1^2, Hz	P_0^2, Hz	P_1^1, Hz	P_1^3, Hz	P_0^1, Hz	P_0^2, Hz	P_1^1, Hz	P_1^2, Hz	P_1^3, Hz
1	0,025	46721,1	154,3	46893	174728	3436	414458	45351	180007	2669	60200	414837
2	0,05	23360,6	77,17	23447	174728	1388	256598	23192	175995	1161	27939	256994
4	0,1	11680,3	38,58	11723	174728	449,9	198389	11659	175042	413,8	12706	198601
8	0,2	5840,14	19,29	5861,7	174728	124,4	180934	5837,5	174806	121	6013,3	181004
10	0,25	4672,11	15,43	4689,3	174728	81,1	178725	4670,8	174778	79,56	4769,3	178771
30	0,75	1557,37	5,144	1563,1	174728	10,39	175177	1557,3	174734	10,35	1566,2	175182
40	1	1168,03	3,858	1172,3	174728	6,387	174981	1168	174731	6,366	1173,6	174983
50	1,25	934,422	3,087	937,87	174728	4,491	174890	934,41	174730	4,477	938,54	174892
60	1,5	778,685	2,572	781,56	174728	3,43	174840	778,68	174730	3,42	781,95	174841
70	1,75	667,444	2,205	669,91	174728	2,765	174811	667,44	174729	2,757	670,15	174811

Let us pay special attention to the frequency P_0^2 on the basis of various micropolar models. This frequency on the basis of the model without the account of transverse shift deformations correspond to formula (39) at which there do not exist values characterizing the bar sizes i.e. it is the frequency not dependent on the bar sizes. According to model (24) - (26) P_0^2 is one of the frequencies, received from equation (36). As Table 1 represents, this character remains unchanged..

5 FORCED OSCILLATIONS OF MICROPOLAR ELASTIC THIN BARS

Let us consider the problem on the forced oscillations of micropolar bars with independent fields of displacements and rotations by general model (24) - (26). In relation to displacement w and rotations Ω_2 and ψ_1 we will receive equations (28). We will attach to these equations boundary conditions of hinged edge case (30). Let us accept that

$$\tilde{p}_3 = p_{30} e^{ipt} \sin\frac{\pi}{a} x_1,\ \tilde{p}_1 = \tilde{m}_2 = 0,\ \text{where } p_{30} \text{ is constant.} \qquad (42)$$

and present the particular solution of boundary problem (28), (30) (with the account of (42)) in the form of

$$w = A_1^1 e^{ipt} \sin\frac{\pi}{a} x_1,\ \Omega_2 = A_1^2 e^{ipt} \cos\frac{\pi}{a} x_1,\ \psi_1 = A_1^3 e^{ipt} \cos\frac{\pi}{a} x_1 \qquad (43)$$

Let us notice that the presented solution satisfy to the boundary conditions of hinged edge (30). Having substituted (42) and (43) in (28), we will receive linear nonhomogeneous algebraic system of equations in relation to amplitudes A_1^1, A_1^2 and A_1^3 of the bar forced oscillations. Solving this nonhomogeneous algebraic system, we will receive values of amplitudes of the forced oscillations.

Of special importance is the problem on the forced oscillations of micropolar bars in the following case

$$\widetilde{m}_2 = m_{20}e^{ipt},\ \widetilde{p}_1 = \widetilde{p}_3 = 0,\ \text{where } m_{20} \text{ is constant.} \quad (44)$$

In this problem we can represent the particular solution by model (24) - (26) in form:

$$w \equiv 0,\ \ \Omega_2 = A_0^2 e^{ipt},\ \psi_1 = A_0^3 e^{ipt} \quad (45)$$

The solution (45) satisfies the boundary conditions (30), and from system (28) (with the view of equation (44)) we will receive

$$(Jp^2 - 4\alpha)A_0^2 + 2\alpha A_0^3 = -\frac{m_{20}}{h}, \qquad 2\alpha A_0^2 + \left[\frac{\rho h^2}{3}p^2 - (\mu+\alpha)\right]A_0^3 = 0 \quad (46)$$

Thus, it is possible to receive amplitudes of the forced oscillations A_0^2, A_0^3.

In case of the considered problem of the forced oscillations, when there takes place (42) for dependences of dimensionless amplitudes (i.e. divided into the maximum values of corresponding sizes of static problem) from the frequency of revolting force (for physical parameters we will take the above presented values, and the bar sizes are $a = 0{,}2\,\text{m}, \delta = h/a = 1/40$), we will find out that in case of the following values of frequencies revolting forces: 21,5 Hz; 2355 Hz; 175750 Hz resonances are obtained.

Let us notice that as far as forced oscillations are considered in case of micropolar material, there appear new resonant frequencies. In this direction the following effect [10] is rather important: in micropolar elastic bar there is acoustic resonance natural frequency of the bar material that is not dependent on its sizes.

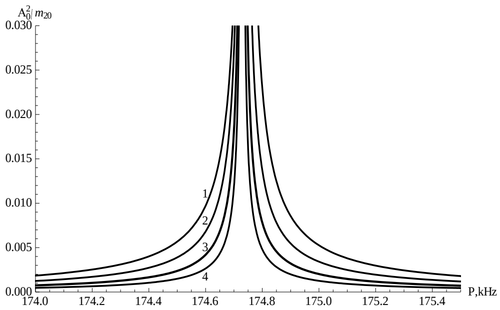

Fig. 1. Natural frequency of bar acoustic resonance not dependent on its sizes.

In case of the considered problem when there takes place (44) we will result dependence of dimensionless amplitude A_0^2/m_{20} (of the quantity Ω_2) on frequency of revoltinforce, at the different sizes of bar (see Fig. 1). The graphics are received for the bar material, which is presented in Table 1, for various values of the bar size a 1) a =0,2 m, 2) a =0,3 m, 3) a =0,5 m, 4) a =0,8 m. In Fig. 1 resonant peak turns out at frequency p =174,7 kHz which practically does not depend on values a.

It is also possible to draw at the conclusions that in the choice of the micropolar material there is a certain possibility to regulate value of low frequency of bar oscillations and by that, achievement of considerable rating of frequencies of natural and forced oscillations of the bar becomes possible. This is important at the resonance phenomenon.

6 THE CONCLUSION

We can draw at the following important conclusions:

- With the help of the method of asymptotic justification the applied dynamic model of micropolar elastic thin bars with independent fields of displacements and rotations with the full account of transverse shift deformations is constructed. The applied dynamic model of micropolar elastic thin bars without the account of transverse shifts is also constructed.
- Problems on the free and forced oscillations of hinged edge of micropolar elastic thin bars are studied. Frequencies of natural oscillations, amplitudes of the forced oscillations and resonance condition are defined. On the basis of the numerical analysis the basic specific features of characteristics of micropolar bars depending on values of various elastic and inertial constants of micropolar material have been brought to light.

• It is shown that in thin bars from micropolar material there is natural frequency of acoustic resonance which does not depend on the bar sizes, and depends only on physical and inertial properties of micropolar material.

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