

## A PROBABILISTIC MODELLING FOR DYNAMIC ANALYSIS OF STEEL-CONCRETE COMPOSITE FOOTBRIDGES

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**Abstract.** *Nowadays, the pedestrian footbridges are being designed with new materials and increasingly daring architectural concepts making these structures slenderer and therefore more susceptible to vibration problems. This fact has changed the serviceability and ultimate limit states associated to their design. Considering all the aspects mentioned so far, in the present paper the human walking is modelled taking into account the random nature of this dynamic load induced by different pedestrians. This uncertainty is related to the variation of the pedestrian weight, step frequency, step length and the dynamic load factors (DLF). Then, the probabilistic approach allows structural engineers to assess the variation of the human walking force generated by different pedestrians and their impact on the dynamic response of footbridges. Finally, the probabilistic walking load model is applied to a steel-concrete composite footbridge in order to calculate the footbridge response under different dynamic loads scenarios. The reference structure investigated is a steel-concrete composite footbridge made of wide flange steel beams with a reinforced concrete deck, built in the new campus of the Institute of Traumatology and Orthopaedics (INTO), located in Rio de Janeiro/RJ, Brazil. A finite element model of the structure was developed using mesh refinement techniques to carry out static and dynamic analysis. Based on an extensive parametric study, the dynamic response spectra of the footbridges expressed in terms of peak accelerations were drawn and compared to the human comfort criteria proposed by several authors and design codes. This probabilistic approach provided a more realistic assessment of the vibration problems induced by human walking.*

## 1 INTRODUCTION

Nowadays, the pedestrian footbridges are being designed with new materials and increasingly daring architectural concepts making these structures slenderer and therefore more susceptible to vibration problems. This fact has changed the serviceability and ultimate limit states associated to their design.

In general, the current design codes and guides [1,2] recommend the use of deterministic models to assess the dynamic behaviour of footbridges. Nevertheless, the human walking is a stochastic phenomenon where the dynamic harmonic force generated at each step depends on each pedestrian weight, step frequency and step length [3].

On the other hand, it is known that footbridges in resonance with dynamic loads induced by humans have their dynamic response amplified considerably when compared to the purely static response. As a result of that, these structures may vibrate excessively causing discomfort to the pedestrians. Bearing that in mind, structural engineers must avoid designing footbridges in resonance with the dynamic load induced by humans [3].

The present research has a main objective to present an analytical probabilistic model to assess the dynamic behaviour of footbridges subjected to pedestrian traffic, taking into account the stochastic nature of the human walking.

The analytical probabilistic approach proposed in this paper was used to assess the dynamic response of a composite internal footbridge built in a public hospital in Rio de Janeiro, Brazil [3]. The findings demonstrate that the peak acceleration calculated using deterministic methods may be overestimated.

## 2 PROBABILISTIC DYNAMIC ANALYSIS METHODOLOGY

In this present research, the probabilistic dynamic analysis method, implemented computationally, assumes that the independent random variables step frequency and step length follow a normal distribution [3].

Hausdorff *et al.* [4] demonstrated that the human step frequency is not constant during walking. This step-by-step variation is called intra-subject step variability whilst the step frequency variation between different pedestrians represents the inter-subject step variability.

The probabilistic model hereby proposed takes into consideration both inter and intra-subject step variability [3]. The Table 1 shows the mean values and respective standard deviations of the random variables adopted in the Monte Carlo simulation to assess the dynamic behaviour of simply supported footbridges.

Variable	Mean ( $\mu$ )	Standard Deviation ( $\sigma$ )	References
Intra-subject step variability (Hz)	-	$1.3 + 0.1\%$ of $\mu$	Hausdorff <i>et al.</i> [4]
Step frequency - $f_p$ (Hz)	2.00	0.20	Bachmann [5]
Step length - $l_p$ (m)	0.71	0.071	Živanović [6]

Table 1: Mean and standard deviation values of the independent variables.

In sequence of the study, the flowchart presented in Figure 1 demonstrates step-by-step the procedures adopted to estimate the modal response of a pedestrian footbridge using the proposed probabilistic method. It is recommended to conduct at least 2000 runs simulating different pedestrians crossing the footbridge span in order to reduce the error [3] embedded in the Monte Carlo method.

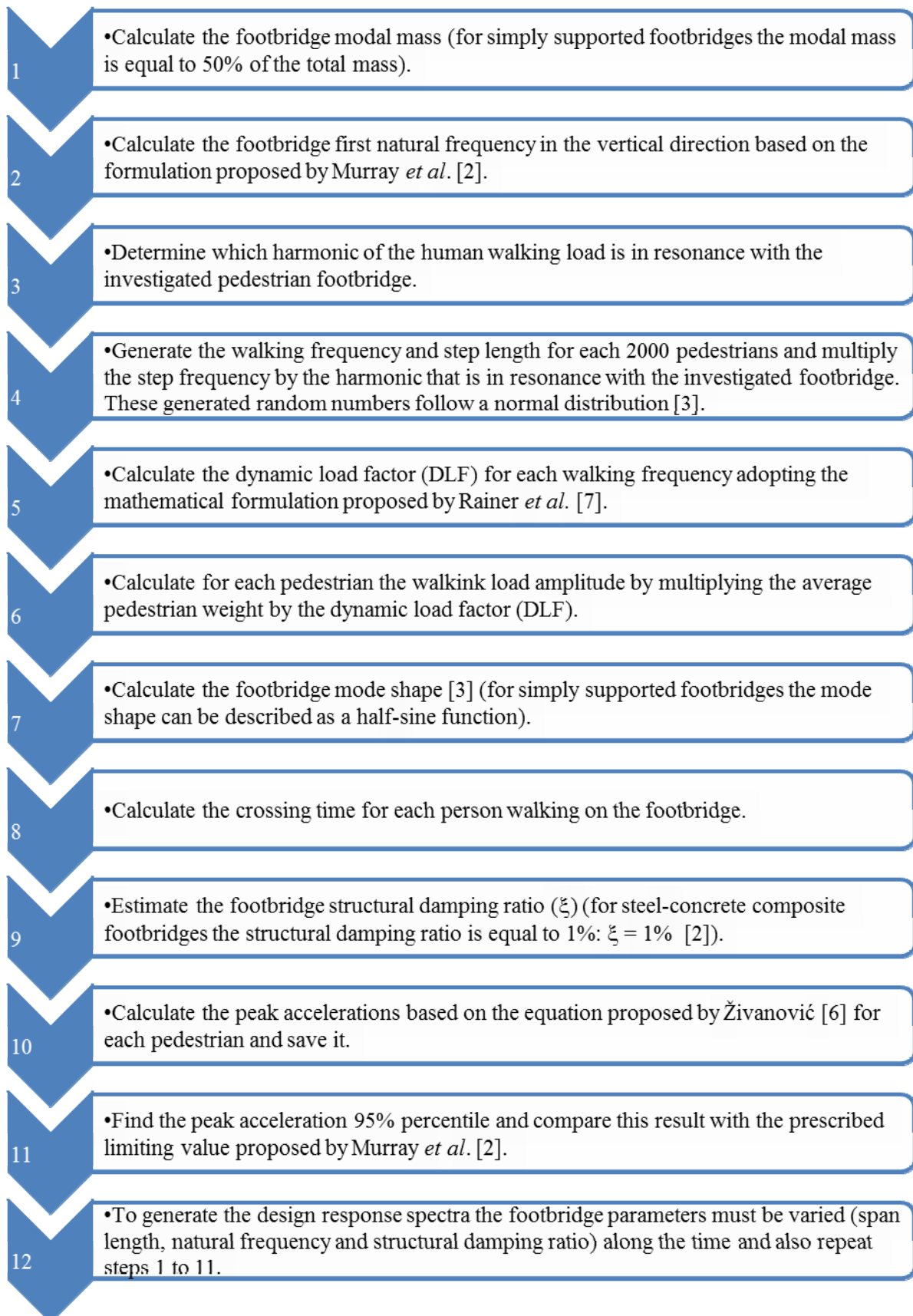


Figure 1: Probabilistic method: step-by-step procedures [3].

### 3 INVESTIGATED STRUCTURAL MODEL

The investigated structural model corresponds to a simply supported steel-concrete composite internal footbridge with a 15 m span [3], as shown in Figure 2. The welded wide flange (WWF) beams are made of ASTM A36 steel. It was adopted a 205 GPa for the steel Young’s modulus. The concrete slab has a 0.10 m thick, with a 25 MPa compressive strength and 28 GPa Young’s modulus. Table 2 presents the structural model geometric properties [3].

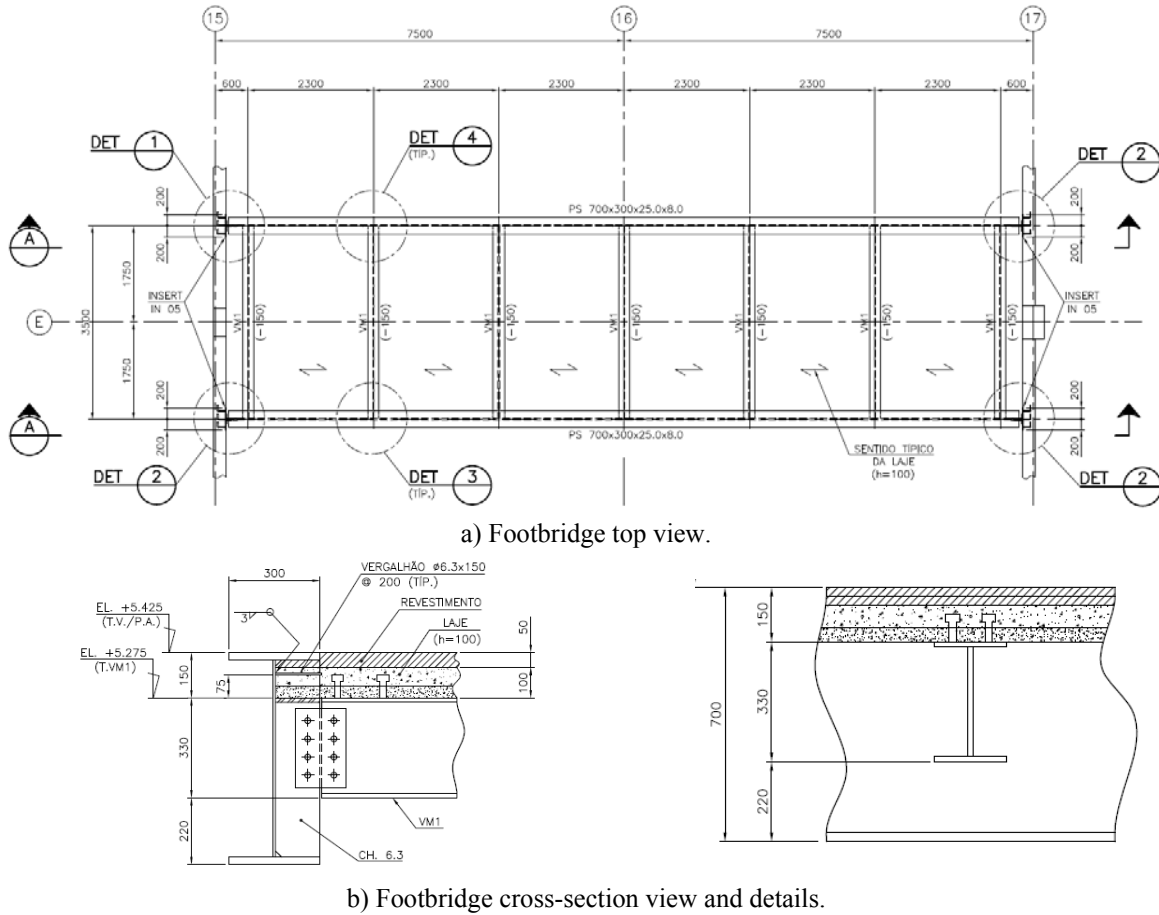


Figure 2. Investigated structural model: steel-concrete composite footbridge.

Profile Type	Height (d)	Flange width ( $b_f$ )	Top flange thickness ( $t_f$ )	Bottom flange thickness ( $t_f$ )	Web thickness ( $t_w$ )
Beams - 700 x 159.9	700	300	25.0	25.0	8.0
Beams - 330 x 45.4	330	200	9.5	9.5	6.3

Table 2: Geometric properties of the welded wide flange (WWF) steel beams (dimensions in mm).

### 4 PROBABILISTIC DYNAMIC RESPONSE SPECTRA

Initially, it must be emphasized that the results obtained from the probabilistic model were obtained applying the Monte Carlo simulation principles as many as 2000 runs [3]. Each run represents a random pedestrian crossing the footbridge with a total mass of 20.000 kg [3]. Also the fundamental frequency, span length, structural damping ratio of the reference footbridge, see Figure 2, were modified, see Table 3. The probabilistic dynamic response spectra correspond to the 95% percentile peak acceleration. Then, it is expected that in 95% of the studied cases the peak acceleration values will be equal or less than spectra values.

Design Parameter	Parametric Variation
Fundamental frequency: $f_{01}$	1 Hz a 10 Hz
Structural damping: $\xi$	0.5% a 2.0%
Footbridge span: L	10 m a 30 m

Table 3: Design parameters varied in the dynamic analysis.

In order to calculate the peak acceleration values, the amplitude of the dynamic force induced by human walking must be determined. The amplitudes of walking harmonic forces are defined as product of a dimensionless coefficient called dynamic load factor (DLF) by the pedestrian weight.

The DLFs were determined using the mathematical formulation proposed by Rainer *et al.* [7]. Evidently, the DLFs used in the Monte Carlo simulation varied according to the investigated pedestrian footbridge natural frequencies so that the reference structure is always close or in resonance with one the harmonics of the human walking force.

Figures 3 to 6 represent the probabilistic dynamic response spectra for the footbridges with span length of 10 m, 15 m, 20 m and 30 m, respectively. The footbridge with a 15 m span corresponds to the structural model investigated, see Figure 3. This structure was built in the public hospital in Rio de Janeiro, Brazil [3], see Figure 2.

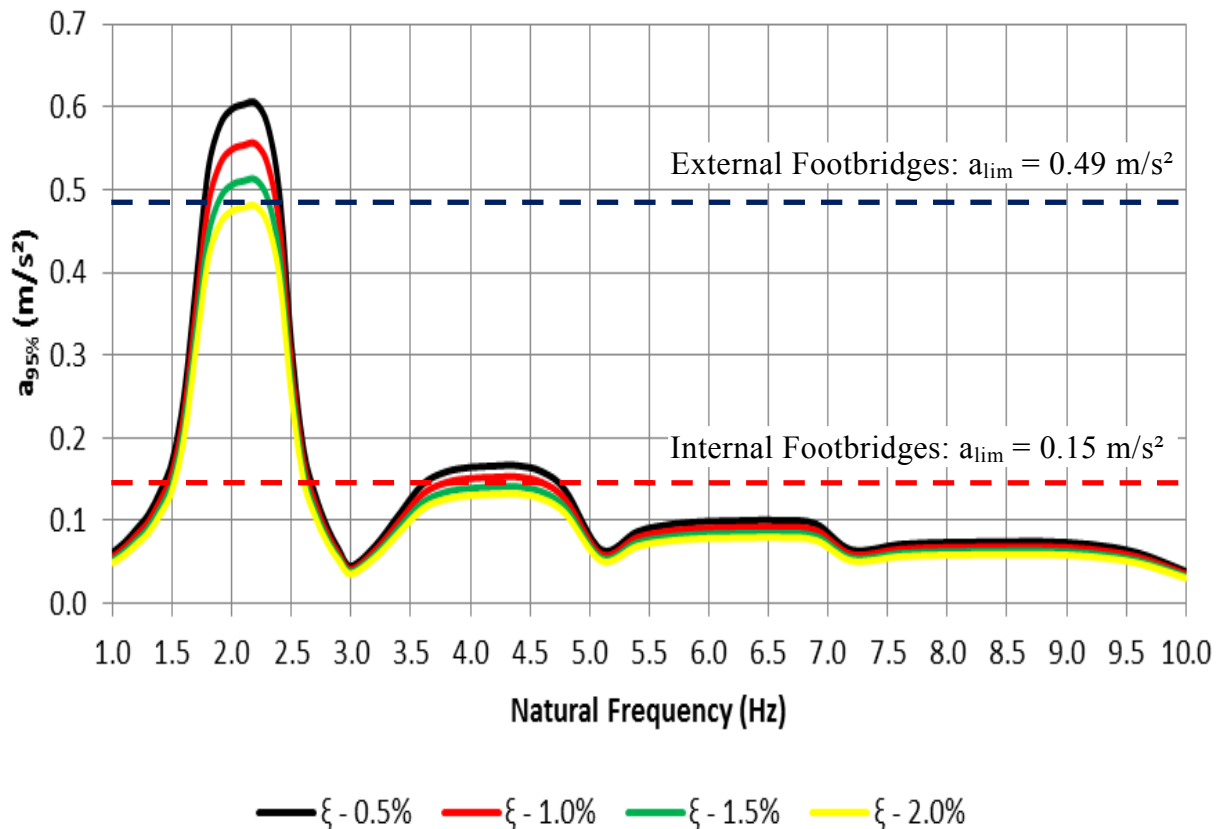


Figure 3: Peak acceleration response spectrum (L = 10 m).

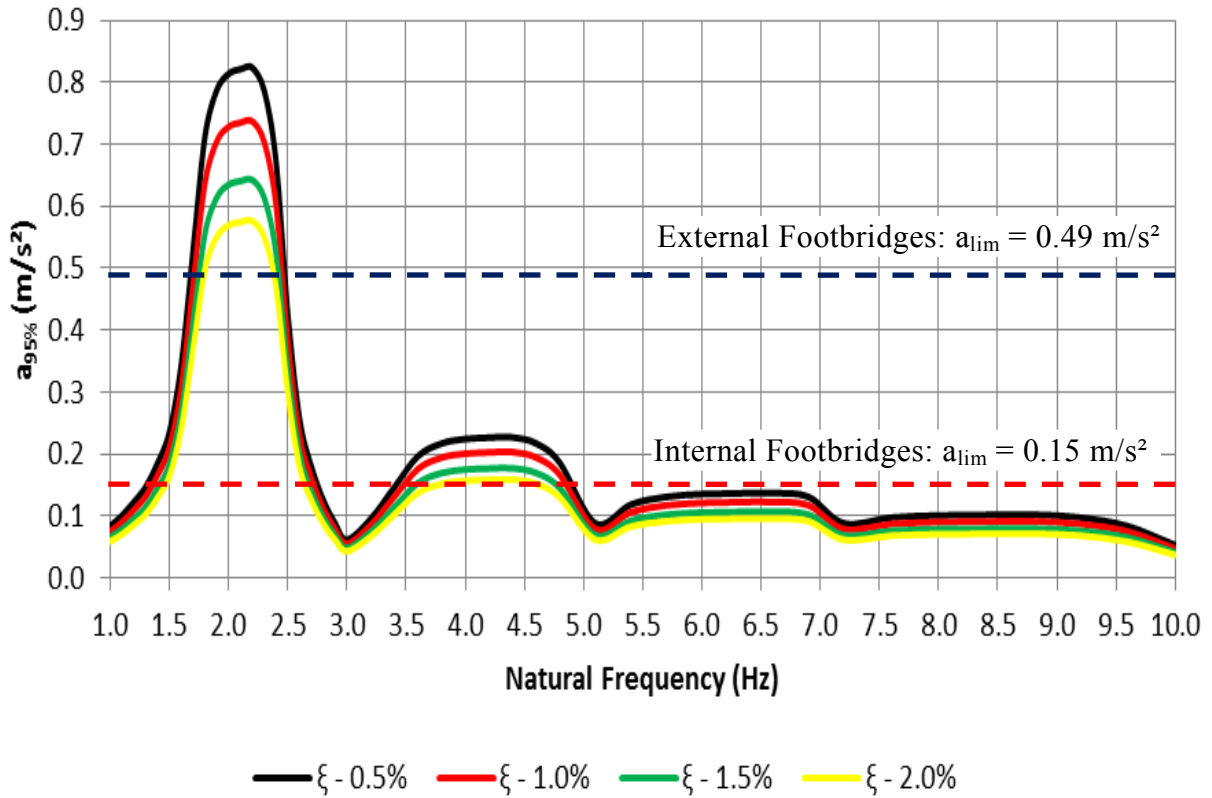


Figure 4: Peak acceleration response spectrum ( $L = 15 \text{ m}$ ).

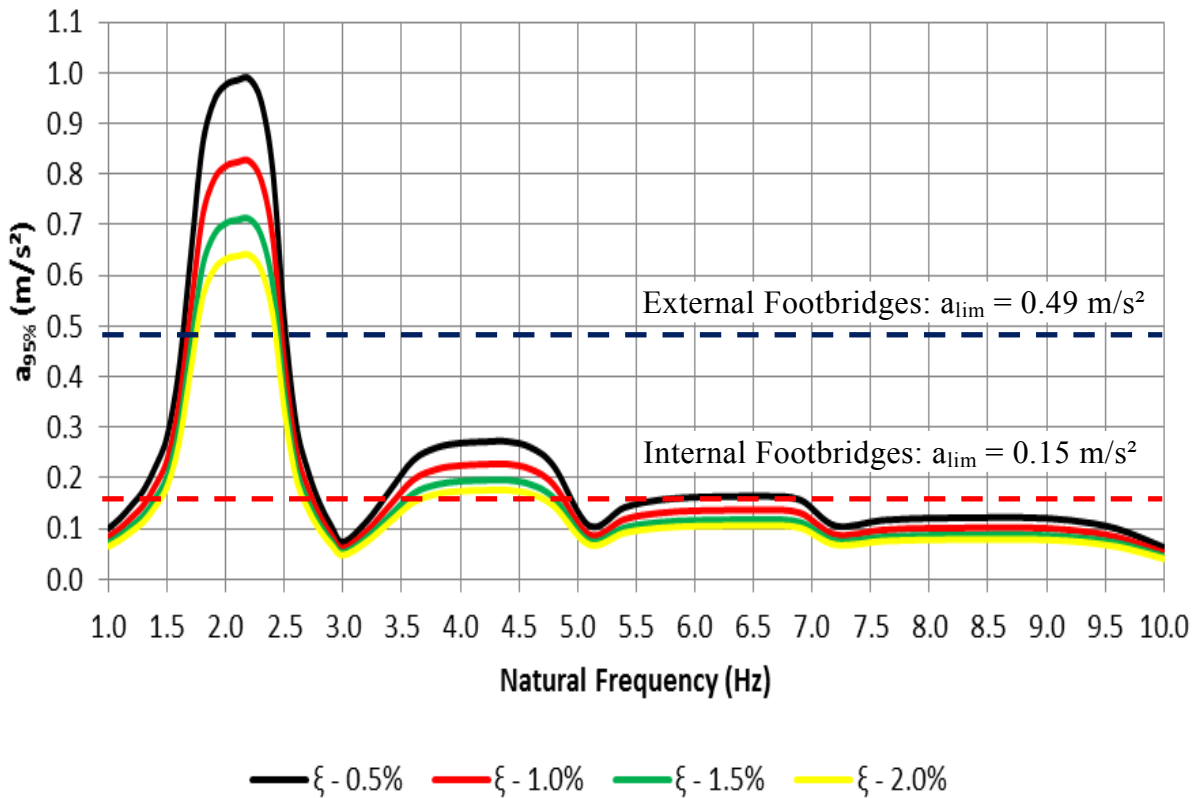


Figure 5: Peak acceleration response spectrum ( $L = 20 \text{ m}$ ).

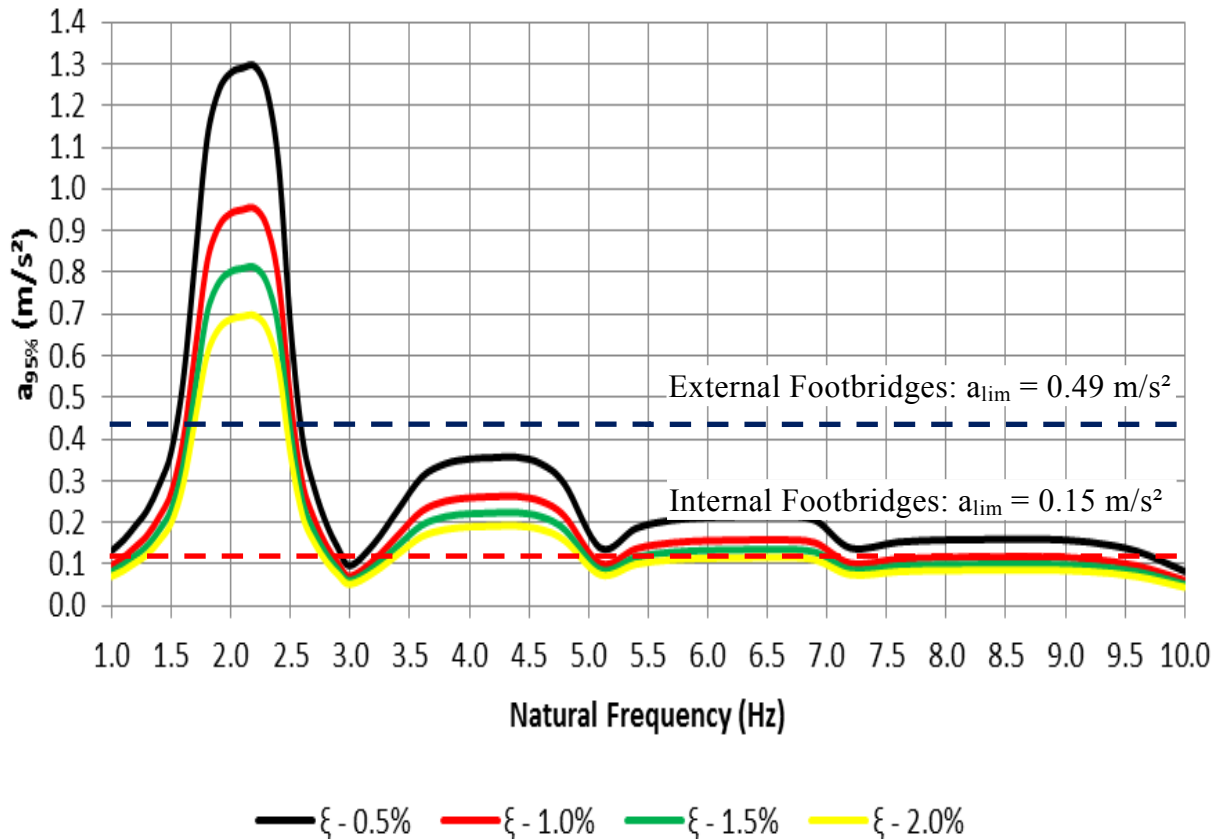


Figure 6: Peak acceleration response spectrum ( $L = 30$  m).

## 5 DISCUSSION OF RESULTS

In order to allow a comparison between the 95% percentile peak accelerations with the human comfort criteria [1,2], it were drawn the red and blue dotted lines representing acceleration limits established by the design codes for external and internal footbridges respectively [1,2]. For external and internal footbridges the acceleration limits [1,2] correspond to  $0.49 \text{ m/s}^2$  e  $0.15 \text{ m/s}^2$ , respectively, see Figures 3 to 6.

The results presented in Figures 3 to 6, demonstrate clearly that the dynamic response of simply supported footbridges subjected to human walking loads vary as a function of the span length. It means that the dynamic response depends on the period of time that the dynamic load acts on the structure.

It can also be observed that the first harmonic of the human walking load is more relevant than the contribution of the other three harmonics (second, third and fourth harmonic) in terms of footbridge vibration response, see Figures 3 to 6. As it can be seen, for almost all studied span lengths, the first harmonic excited the footbridges above the human comfort criteria limit [1,2] ( $a_{95\%} > a_{lim} = 0.49 \text{ m/s}^2$  and  $a_{95\%} > a_{lim} = 0.15 \text{ m/s}^2$ ), see Figures 3 to 6.

From the Figures 3 to 6 it can also be concluded that the second, third and fourth harmonic of the human walking do not excite these reference external footbridges above the design limits [1,2] ( $a_{95\%} < a_{lim} = 0.49 \text{ m/s}^2$ ), considering the adopted structural damping ratios ( $\xi = 0.5\%$ ,  $1\%$ ,  $1.5\%$  and  $2\%$ ).

Similar analysis may be carried out for internal footbridges. For instance, the second harmonic of the human walking excited the reference footbridges with the analysed span lengths and structural damping ratios ( $\xi = 0.5\%$  and  $1\%$ ) above the human comfort criteria [1,2], see Figures 3 to 6.

By increasing the simply supported footbridge span lengths, the harmonics of the human walking start exciting these structures above the limits suggested by the design codes, for the structural damping ratios adopted in the present probabilistic method ( $\xi = 0.5\%$ ,  $1\%$ ,  $1.5\%$  e  $2\%$ ).

## 6 CONCLUSIONS

The probabilistic method proposed in this paper was used to calculate the peak accelerations for a reference footbridge with a total mass of 20.000 kg [3]. An extensive parametric analysis was carried out by varying the reference footbridge span length, structural damping ratio and natural frequency. Finally, based on a parametric analysis the 95% percentile peak acceleration ( $a_{95\%}$ ) was calculated.

The probabilistic response spectra demonstrate that the dynamic response of simply supported footbridges (expressed in terms of peak accelerations) vary as a function of the span length, meaning that the footbridge dynamic behaviour depends on the period of time that dynamic load acts on these structures.

The steady-state peak acceleration for footbridges in resonance with the human walking load calculated based on the use of deterministic methods are greater than the values obtained using the probabilistic approach hereby suggested that takes into account the footbridge span length.

For instance, for a footbridge with a total mass of 20.000 kg [3], span length equal to 15 m, 1% structural damping ratio and natural frequency equal to 2 Hz, the deterministic steady-state peak acceleration is equal to  $1.5 \text{ m/s}^2$  [7] whilst the respective value obtained based on the proposed probabilistic method is equal to  $0.72 \text{ m/s}^2$ .

The deterministic methods proposed by Murray *et al.* [2] and Rainer *et al.* [7] also take into consideration the footbridge span length. However, the structure is always placed in resonance with the harmonics of the human walking load. Therefore, these deterministic methods do not carry out sensitivity analysis using various combinations of different assumptions concerning the values of parameters associated with pedestrian step frequency and step length.

Alternatively, the footbridge dynamic response may be calculated adopting the average human walking step frequency equal to 2.0 Hz [5]. In this case, the footbridge would only be in resonance with the dynamic load if one of its natural frequencies was equal to 2.0 Hz. Then, adopting the average human walking step frequency of 2.0 Hz [5] to calculate the dynamic response of footbridges, the peak acceleration might be underestimated since it is not being taking into account the inter-subject step variability. The step frequency variation amongst pedestrian may place certain footbridges in resonance with the walking load generated by a percentage of individuals.

Another aspect that is not incorporated into the deterministic approach regards the intra-subject step variability. The probabilistic response spectra demonstrate that the peak accelerations calculated for footbridges away from resonance are greater than the equivalent values obtained based on the use of deterministic methods. That occurs due to the fact that in the probabilistic approach the step frequency follows a normal distribution and a percentage of these pedestrians may generate dynamic walking load in resonance with the footbridge.



## 7 ACKNOWLEDGMENTS

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