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DYNAMIC ANALYSIS OF A CONSTRAINED ROTATING FLEXIBLE EXTENSIBLE LINK WITH SEMI-PERIODIC IMPACT

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Abstract. *In this paper the modeling of an extensible mechanism with a rigid crank and a flexible link subject to semi-periodic impact is considered and its dynamical behavior analyzed. The flexible extensible link having one end constrained to a predefined trajectory, rotate with a constant angular velocity in a horizontal plane. Semi-periodic impact occurs between the non-constrained end of the flexible extensible link and a rigid support. The dynamic evolution of the system is investigated and the flexural response of the flexible link analyzed under the combined effect of flexibility, semi-periodic impacts and clearance.*

1 INTRODUCTION

The modelling and simulation of rotating mechanical and robotic systems have received attention for several years. The study of their dynamics was considered important both from a design perspective, including applications such as robot manipulators and helicopter rotors, as well as for their dynamic stability. Since the majority of those mechanical models contain flexible or rigid parts connected by joints, their dynamical response is affected by the parts deformation, clearance or impact between the components, which results in a medium to low ability to perform precision manipulation, increased components and assembly vibration and subsequently, high levels of noise. Due to the dynamical stress caused by the motion and the impact of the parts which affect the vibration characteristics of the mechanical system the mechanical system may fail or perform at a very low capacity.

Interesting reviews about impact dynamics and control of flexible-joint and dual-arm robots and extensible members can be found in the papers given by [1-4]. The impact control of a flexible link and the combined vibration control of a flexible linkage mechanism for damping its vibrations and impact effects have been discussed in [5, 6]. The linear control a slender flexible beam - attached to a rigid link of variable length - undergoing rotations and periodic impacts was studied in [7].

Rotating extensible flexible beams have also been studied as part of the modelling, simulation and control of robotic systems. The dynamic behaviour of a translating flexible beam with a prismatic joint and rotational motion was studied in [8] and an experimental verification was considered in [9]. A dynamic finite element modelling of a translating and rotating flexible link has been studied in [10], and a study regarding the free vibration of an extensible rotating beam was considered in [11].

Classical examples of flexible mechanical systems, their forward, inverse and impulsive dynamics, and their stability have been presented in [12-14]. The study presented in [15] presents a dynamic analysis of a planar mechanism with slider joints and clearance. The simulation of the non-smooth translational joints with clearance has received attention in the work of Zhuang and Wang [16]. A dynamical analysis of some classical mechanical systems with lumped masses and impact was performed in [17, 18]. Mechanical systems with impact, clearance and different types of contact force models has considered in [19-23].

In this paper the modelling and simulation of an extensible mechanism with a rigid crank and a constrained flexible link subject to semi-periodic impacts was considered. Simulations have been performed a double elliptic-circular/elliptic constrained trajectory in order to explore mechanism behaviour under semi-periodic impacts. A clearance vs. a non-clearance model of the extensible mechanism with a flexible link was considered in order to analyse the effect of clearance on the dynamical behaviour of the system.

2 SYSTEM MODEL

A flexible extensible link composed of a rigid guide of length l_{CD} and a flexible link denoted by *PS*, rotate about its fixed end *C* as shown in Fig. 1. A Cartesian reference frame *Oxy* having the origin at *O* and the unit vectors **i** and **j** is considered. The flexible link, modelled as in [24] using *n* successive equal rods P_iP_{i+1} (where $P_1=P$ and $P_n=S$) connected with torsional springs, can slide inside the guide *CD*. Each one of the rigid rods of the flexible link has the mass $m_i = m$ and moment of inertia J, and each one of the springs used to model link flexibility has the stiffness $k = EJ/l_{PS}$ computed as in [24, 25], where *E* is the Young modulus and

$l_{PS} = \sum_{i=1}^{n} l_{P_iP_{i+1}}$ is the length of the flexible link. The length l_{CP} represents the distance between the end C of the guide and the constrained trajectory of the system.

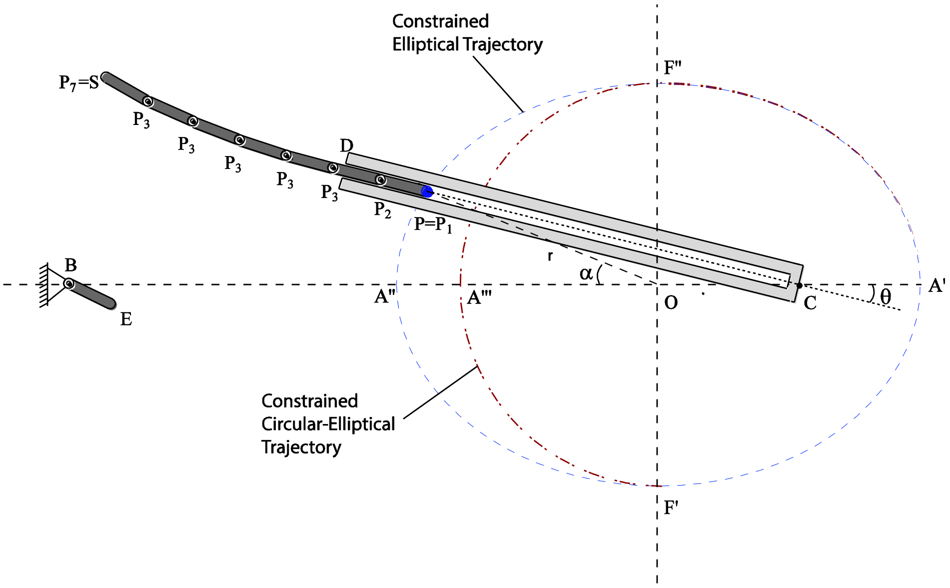


Figure 1: Constrained Flexible Extensible Link with Rigid Support and semi-periodic Impacts Model. An elliptic-circular/elliptic double trajectory is considered.

The constrained trajectory is a double elliptic-circular/elliptic trajectory composed of two trajectories one elliptical and one half elliptical half circular as shown in Fig. 1. The elliptical trajectory has the centre at the origin O of the Cartesian frame and has the transverse diameter of *A'A"* of length $d_{A'A''}$, and the conjugate diameter *F'F"* of length $d_{F'F''}$. The circular/elliptic trajectory has the right side a half ellipse and the left side a half circle having the diameter equal with the conjugate diameter *F'F"* of the elliptical trajectory. The rigid rod P_1P_2 is linked to the constrained trajectory with a slot-joint at P_1. The angles between the links P_iP_{i+1} and $P_{i+1}P_{i+2}$ denoted by θ_i are named relative angles (for any $i=\overline{1,n-1}$), while the angles between the links P_iP_{i+1} and the horizontal direction denoted by Θ_i are named absolute angles. The angle between the guide and the horizontal direction is denoted by θ, and in the case of a flexible link with no clearance is always equal with the angle θ_1. A motor torque $\mathbf{M}_1$ acts on end C of the rigid guide of the mechanical system.

The impacted rigid link *BE* of length l_{BE} can rotate around the fixed point *B* as shown in in Fig. 1. The rigid link is connected to the ground through a spring having the stiffness k_{BE}, such that, after the impact between the flexible link and the rigid fixed (impacted) link, the link oscillations damped rapidly to zero before a new impact will occur.

3 DYNAMIC MODEL OF THE FLEXIBLE AND IMPACTING LINK

The dynamic model for the constrained flexible link as well as for the rigid impacting link can be expressed based on the position, velocity and acceleration vector of the centre of the mass C_i of each rigid rod P_iP_{i+1}, $i=\overline{1,n}$ of the link and rigid impacting link respectively. The position centre of link P_iP_{i+1} is given by $\mathbf{r}_{C_i} = x_{C_i}\mathbf{i} + y_{C_i}\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the associated Cartesian reference frame *Oxy*. The velocity and acceleration vector of the center of the mass C_i of each rigid rods P_iP_{i+1} of the constrained flexible extensible link is the derivative and respectively the double derivative with respect to time of the position vector r_{C_i} and is given by $\mathbf{v}_{C_i} = \dot{\mathbf{r}}_{C_i} = \dot{x}_{C_i}\mathbf{i} + \dot{y}_{C_i}\mathbf{j}$ and respectively $\mathbf{a}_{C_i} = \ddot{\mathbf{r}}_{C_i} = \ddot{x}_{C_i}\mathbf{i} + \ddot{y}_{C_i}\mathbf{j}$.

3.1 Flexible Link Dynamic Model with no Clearance

For the no clearance model, when the flexible link translates parallel to its support, the dynamics of the flexible extensible link can be expressed using the Lagrange differential equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \tag{1}$$

where Q_i are the generalized forces, T is the total kinetic energy of the system expressed as $T = \sum_{i=1}^{n} T_i = \frac{1}{2}\sum_{i=1}^{n}\left(m_i \mathbf{v}_{C_i}^2 + I_{C_i}\omega_i^2\right)$, T_i is the kinetic energy of each i^{th} link, $q_i = \theta_i$ are the generalized coordinates, the subscript i represents the number of the generalized forces/coordinates. The model described in [17] was used to describe the generalized forces acting on each link.

3.2 Flexible Link Dynamic Model with Clearance

The same clearance model considered in [24], where the flexible link of the extensible can translate and rotate about its support, was used for this study. Due to the clearance model, the flexible link may impact the rigid support as shown in Fig. 2.

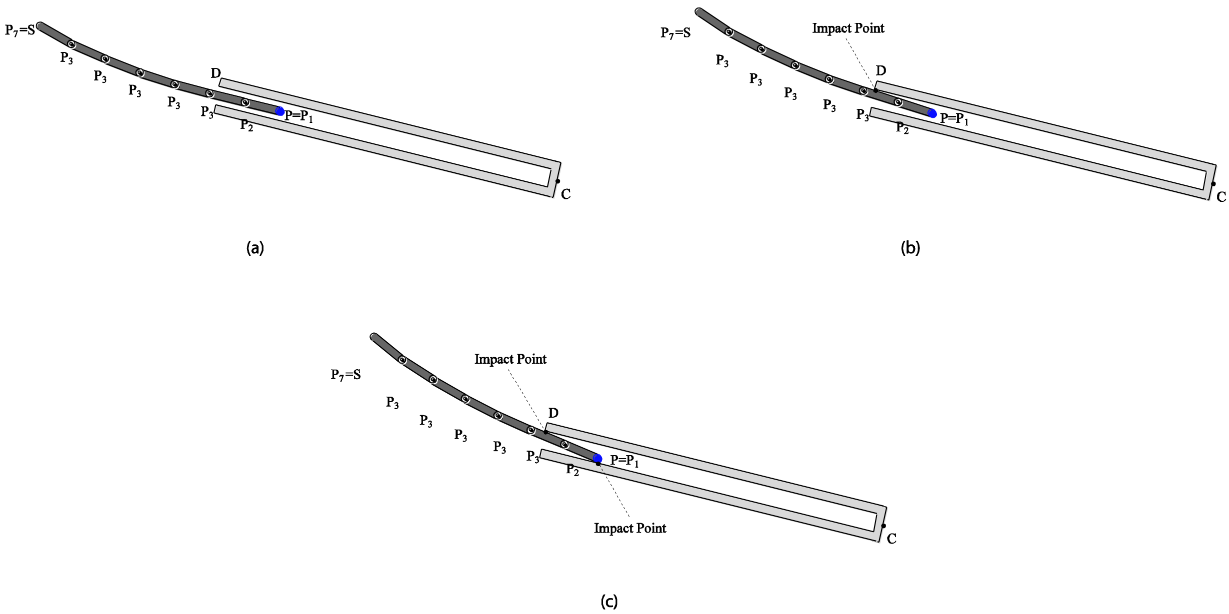


Figure 2: Constrained Flexible Extensible Link with Rigid Support and Clearance Model, (a) No Impact, (b) Impact on one Point, (c) Impact on two Points

The impact model between the flexible link and the guide is shown in Fig. 5. The main impact cases are a) no impact between the flexible link and the guide (link support), b) impact of the flexible link on a single point of the guide, and, c) impact of the flexible link with two points of the guide. For this study, multiple impacts have not considered but only presented since impacts at the same time instant can be statistically excluded. The discontinuous model, used in this paper to evaluate impacts, assumes that the impact occurs instantaneously and that no change of the system configuration occurs during contact. The integration of equations of motion is halted at the time of impact and a momentum balance is performed to calculate the post impact velocities of the system components. The restitution coefficient is employed to quantify the dissipation energy in the process. Different types of impact and contact force models have been discussed in [7, 17] and [19-23].

To derive the equation of motion with impact a similar approach as in [7, 17] was considered. The impact differential equation of motion can be written as

$$\left(\frac{\partial T}{\partial \dot{u}_k}\right)_{t_s} - \left(\frac{\partial T}{\partial \dot{u}_k}\right)_{t_a} = P_k , \tag{2}$$

where T is the total kinetic energy of the system, $\frac{\partial T}{\partial \dot{u}_k}$ are the generalized momenta, P_k are the generalized impulses associated with the coordinate u_k, t_a and t_s represents the moments of time of approach and separation, respectively before and after the impact. Considering $\int_{t_a}^{t_s} \mathbf{R}dt = R_y \mathbf{j}$ the force exerted during the impact (only a vertical component is considered in this study) by the flexible ling at contact, the generalized impulses can be expressed by $P_k = \sum \frac{\partial \mathbf{v}_N}{\partial q_k} \int_{t_a}^{t_s} \mathbf{R}dt$ where the velocity of the impact point $\mathbf{v}_N$ can be expressed as $\mathbf{v}_N = \mathbf{v}_M + \boldsymbol{\omega} \times \mathbf{r}_N$. Using Newton's coefficient of restitution *e*, the velocity of approach and separation for the impact points *N* (on the flexible link) and *M* (on the impacted link) can be expressed as

$$\mathbf{v}_a = \mathbf{v}_N\big|_{t_a} - \mathbf{v}_M\big|_{t_a}, \quad \mathbf{v}_s = \mathbf{v}_N\big|_{t_s} - \mathbf{v}_M\big|_{t_s}, \tag{3}$$

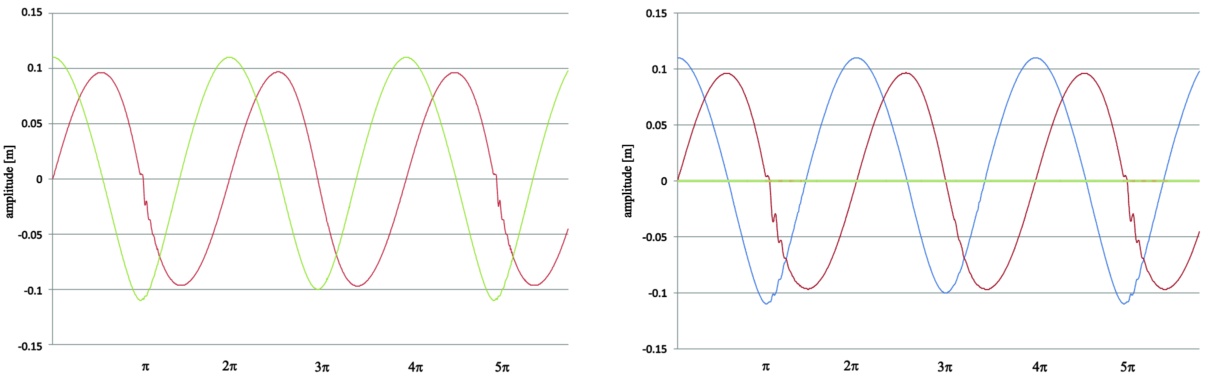
where $\mathbf{v}_N\big|_{t_a}$ and $\mathbf{v}_M\big|_{t_a}$, $\mathbf{v}_N\big|_{t_s}$ and $\mathbf{v}_M\big|_{t_s}$ are the flexible link velocity and the impacted link velocity at time t_a and t_s before and after the impact. Using Eq. (3) and the definition of the coefficient of restitution *e* (using Newton's formulation), one can write

$$-e\mathbf{v}_a = \mathbf{v}_s . \tag{4}$$

4 SIMULATIONS AND RESULTS

The results from the computer simulations are presented in this section using the following numerical values: the length of the guide l_{CD} = 0.09 m, the length of the flexible link l_{PS} = 0.07 m, and the clearance model, and the clearance between the guide and the extensible link (one side) of $5 \cdot 10^{-3}$ m, chosen to accentuate the impact effect between the guide and the flexible link. The next material properties are considered for the guide and for the moving parts: density (kgm^{-3}) 7850, Young modulus $2 \cdot 10^{11}$ Pa and Poisson Ratio 0.3. The rotating end *C* of the guide is located on the *Ox* axis at 0.02 m from the origin *O* of the Cartesian reference frame, where the centre of the elliptic-circular/elliptic constrained trajectory is located. The principal axes of the elliptic constrained trajectory have the next length: the transverse diameter of 0.08 m, the conjugate diameter of 0.06 m, and the semi-circle radius of 0.03 m. The impacting rigid link is located at 0.108 m from the origin *O* of the Cartesian reference frame.

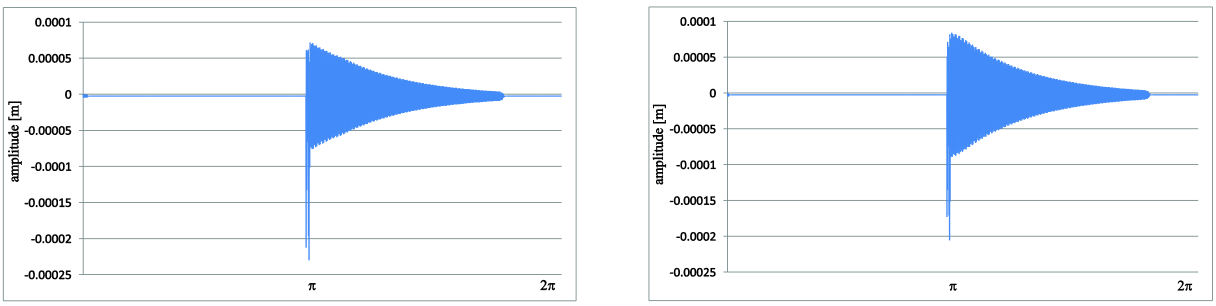
The dynamical behaviour of the extensible mechanism with a flexible link, no clearance and semi-periodic impacts due to its constrained elliptic-circular/elliptic trajectory is shown in Fig. 3.a, that is, the *Ox* and *Oy* trajectories of the end *S* of the constrained extensible link plotted vs. the crank angle. From Fig. 3.a one can observe the moment of impact and the immediate perturbation the system exhibit due to the impact, as well as the time frame in which the system damped the perturbation. Since perturbation of the trajectory is an important factor which affects system dynamics, the behaviour of the extensible flexible link considered for the elliptic-circular/elliptic constrained trajectory (shown in Fig. 1) and subject to semi-periodic impact was considered for a clearance vs. a non-clearance model.



(a) (b)

Figure 3: Dynamical behaviour of the extensible flexible link with semi-periodic impacts. The *Ox* (green and blue colour) and *Oy* (red colour) trajectories of the end *S* of the constrained extensible flexible link for: (a) no clearance, (b) clearance

The dynamical behaviour of the extensible flexible link with clearance and semi-periodic impacts is shown in Fig. 3.b, that is, the *Ox* and *Oy* trajectories of the end *S* of the constrained extensible link plotted vs. the crank angle. It was observed that the clearance model add more excitation to the system as one can see on both the excitation time as well as the amplitude of the excitations which can be clearly observed on Fig. 3.a and Fig. 3.b.



(a) (b)

Figure 4: Divergence *d* of the trajectories of the motion (a) no clearance of the flexible link, (b) clearance of the flexible link

The dynamical behaviour of the impacted rigid link is shown in Fig. 4.a and Fig. 4.a for the clearance vs. a non-clearance model respectively. One can observe that the magnitude of oscillations decrease in time because of the damping as well as because the elastic part is retracting inside the rigid guide. It was also observed that for the clearance model, the initial amplitude of the oscillations is a little bit higher when compared with the no-clearance model.

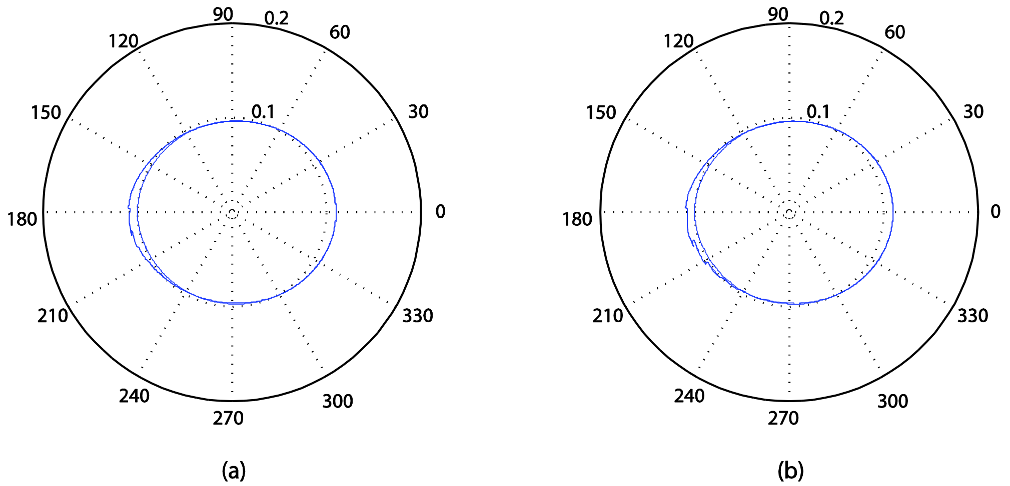


Figure 5: Dynamical behaviour of the extensible flexible link with semi-periodic impacts plotted vs. the crank angle in a polar coordinate system. Trajectory of the end of the constrained extensible flexible link with: (a) no clearance, (b) clearance

The dynamical behaviour of the extensible mechanism with a flexible link, no clearance and semi-periodic impacts are plotted vs. the crank angle in a polar coordinate system as shown in Fig. 5. From Fig. 5.a and Fig. 5.b one can observe that the trajectories of the extensible mechanism with a flexible link and with clearance as well the one without clearance diverges in time, that is, a separation of nearby trajectories and a clear sign of nonlinear motion. It was observed also that the clearance model add more excitation to the system, both the amplitude of the excitations and trajectories separations are clearly visible in this case.

One can conclude that the time evolution of the system is mainly affected by the semi-periodic impacts as well as by link flexibility, clearance and velocity, that is, the combined effect of link flexibility and clearance accentuate trajectory divergence and affect system stability.

5 CONCLUSIONS

In this paper the modelling of an extensible mechanism with a double constrained flexible link and a rigid crank subject to semi-periodic impacts is presented and its behaviour analysed. Accurate simulations for an elliptic-circular/elliptic constrained trajectory are performed. A dynamical analysis is carried out in order to compare the dynamical response of the flexible link with clearance vs. no clearance under the combined effect of the flexibility and semi-periodic impacts. Trajectories divergence has been observed for the constrained extensible mechanism with and without clearance and semi-periodic impacts, however, it was observed that the system behave more unstable in the clearance case. It was concluded that parameters such as clearance, rotational velocity, impact and flexibility have a great influence on the dynamical stability of mechanical system performance and stability. Experimental tests will be performed in order to validate and generalize the simulations reported in this study.

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