A NEW SIMPLE PASSIVE APPROACH FOR VIBRATION MITIGATION OF 3-DIMENSIONAL STRUCTURES

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Abstract. Exact optimal classical closed-open loop control is not achievable for the buildings under seismic excitations since it requires the whole knowledge of earthquake in the control interval. This has motivated the researchers to develop suboptimal control policies. This study proposes some representative simple methods to obtain the suboptimal passive damping and stiffness parameters from the optimal control gain matrix since it is not possible to add the exact optimal damping and stiffness parameters to the structure in practice [1]. Proposed method is applied to a 3-dimensional tier building structure. For this study, a 3-story steel building designed for the SAC project Los Angeles, California region is considered [2]. This structure is idealized with two translational and a single rotational degree of freedom in each story. The structure is tested under a unidirectional real earthquake excitation. Dynamic equations of motion are derived in matrix form for the three dimensional structure. The dynamic response of the 3-story three-dimensional building with its base excited in one direction, including damping effects was found numerically. The behaviour of the building with no control and with simple passive approach was investigated and compared with each other. Although the proposed approach is intrinsically passive and has no adaptive property against changing dynamic effects it is shown numerically that modifying the damping and the stiffness in a suboptimal way may suppress the uncontrolled vibrations and rotations. The proposed simple approach can be a new way for structural vibration mitigation without implementing any supplemental devices to structures.
1 INTRODUCTION

Protecting structures from earthquake induced vibrations has been one of the major problems of earthquake engineering. Together with some important buildings such as hospitals, power plants, emergency centres etc. require stiffer seismic protection. These types of buildings can’t be constructed by conventional design methods. They contain important equipment which is very sensitive to vibration. This equipment can lose their facility when they are exposed to strong earthquake excitations. Aforementioned circumstances pioneered the research of new concepts for seismic protection. Since the early 1970’s new techniques for protecting structures have been introduced and widely researched. These alternative techniques are passive, semi active and active control methods [3-5]. Among these methods active control is superior to the others in reducing the vibrations [6-8]. Although implementing active control devices to the structures improves the structural behaviour significantly, there are still some uncertainties about active control such as modelling error and spillover effects, structural reliability, computational time delays, energy losses during earthquakes and phase lag effects [9]. In addition implementing active control devices to the structures is a very costly way of protecting structures.

Besides active control, passive control devices are also being used as structural control elements for protecting structures from earthquakes and severe winds. Among passive control devices base isolations are mostly researched and widely applied in practice [10-11]. Despite these devices are effective in reducing the superstructure vibrations, when they are implemented to multi-story buildings they can make overturning effects and cause totally collapse of the structure. In addition central problem of base isolation is under some earthquake excitations the structure can go excessive base displacements [12].

Another commonly used control device is tuned mass dampers [13-14]. Although they are widely researched the effectiveness of these devices are limited due to the mistuning effect [15]. If the tuning frequency of the mass damper differs from the main frequency of the structure, tuned mass damper will have no effect on reduction of seismic responses.

Main purpose of this study is to overcome the aforementioned negative effects of the passive and active control systems by a simple approach. This approach can be applied to the structures without implementing any control devices. This approach is based on modifying stiffness and damping parameters of the uncontrolled structure by appropriate amounts with solving the Riccati equation in closed loop classical optimal control (CLOC) and with the help of the control gain. To illustrate this approach simulations of a 3-dimensional structure have been performed under CLOC control law. By the help of these simulations additional damping and stiffness parameters of the structure are obtained. Numerical verifications are carried out by a 3-story steel building designed for the SAC project Los Angeles, California region. This structure is idealized with two translational and a single rotational degree of freedom in each story. Tier building formulation is used while idealizing the structure in 3-dimensional form. More realistic and complex analysis than widely used shear structure analysis in structural control studies is achieved by this idealized 3-dimensional model. A fully active tendon controller system is implemented to the structure to obtain the additional stiffness and damping values. The structure is tested under El Centro 1940 (NS component) earthquake excitation. Further to the work described above the dynamics equation of the motion of the 3 dimensional tier building and formulation of the proposed simple approach in addition its application to 3 dimensional structures are given in the paper.
2 TIER BUILDING FORMULATION

The matrix formulation of 3D multi-story tier buildings was developed by [16-17] and has been used extensively over the years, more recently by [18]. The schematic diagram of the three story idealized tier building is given in Figure 1. In Figure 1 $h_1$, $h_2$ and $h_3$ denote the height of the stories respectively from bottom to top, $l_x$ and $l_y$ denote the length of the floors in $x$ and $y$ directions.

In tier building formulation, each story is treated as a rigid body with 3 degrees of freedom (DOF) per floor. The detailed formulation of tier buildings can be found in [16-17]. To integrate our fully active tendon controlled model into a multi-DOF multi-story structural model, we propose the use of the well-developed and relatively simple matrix formulation of the tier buildings and incorporate into it the active tendon controllers with CLOC algorithm. This would enable us to obtain the additional stiffness and damping of the 3-dimensional structure by the help of CLOC.

The equations of motion for this structure can be expressed as;

$$[M] \times \{\ddot{X}\} + [C] \times \dot{X} + [K] \times X = [A] \quad (1)$$

where $X$ is the relative displacement vector (as a function of time), $\dot{X}$ and $\ddot{X}$ are relative velocity and acceleration vectors respectively. $M$, $C$ and $K$ are the $3 \times 3$ mass-inertia, damping and stiffness matrices respectively where $n$ is the number of the stories. $A$ is the action vector. The action vector can either be a dynamic force or a real earthquake excitation. The mass and stiffness matrices of a two story model structure can be defined as;

$$C = a_0 M + a_1 K \quad (2)$$

$a_0$ and $a_1$ in Eq. (2) can be obtained by using natural frequencies of the building and the damping coefficient as

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{2}{\omega_0^2 - \omega_n^2} \begin{bmatrix} \omega_0 & -\omega_n \\ -1/\omega_n & 1/\omega_n \end{bmatrix} \begin{bmatrix} \xi_m \\ \xi_n \end{bmatrix}$$

$$\begin{bmatrix} \omega_m \\ \omega_n \end{bmatrix}$$

where $\omega_n$ and $\omega_m$ are the corresponding natural frequencies and $\xi_m$, $\xi_n$ are the viscous damping coefficients of the corresponding natural frequencies.

Figure 1: 3D tier building under one dimensional earthquake and control
3 CLASSICAL LINEAR OPTIMAL CONTROL FOR 3D BUILDINGS

A 3 dimensional tier building structure given in Figure 1 under the one-dimensional earthquake acceleration and the control (one dimensional control force but control force can be applied either in two and three dimensions) is idealized by a $3n$-degree of freedom system. Equation of motion of the structure can be described as

$$M \ddot{X}(t) + C \dot{X}(t) + KX(t) = D_1 f(t) + D_2 U(t), \quad t \in (t_0, t_1)$$

(4)

where $X(t) = (X_1, X_2, \theta_1, X_n, \theta_{n/2})^T$ is the $3n$-dimensional response vector denoting the relative displacements in two directions and rotation $(\theta_1, \ldots, \theta_{n/2})$ of the each story (unit with respect to the ground; $D_1$ is a $(3n \times 1)$ location matrix of excitation and given by $D_1^T = -(m_1, m_2, m_3, \ldots, m_n)$ where $m_1$ and $m_2$ are the mass in two directions and $m_3$ is the rotational mass; $D_2$ is the $(3n \times r)$-dimensional location matrix of $r$ controllers; $U(t)$ is the $r$-dimensional active control force vector and described as $U^T(t) = (u_1(t), \ldots, u_r(t))$ and scalar function $f(t)$ is the one dimensional earthquake acceleration. Initial conditions of the structure can be given as

$$X(t_0) = X^{(0)}; \quad \dot{X}(t_0) = X^{(1)}$$

(5)

in which $X^{(0)} = X^{(1)} = 0$ in practice. Upon introducing a $6n$-dimensional state vector (for 3 dimensional structures) $Z = (X, \dot{X})^T$, Eq. (4) can also be casted into a first order matrix equation with dimension $6n$ in the following form:

$$\dot{Z}(t) = A Z(t) + B U(t) + D f(t), \quad t \in (t_0, t_1)$$

(6)

in which

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ [M^{-1}D_2] \end{bmatrix}; \quad D = \begin{bmatrix} 0 \\ [-\eta] \end{bmatrix}$$

(7)

such that $I$ is an $(3n \times 3n)$ identity matrix and the $\eta = (1, \ldots, 1)^T$ is an $n$-dimensional constant vector. Initial condition of the system Eq. (6) can be written with using Eq. (5) as follows;

$$Z(t_0) = Z^0 = \begin{bmatrix} X^{(0)} \\ X^{(1)} \end{bmatrix}$$

(8)

Solution of the system given by Eq. (6) with the initial conditions given in Eq.(8) can be found in the following form;

$$Z(t) = e^{A(t-t_0)} Z^0 + \int_{t_0}^t e^{A(t-s)} q(s) ds$$

(9)

where

$$q(t) = B U(t) + D f(t)$$

(10)

following equation is easily obtained from Eq. (9):

$$Z(t) = e^{A\Delta t} Z(t - \Delta t) + \int_{t-\Delta t}^t e^{A(t-s)} q(s) ds$$

(11)

where $\Delta t$ is the sampling interval. After evaluating the integral on the right hand side of Eq. (11) with the aid of trapezoidal rule, $Z(t)$ can be expressed as

$$Z(t) = e^{A\Delta t} Z(t-\Delta t) + (\Delta t/2) e^{A\Delta t} q(t-\Delta t) + (\Delta t/2) [B U(t) + D f(t)] + O(\Delta t^3)$$

(12)
in which \( O(\Delta t^3) \) denotes the quantity \( g(\Delta t) \) which satisfies the condition \( |g(\Delta t)| \leq C_0 \Delta t^3 \), \( C_0 = \text{constant} > 0 \) for \( \Delta t \to +0 \).

In the classical optimal control law; the classical integral type quadratic performance measure

\[
J = \int_0^{t_1} (Z^T Q_C Z + U^T R_C U) \, dt
\]  

(13)

is minimized; where \( t_1 \) is the duration longer than that of earthquake and the resulting linear optimal control law is obtained as

\[
U(t) = -\frac{1}{2} R_C^{-1} B^T P(t) Z(t)
\]  

(14)

where \( P \) is the solution of the following nonlinear matrix Riccati equation,

\[
\dot{P}(t) + P(t) A - \frac{1}{2} P(t) B R_C^{-1} B^T P(t) + A^T P(t) + 2 Q_C = 0 ; \quad P(t_1) = 0
\]  

(15)

and the subscript C refers to classical linear optimal control; \( Q_C \) and \( R_C \) are positive semi-definite and positive definite weighting matrices. In general, numerical values for the elements of \( Q_C \) and \( R_C \) matrices are assigned according to the relative importance of the state variables and the control forces in the minimization procedure in order to adjust the power requirements in the actuators. If we want to achieve a significant decrease in structure response in time domain, we must assign larger values to the elements of the weighting matrix \( Q_C \) than those of the weighting matrix \( R_C \). The opposite is true when the elements of \( R_C \) are large in comparison with those of \( Q_C \). We can also express Eq. (14) as

\[
U(t) = G_C Z(t)
\]  

(16)

where the control gain matrix \( G_C (3n \times 6n) \) is

\[
G_C(t) = -\frac{1}{2} R_C^{-1} B^T P(t)
\]  

(17)

The Riccati matrix \( P(t) \) obtained from Eq. 15 does not yield an optimal solution since the excitation term \( f(t) \) vanishes within the control interval \([0, t_1]\) [5], or it yields a solution which corresponds to white noise disturbance [19]. It is also known that the Riccati matrix \( P(t) \) can be assumed to be almost constant in practice for structural control applications. Therefore, it is concluded from the Eqs. (15) and (17) that once the numerical values are assigned to the elements of the weighting matrices for a given structure, then the corresponding Riccati matrix and the gain matrix are constant and independent from the earthquake excitations [20].

4 PROPOSED PASSIVE APPROACH

Using the sub-matrices \( G_1 \) and \( G_2 \), the control gain \( G_C \) given by Eq. (17) can be expressed

\[
G_C = [G_1 \quad G_2]
\]  

(18)

substituting Eq. (15) into Eq.(13) gives [21-22]

\[
U = [G_1 \quad G_2] \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix}
\]  

(19)
where $G_1$ and $G_2$ are $(3n \times 3n)$ dimensional sub-matrices of $G_C$. Substituting Eq. (19) into Eq. (4) yields the following equation of the motion of the 3 dimensional structure

$$M \ddot{X}(t) + [C - D_2 G_2] \dot{X}(t) + [K - D_2 G_1]X(t) = D_1 f(t)$$

It is clear from Eq. (20) that the control force modifies the damping and stiffness matrices of the uncontrolled structure. This modification can be expressed as

$$K_{opt} = -D_2 G_1; \quad C_{opt} = -D_2 G_2$$

In most cases, these optimal stiffness and damping matrices obtained from classical linear optimal control law can’t be directly added to the damping and stiffness matrices of the structure. It seems not possible to extract the additional story damping and stiffness parameters which are equivalent to the exact optimal stiffness and damping matrices. If this is possible, then there will be no need for the implementation of active devices. However, since this is not possible in general, this study proposes some simple representative approaches to determine the additional story damping and stiffness parameters which are not exact optimal but suboptimal. If the exact optimal stiffness and damping matrices have an appropriate form, they are added directly to the stiffness and damping matrices of the structure. If not, the eigenvalues of the optimal stiffness and damping matrices are added to the corresponding stiffness and damping coefficients of the 3 dimensional structures as follows;

$$\text{Eig}_i(-D_2 G_2) = Ca_i; \quad i = 1...3n; \quad \text{Eig}_i(-D_2 G_1) = Ka_i; \quad i = 1...3n$$

where $Ca_i$ and $Ka_i$ represent the $i$th additional translational and rotational damping or stiffness of the stories. Where $n$ represents the $n$th story. Or, average damping and stiffness parameters can be assigned to stories as follows;

$$\sum_{i=1}^{n} \text{Eig}_i(-D_2 G_2) \over 3*n = Ca_j; \quad \sum_{i=1}^{n} \text{Eig}_i(-D_2 G_1) \over 3*n = Ka_j; \quad j = 1..3n$$

Or, the maximum damping and stiffness parameters can be assigned to the first floor while the minimum damping and stiffness parameters can be assigned to the top floor, the other damping and stiffness parameters are assigned larger to smaller from the first floor to the top floor

$$[\text{Eig}_i(-D_2 G_2)]_{\text{max,mini}} = Ca_{1...3n}; \quad [\text{Eig}_i(-D_2 G_1)]_{\text{max,mini}} = Ka_{1...3n}$$

5 NUMERICAL EXAMPLE

As a numerical example, a 3-story steel building designed for the SAC project Los Angeles, California region is considered [2]. The 2D building frame, plan, elevation, orientation and material properties are given in Figure 2 (the figure is taken as original drawing from [2]). This 2D building frame is converted to a 3 dimensional tier building by considering the matrix formulation of the tier buildings which has been defined in previous section briefly and detailed in [17-18]. For the 3 dimensional three story tier building there are totally nine degrees of freedom. Three degrees of freedom (DOF) for each story (2 translational and 1 rotational DOF).
Schematic figure of the 3D tier building under one directional earthquake in NS direction and the orientation of the building are given in Figure 3. As it has been presented in Figure 3 that DOF1-4-7 in the 3D tier building model correspond to the NS direction of the first, second and third stories of the 2D benchmark structure respectively. The first three natural frequencies of the 2D benchmark building structure are 0.99, 3.06 and 5.83 Hz [2]. While the natural frequencies of the 3D tier building are calculated as 1.00, 1.58, 3.00, 3.68, 4.38, 5.87, 6.51, 10.4051 and 15.19. The mode shapes of the two models are compared and the matching frequencies are defined. The frequencies for 2D and idealized 3D tier building in NS direction match each other very well 0.99-1.00, 3.06-3.00 and 5.83-5.87 for 2D and 3D structure respectively.

After comparing the frequencies in NS direction the 3 dimensional tier building behavior is dynamically analyzed under El Centro 1940 earthquake (NS component). Programs and simulations which were developed in MATLAB and MATLAB-SIMULINK and verified in ANSYS are used to carry out all these dynamical analysis of the 3D tier building with or
without control forces. El Centro 1940 earthquake (NS component) is applied to the structure in NS direction. The eccentricity of the structure is calculated to be 0.1 m in NS direction while it is 5.22 m in EW direction.

To obtain the additional stiffness and damping parameters, the dynamic behavior of the building is first investigated considering a full active tendon control system which is implemented by means of actuators exerting forces at each floor in NS direction.

The weighting matrices $Q_C$ and $R_C$ are partitioned as for the classical linear optimal control

$$Q_C = \begin{bmatrix} K & C \\ C & M \end{bmatrix} ; \quad R_C = 0.5 \times 10^{-4} I_{(3 \times 3)}$$ (25)

$K_{opt}$ and $C_{opt}$ for the 3 dimensional building are obtained according to Eq. (21) and they found to be different in arrangement from the stiffness and damping matrices of the tier building. Since it is not possible to add these matrices directly to the stiffness and damping matrices of the 3D tier building, as a possible approach, the corresponding eigenvalues are added by using Eq. (22). As a fully active tendon controller system is implemented in NS direction with single active control force in each story we obtained three eigenvalues to be added to stiffness and damping matrices of the structure. Real parts of the eigenvalues of these matrices are calculated as; $Eig(K_{opt})=\{326, 2296.2, 2296.2\}$ and $Eig(C_{opt})=\{68, 68, 86\}$. The first, second and third eigenvalues are added to the stiffness and damping parameters of the first, second and third stories respectively in NS direction. This case is named as C1. Comparison of maximum absolute responses in NS direction, EW direction and rotations for uncontrolled 3-dimensional structure, structure with CLOC and C1 case are given in Figure 4. As it can be indicated from Figure 4 that C1 case is effective in decreasing maximum displacements in NS direction. As it is a totally passive approach CLOC case is superior to C1 case. The reduction of maximum story displacement responses for C1 case in NS direction is at the extent of %30 while it is %40 for CLOC case. As the active control and simple passive control approach is implemented to the structure in NS direction there is not much reduction in uncontrolled story displacements in EW direction and rotations. The uncontrolled story displacement reduction percentage in EW direction and rotations these directions are at similar extent for both C1 and CLOC cases. The reduction percentage for uncontrolled EW displacements and rotations for CLOC is $\approx$ %10 while it differ %5-%8 for C1 case.

![Figure 4: Maximum responses of 3-dimensional structure](image-url)
6 CONCLUSION

A new simple passive approach for vibration mitigation of structures is presented in this paper. This simple passive control approach addresses the question that how much damping and stiffness must be added the structure to strengthen the structures against earthquakes. The required damping and stiffness parameters are obtained from the optimal control gain matrix derived from the solution of classical closed loop control (CLOC) with white noise disturbance. This simple approach was applied to shear building structures and the effectiveness of the approach was shown in the previous study considering shear buildings [1]. In this study the approach is applied to a 3 dimensional tier building structure considering a single earthquake input. The approach is implemented to the 3-dimensional structure only in one direction. It is shown by numerical simulation results that the proposed passive control approach suppresses the uncontrolled responses significantly and shows a reasonably close and similar performance to the controlled case. The study also shows that without implementing any active device to the structure, modifying the stiffness and damping parameters of the existing structure in reasonable amounts derived from optimal control approach can improve the structural seismic performance. However, it is an expected result that an active system is always superior to a passive system provided that the required active control forces are generated in time disregarding the practical issues such as actuator saturations, time delay effects, spillover effects and possible uncertainties. The reason for the comparison of a passive system to an active system in this study is that the additional passive damping and stiffness parameters required for the proposed passive control approach are calculated based on the optimal active control gain. So, it is a contribution to passive control studies provided that the proposed passive control approach has a similar and close enough performance to active control performance without implementing any active device.

The results presented in this study are preliminary for implementing this passive approach for 3-dimensional structures and further study will be carried out by implementing the active control in both directions as well as EW and rotational and using that implementation scheme to obtain the additional stiffness and damping in both directions.

REFERENCES


