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ON THE ACCURACY AND COMPUTATIONAL COST OF TIME HISTORY ANALYSIS OF RESIDENTIAL BUILDINGS BY THE QUASI-WILSON-THETA METHOD

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Abstract. *Quasi-Wilson-Theta is a time integration method in 2011 officially introduced in the literature. In this paper, the quasi-Wilson-Theta method is compared with the original Wilson-Theta method from the points of view of accuracy and computational cost, and via the computational analysis of a real building. The consequence is adequate performance in the range of numerical stability.*

1 INTRODUCTION

Considering the semi discretized equations of motion [1-4], i.e.

$$\begin{array}{l} \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{f}_{\text{int}}(t) = \mathbf{f}(t), \qquad 0 \le t \le t_{end} \\ \text{Initial conditions:} \left| \begin{array}{l} \mathbf{u}(t=0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_0 \\ \mathbf{f}_{\text{int}}(t=0) = \mathbf{f}_{\text{int}_0} \end{array} \right. \\ \text{Additional constraints: } \mathbf{Q} \end{array} \qquad (1)$$

where, t and t_{end} imply the time and the duration of the dynamic behavior; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\text{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$ denote the vectors of displacement, velocity, and acceleration; $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$ define the initial status of the model; and finally, $\mathbf{Q}$ represents some restricting conditions, e.g. additional constraints in problems involved in impact or elastic-plastic behavior [5, 6], all in view of the degrees of freedom set for the model, time integration is the most versatile analysis method [7, 8], started from mid twentieth century, with the efforts of J.C. Houbolt, N.M. Newmark, etc., and continued by R.W. Clough, E.L. Wilson, K.J. Bathe, R.L. Taylor, T.J.R. Hughes, and many others [9-14]. In 2011, the quasi-Wilson-Theta method is introduced [15] as an inferior version of the Wilson-Theta method, with first order of accuracy, slightly less computational cost, and numerical stability and omission of higher modes as addressed in Figures 1 and 2, where, Δt is the integration step size, T is the natural period under consideration, θ stands for the parameter of the method, and ρ introduces the spectral density [17].

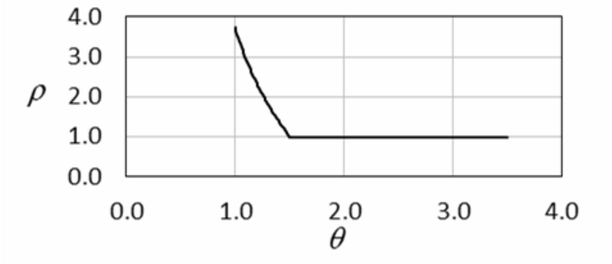

Figure 1: Numerical stability for the quasi-Wilson-Theta method [16].

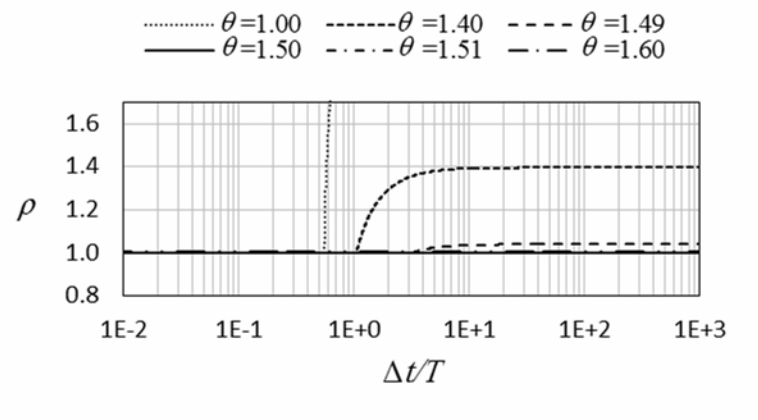

Figure 2: Omission of higher modes by the quasi-Wilson-Theta method [16].

After several studies on the quasi-Wilson-Theta method [14-16], herein, for the first time, attention is paid to the performance of the new method in real analyses, and the performance is compared with the Wilson-Theta method, from the points of view of accuracy and computational cost. In Section 2, the performance is studied theoretically. In Section 3, a real building, designed according to the Iranian codes [18, 19], is introduced, together with four major historical earthquake records in Iran [20]. In Section 4, the results of the analyses, by the Wilson-Theta and quasi-Wilson-Theta methods, are discussed; and finally the paper is briefly concluded, in Section 5.

2 THEORY

In view of the amplitude decay and period elongation plots displayed in Figures 3 and 4, and the fact that step by step time integration is originally based on the assumption of linear behaviour at each step, the difference of the two methods, is negligible for small values of Δt.

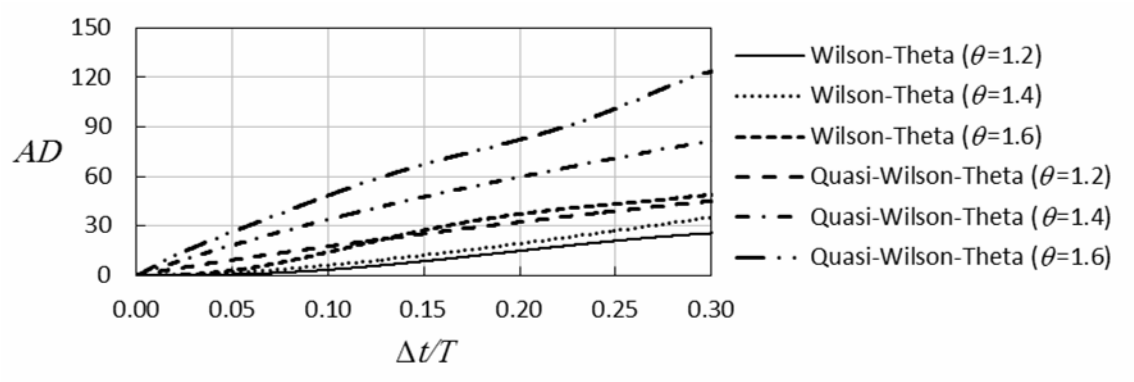

Figure 3: Amplitude decay [16, 21].

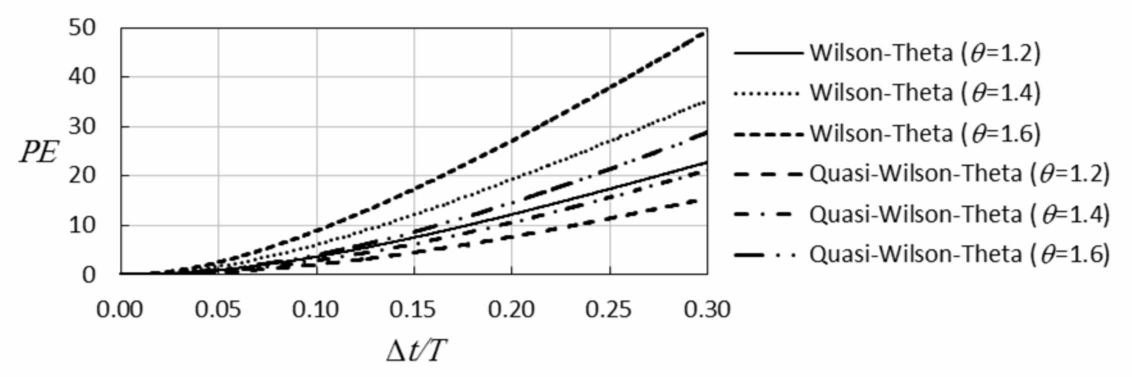

Figure 4: Period elongation [16, 21].

In seismic analyses, the $\mathbf{f}(t)$, in Eq. (1), originates in the strong ground motions, e.g.

$$\mathbf{f}(t) = -\mathbf{M}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}\ddot{u}_g \tag{2}$$

($\ddot{u}_g$ is the ground acceleration available as a digitized record), and the integration steps, Δt, are conventionally set equal or smaller than the earthquake digitization step, Δt_f [22, 23,24],

$$\Delta t \le \Delta t_f \tag{3}$$

This is while according to the recent progresses in seismological instrumentations [25],

$$\Delta t_f \le 0.01 \tag{4}$$

In view of Eqs. (3) and (4) and Figures 3, 4 and the range of major natural periods of building structural systems [26], it is reasonable to expect acceptable change of accuracy in replacing the quasi-Wilson-Theta method, instead of the Wilson-Theta method, in the analyses. Numerical evidence of this claim is reported in Sections 3 and 4.

3 THE STRUCTURAL SYSTEM

The fifteen story residential building with steel dual structure, designed according to the Iranian codes [18, 19] (see Figure 5), is modelled, as the shear frame introduced in Table 1. The strong ground motions addressed in Figure 6 are from major earthquakes in Iran [20], well considered in the Iranian codes [18, 27]. In order to make an idea about the performance of the quasi-Wilson-Theta method in real analyses, the structural system is considered subjected to the seismic excitations in Figure 6, which in order to take into account the changes of the θ, three values are assigned to θ,

$$\theta = 1.2,\ 1.4,\ 1.6 \tag{5}$$

In view of the exact responses reported partially in Figure 7,

$$\frac{T}{10} = 0.01,\ 0.02,\ 0.02,\ 0.01 \tag{6}$$

respectively, for the four strong motions (T, in Eq. (6), stands for the least dominant period of the vibration), and the following comment for assigning values to Δt

$$\Delta t = \mathrm{Min}\left(\frac{T}{10}, h_s, \Delta t_f\right) \tag{7}$$

(the new parameter, h_s, implies the restriction imposed by numerical stability [16, 17]), the analyses are carried out with steps addressed in Tables 2 and 3.

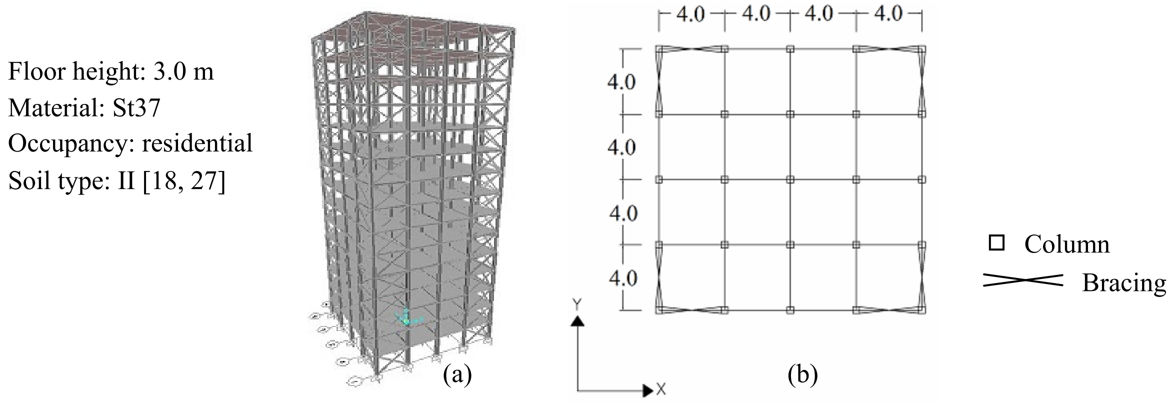

Figure 5: The structural system under consideration [29]: (a) schematic view, (b) plan (2D view from top).

Story	Mass (Kg)	Stiffness (N/m)	Damping ratio
1	222992	1.8638E9	0.05
2	228302	1.8638E9	
3	228302	1.8638E9	
4	231057	1.6307E9	
5	231057	1.6307E9	
6	225858	1.2454E9	
7	225858	1.2454E9	
8	224049	1.1530E9	
9	224049	1.1530E9	
10	224049	1.1530E9	
11	224049	1.1530E9	
12	218271	8.4128E8	
13	218271	8.4128E8	
14	218271	8.4128E8	
15	150935	8.4128E8	

Table 1: Characteristics of the shear frame model.

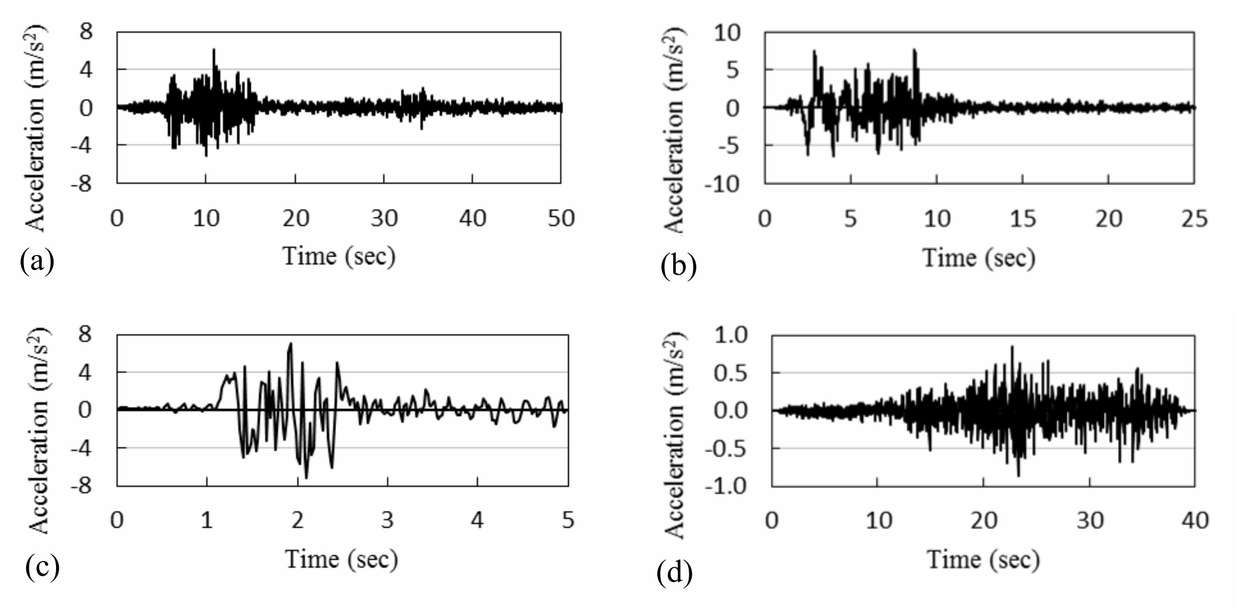

Figure 6: The strong ground motions under consideration: (a) Abbar (1990), (b) Bam (2003), (c) Naghan (1977), (d) Tabas (1978) [20].

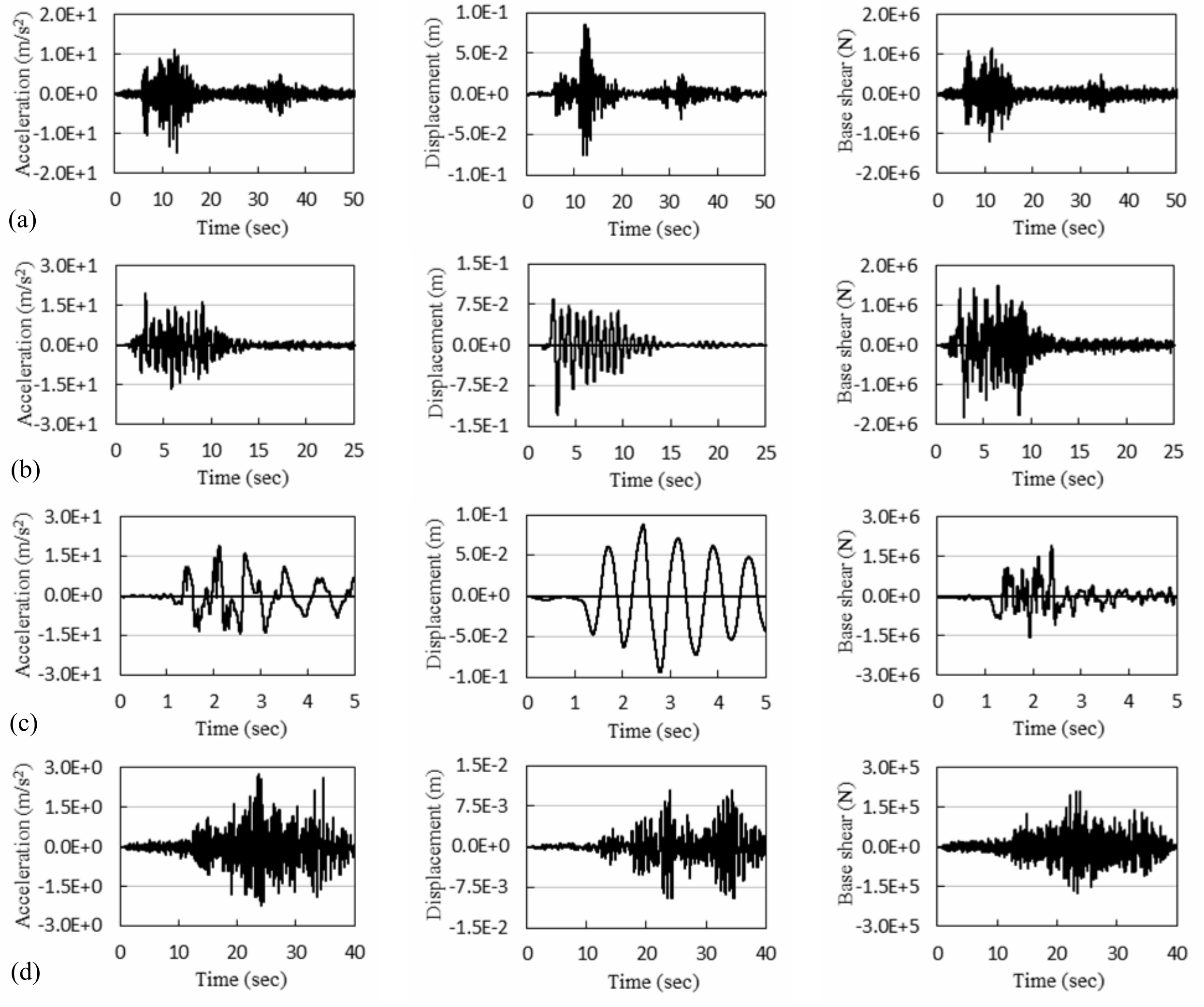

Figure 7: The exact top acceleration, mid-height displacement, and base shears: (a) Abbar (1990), (b) Bam (2003), (c) Naghan (1977), (d) Tabas (1978) [20].

Excitation	θ=1.2	θ=1.4	θ=1.6
a	0.005	0.005	0.005
b	0.005	0.005	0.005
c	0.02	0.02	0.02
d	0.01	0.01	0.01

Table 2: Integration steps in analysis with Wilson-Theta method.

Excitation	θ=1.2	θ=1.4	θ=1.6
a	0.005	0.005	0.005
b	0.005	0.005	0.005
c	0.02	0.02	0.02
d	0.01	0.01	0.01

Table 3: Integration steps in analysis with quasi-Wilson-Theta method.

4 NUMERICAL OBSERVATION

In view of the explanations in Section 3, the responses obtained for the analyses are reported, in Figures 8-10, for Excitation a, and, in Tables 4-6, for Excitations b-d. The computational costs are meanwhile compared in Table 7 in terms of analysis run time (the difference between the memories is trivial). Apparently, while the accuracies are in some cases better when implementing the Wilson-Theta method, and in some cases better when implementing the quasi-Wilson-Theta method (pictorially unrecognizable in all cases), the computational cost are in all cases less in implementation of the quasi-Wilson-Theta method. This implies the good performance of quasi-Wilson-Theta method in practical real analyses.

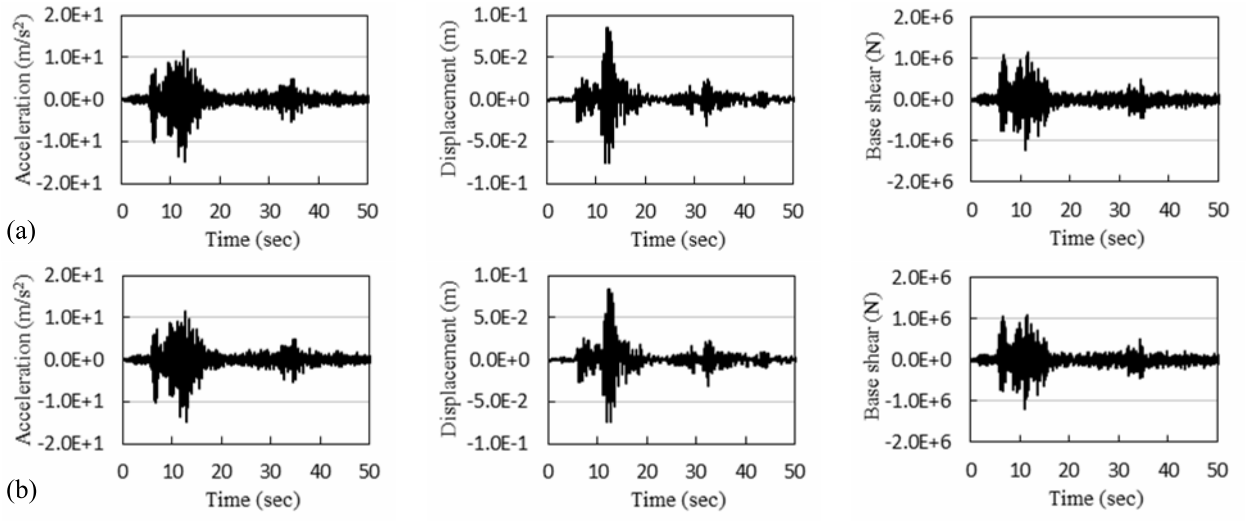

Figure 8: The top acceleration, mid-height displacement, and base shears, obtained from the analyses, against the Excitation a, with θ=1.2: (a) Wilson-Theta, (b) quasi-Wilson-Theta.

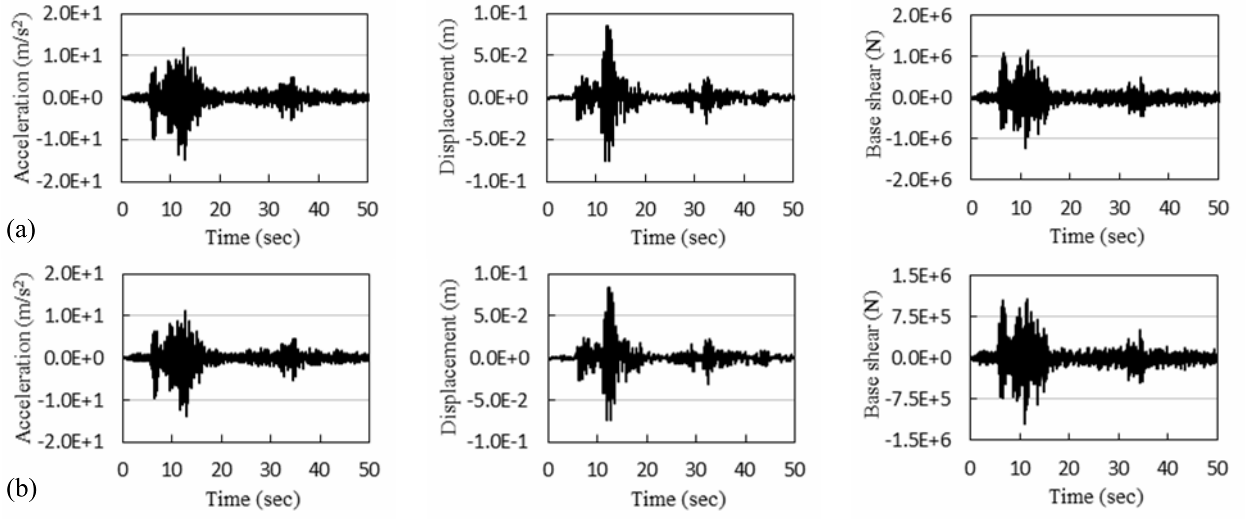

Figure 9: The top acceleration, mid-height displacement, and base shears, obtained from the analyses, against the Excitation a, with θ=1.4: (a) Wilson-Theta, (b) quasi-Wilson-Theta.

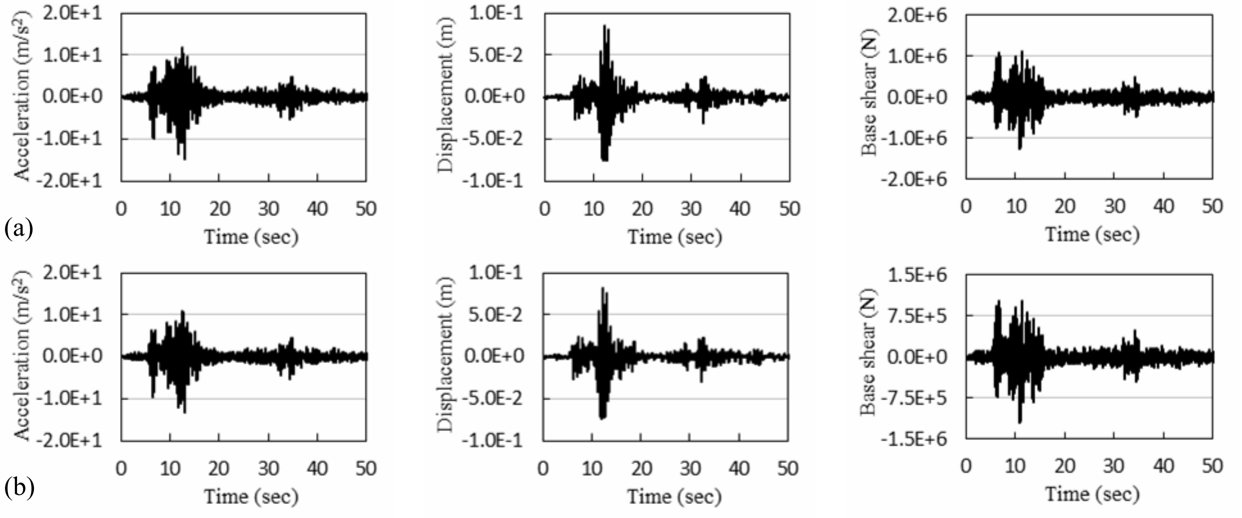

Figure 10: The top acceleration, mid-height displacement, and base shears, obtained from the analyses, against the Excitation a, with θ=1.6: (a) Wilson-Theta, (b) quasi-Wilson-Theta.

	Top acceleration	Mid-height displacement	Base shear
Wilson-Theta (θ=1.2)	3.019	0.347	2.614
Quasi-Wilson-Theta (θ=1.2)	7.539	2.991	5.549
Wilson-Theta (θ=1.4)	5.312	0.619	4.617
Quasi-Wilson-Theta (θ=1.4)	13.695	5.766	9.885
Wilson-Theta (θ=1.6)	8.047	0.957	7.017
Quasi-Wilson-Theta (θ=1.6)	18.910	8.361	13.525

Table 4: Errors of analyses against Excitation b, with the L2 norm (%).

	Top acceleration	Mid-height displacement	Base shear
Wilson-Theta (θ=1.2)	24.144	04.076	33.973
Quasi-Wilson-Theta (θ=1.2)	19.237	09.197	25.957
Wilson-Theta (θ=1.4)	30.557	07.043	45.888
Quasi-Wilson-Theta (θ=1.4)	27.208	16.809	33.087
Wilson-Theta (θ=1.6)	37.323	10.515	58.679
Quasi-Wilson-Theta (θ=1.6)	33.701	23.464	40.910

Table 5: Errors of analyses against Excitation c, with the L2 norm (%).

	Top acceleration	Mid-height displacement	Base shear
Wilson-Theta (θ=1.2)	9.633	2.157	8.812
Quasi-Wilson-Theta (θ=1.2)	14.644	6.762	11.019
Wilson-Theta (θ=1.4)	16.342	3.799	14.586
Quasi-Wilson-Theta (θ=1.4)	24.480	12.395	17.850
Wilson-Theta (θ=1.6)	23.606	5.735	20.489
Quasi-Wilson-Theta (θ=1.6)	31.938	17.320	23.034

Table 6: Errors of analyses against Excitation d, with the L2 norm (%).

Excitation	a	b	c	d
Wilson-Theta (θ=1.2)	43.462	21.746	4.025	31.465
Quasi-Wilson-Theta (θ=1.2)	30.685	13.743	2.730	21.856
Wilson-Theta (θ=1.4)	43.946	19.562	3.963	31.060
Quasi-Wilson-Theta (θ=1.4)	30.639	15.101	2.730	21.918
Wilson-Theta (θ=1.6)	43.946	21.715	3.947	31.107
Quasi-Wilson-Theta (θ=1.6)	31.496	13.743	2.730	22.012

Table 7: The computational costs in implementation of Wilson-Theta and quasi-Wilson-Theta methods.

5 CONCLUSION

In spite of the inferior theoretical characteristics of the quasi- Wilson-Theta method compared to the Wilson-Theta method, in implementation in analysis of mid-rise residential buildings structures, designed according to building design codes, the quasi-Wilson-Theta method can be superior. From the point of view of accuracy, the difference between the responses time histories can be pictorially unrecognizable. Still, more study is essential.

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