NUMERICAL AND EXPERIMENTAL STUDY OF A BIT-ROCK VIBRO-IMPACT SYSTEM

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ABSTRACT

In order to obtain the strong axial impact force for rotary-percussive drilling technology, the dynamics of a bit-rock vibro-impact system is studied to investigate the dynamic evolution mechanism of percussive drilling. On the one hand, a dedicated experimental setup is built up, see Fig.1 (a), and both the linear and nonlinear bit-rock interactions are investigated [1]. On the other hand, a bounded mathematical model for the experimental rig developed via oscillation reconstruction [2] is analyzed by both numerical integration and numerical continuation, see Fig.1 (b). Based on the codimension-one bifurcations detected during one-parameter continuation, the two-parameter bifurcation curves are traced by two-parameter continuation. The influences of three control parameters (frequency and amplitude of excitation, and static force applied on the bit) and the rock stiffness combination [3] are studied experimentally and numerically, and the correspondence between the experimental result and the numerical simulation is verified primarily, see Fig.1 (c). Moreover, bifurcation analysis and phase-plane analysis indicate that the dynamic responses of this system are mainly period-one motions with varied impact numbers, which can further develop into period-two motions or even chaos via period-doubling bifurcations, but will go back to period-one motion via inverse period-doubling bifurcations or fold bifurcations. The vibration condition of the bit determines the axial impact force, to obtain the most powerful percussion, the ideal vibration of the bit is the period-one one-impact motion. Therefore, high excitation frequency and amplitude are suggested, but high static force should be avoided since it will trigger harmful chattering for percussive drilling.

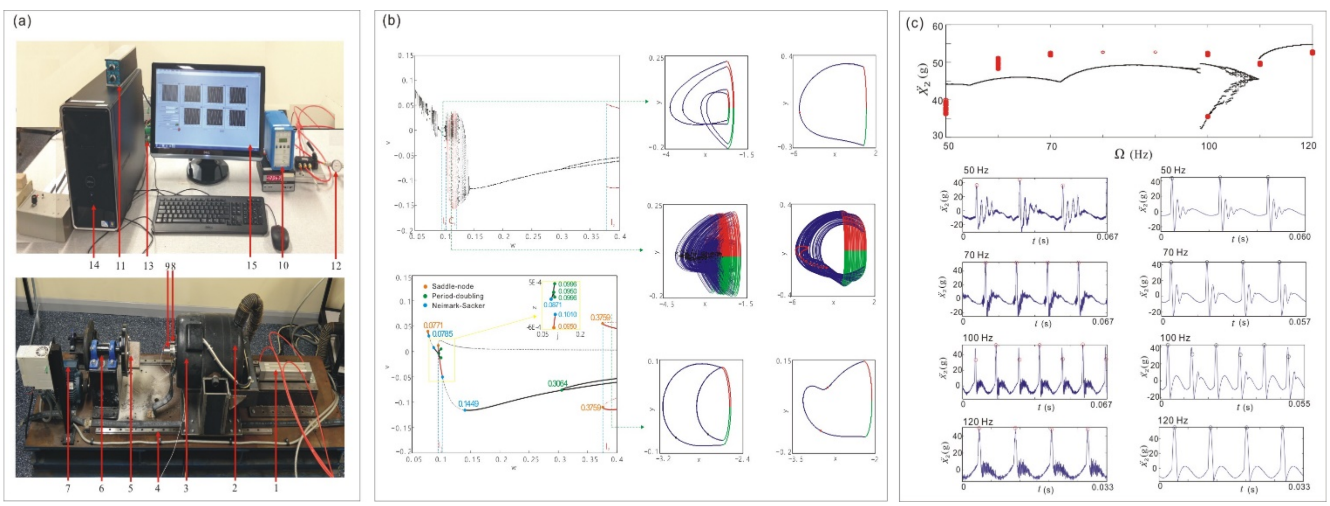

Figure 1: Numerical study and experimental verification of a bit-rock vibro-impact system

Keywords: non-smooth, vibro-impact, bifurcation, numerical continuation, rotary-percussive drilling

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A NUMERICAL CONTINUATION APPROACH FOR THE STUDY OF PIECEWISE-SMOOTH SYSTEMS WITH DELAY

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ABSTRACT

In this presentation we will propose a numerical approach in order to analyze non-smooth DDEs via path-following (continuation) methods. Numerical continuation is a well-established technique that permits an in-depth analysis of a system dynamics, under parameter variations. In particular, it allows tracing certain invariant sets (such as equilibria, periodic orbits, homoclinic orbits, etc.) as selected system parameters vary, usually via a predictor-corrector approach [1]. Software packages for continuation analysis have been developed in the past, in particular for smooth ODEs (AUTO, CONTENT, MATCONT), piecewise-smooth ODEs (SLIDECONT, TC-HAT, COCO) and smooth DDEs (DDE-BIFTOOL, and PDDE-CONT). To the best of our knowledge, however, no software package for continuation analysis of non-smooth DDEs has been developed sofar, and this is precisely the main motivation of the present work.

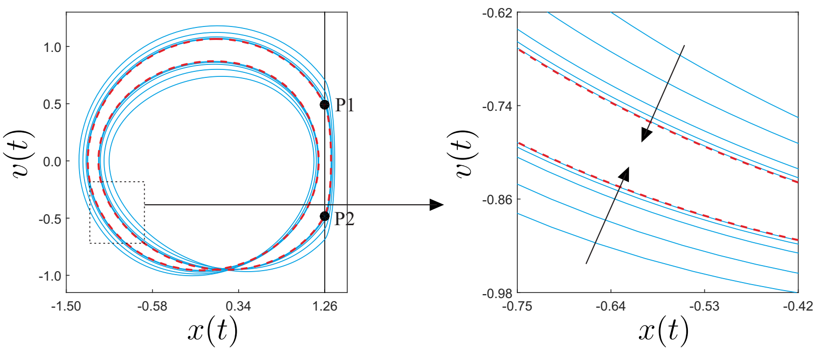

Figure 1: Family of approximating solutions (in blue) as the step-size decreases (arrows indicate the direction of decreasing step-size). Here, the exact solution of the impact oscillator with time-delayed control is plotted in red. The position and velocity of the oscillating mass is given by $x(t)$ and $v(t)$, respectively.

The proposed numerical approach is based on the *chain method* [2], in combination with a higher-order approximation scheme of the original DDE by considering a finite sequence of Taylor expansions. Let us consider a DDE of the form

$$\dot{x}(t) = f(t, x(t), x(t - \tau_d)), \tag{1}$$

which defines a system of delay differential equations (DDEs) with constant delay $\tau_d > 0$, where $f : \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$ is a sufficiently smooth function. Let us now take $N \in \mathbb{N}$ sufficiently large and define the grid points $t_i := i\frac{\tau_d}{N}$, $i = 0, \ldots, N$. Furthermore, define $u_i(t) := x(t - t_i)$ for all $t \geq 0$, $i = 0, \ldots, N$. In this setting, we obtain via Taylor expansion that

$$u_{i-1}(t) = x\left(t - \left(t_i - \frac{\tau_d}{N}\right)\right) = u_i\left(t + \frac{\tau_d}{N}\right) = \sum_{k=0}^{M} \frac{1}{k!} u_i^{(k)}(t) \left(\frac{\tau_d}{N}\right)^k + \mathcal{O}\left(\left(\frac{\tau_d}{N}\right)^{M+1}\right), \quad \text{and} \tag{2}$$

$$\dot{u}_0(t) = f(t, u_0(t), u_N(t)), \tag{3}$$

for all $t \geq 0$, $i = 1, \ldots, N$, $M \geq 1$. After neglecting the $\mathcal{O}$-terms, we obtain from (2) a system of dN differential equations of order M. In this way, a piecewise-smooth dynamical system with constant delay can be approximated by a piecewise-smooth system of ODEs of large dimension, which then allows the study of the resulting model in the framework of hybrid dynamical systems [3]. The effectiveness of the proposed numerical scheme is tested on a case study given by a periodically forced impact oscillator driven by a time-delayed feedback controller, see Fig. 1.

Keywords: Delay differential equation, Piecewise-smooth dynamical system, Numerical continuation.

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EFFECT OF STOCHASTIC SWITCHING SURFACE IN NONSMOOTH SYSTEMS

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In recent years the bifurcations occurring in switching dynamical systems have been the subject of great interest, as many physical systems (examples: impact oscillators, relay-feedback systems), engineering systems (examples: power electronics circuits, walking and hopping mechanisms in robotics, switching circuits) and biological systems (gene regulatory networks and neurons in computational neuroscience) are modelled by switching dynamical systems. Piecewise-smooth (PWS) dynamical systems show many phenomena that do not occur in smooth dynamical systems, such as *border collision bifurcations, robust chaos*. Detail investigation have been done in 1D, 2D and higher dimensional piecewise smooth systems in deterministic cases.

These physical systems always exist in noisy environments, and therefore it is necessary to study the dynamics of such systems taking into account the effect of various possible noise terms that can significantly affect the system dynamics. However, investigations into systems that are both piecewise-smooth and involve noise are relatively uncommon. Very few such studies [1] have been reported in the literature where the noise term appears in the functional form of the system, and hence it affects only the subsequent iterates but does not affect the position of the border.

However this assumption may not always be true and there may arise situations where the border is not fixed due to the presence of some special kind of noise in the system. Such variation of the border may be caused due to mechanical reasons (loosely fitted panels or dividers), due to variations in the environment (effects of change of pressure or temperature), due to some defined rule of measurements [2]. In these cases we have to consider a border moving stochastically, in a bounded region of the phase space.

Mandal & Banerjee [3] have considered a piecewise smooth (PWS) one-dimensional map, having stochastically varying border, which is allowed to move in a small bounded region of the phase space while retaining the deterministic dynamics on each compartment of the phase space. They have shown the effect of stochastically moving border on the basin of attraction.

In this work, we consider a two-dimensional piecewise smooth map with stochastically moving border. The system description is

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{cases} \begin{pmatrix} \tau_l & 1 \\ -\delta_l & 0 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \text{if } x_n \leq \xi_n \\ \begin{pmatrix} \tau_r & 1 \\ -\delta_r & 0 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \text{if } x_n \geq \xi_n \end{cases} \tag{1}$$

where ξ_n denotes the stochastic noise term. In keeping with the mechanical and measurement sources of noise we assume that the switching surface varies randomly but within specific bounds.

The question we are addressing is: What happens when the border stochastically move? We have done a systematic analysis of the effects of stochastically moving border in higher dimensional piecewise smooth maps.

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EFFECTS OF TEMPERATURE AND STRAIN RATE ON MECHANICAL BEHAVIOUR OF 3D PRINTED FLEXIBLE THERMOPLASTIC POLYURETHANE HONEYCOMB BASED STRUCTURES

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ABSTRACT

The main goal of this work is a presentation of the investigations results concerned on mechanical behavior of 2D honeycomb based structures under compression loading, with a particular emphasis on the temperature and strain rate effects. Regular honeycomb and four additional variants with gradually changed topologies (Fig.1a) were manufactured additively with the use of FFF (Fused Filament Fabrication) 3D printing technique (Fig.1b). The material used during fabrication process was thermoplastic polyurethane TPU 95 which demonstrate a hyper-elastic mechanical properties (Fig.1c). The adopted investigation methodology included a manufacturability study, strength tests of the base material and experimental compression tests of developed regular cellular structures with consideration two loading scenarios. The first one, refers to quasi-static tests where additionally the influence of temperature conditions (-20 ^oC, 0 ^oC, +25 ^oC) was taken into account. The other approach, related to dynamic compression tests was carried out with the use of split Hopkinson pressure bar laboratory set up in a direct configuration. Based on carried out experimental studies it has been found that applied TPU 95 Polyflex material indicate a non-linear stress strain relationship and very high flexibility (elongation up to 380 %). Nevertheless, depending on the temperature conditions the stress-strain curves are varying significantly. In turn, dynamic compression tests of regular honeycomb and four variants with gradually changing topologies revealed that buckling and bending were the main mechanisms responsible for the deformation of developed structures. Furthermore, it was found that proposed TPU 95 material indicate a high strain rate sensitivity.

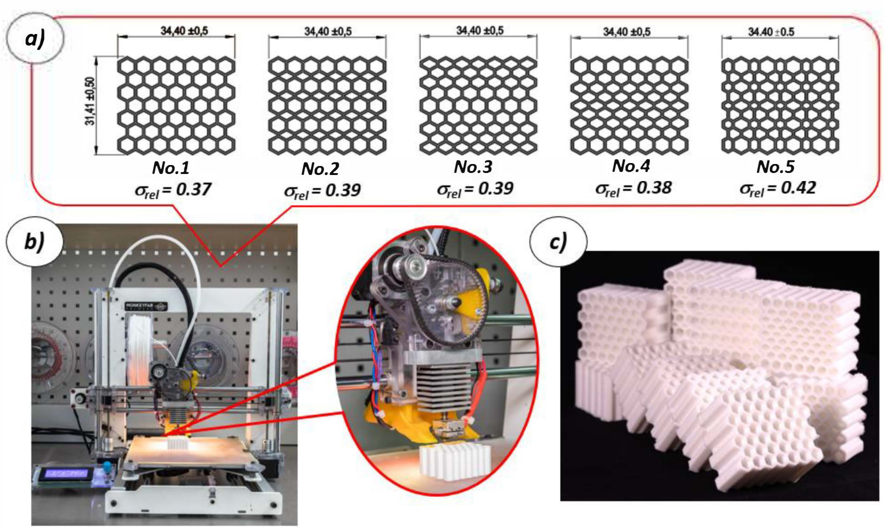

Figure 1: The main view of studied regular cell structures:
a) structure topologies,
b) FFF 3D printing process, c) fabricated TPU 95 structure specimens

Keywords: 3d printing, Fused Filament Fabrication, compression tests, honeycomb structures, hyper elasticity, thermoplastic polyurethane, split Hopkinson pressure bar

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USING HIGH-SPEED VIDEO RECORDING TO DETERMINE ROTARY POSITION AND SPEED IN A DOUBLE PENDULUM MOTION

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ABSTRACT

Measurement of position and velocity of rotating objects rely on installation of some additional device on them, like rotary encoder. Such a device can significantly change object's properties like mass moment of inertia, location of center of gravity etc. therefore dynamic properties also change. Non-contact methods appear not to have the above mentioned drawback but using external sensors like Hall devices also have a load influence to the system when physical properties cause forces and moments comparable in values. To measure the angular position (and possibly angular velocity) of the object video measurement technique stands as a non-contact alternative. The rotational motion can be recorded by a high-speed camera and then using tracking software kinematics of the chosen marked points traced. Digitized results allow for determination of angular position of selected elements and further by differentiation rotary speed can be calculated. Another advantage is possibility of recording maps called position maps which have half of the information stored in well known Poincaré maps. The experimental results are compared with simulations for different type of motion realized by set of pendulums. Figure presents two examples of experimentally recorded data collected by use of different digital cameras in similar conditions. Points represent markers located at excited axis of the large pendulum, axis and center of mass of the smaller one.

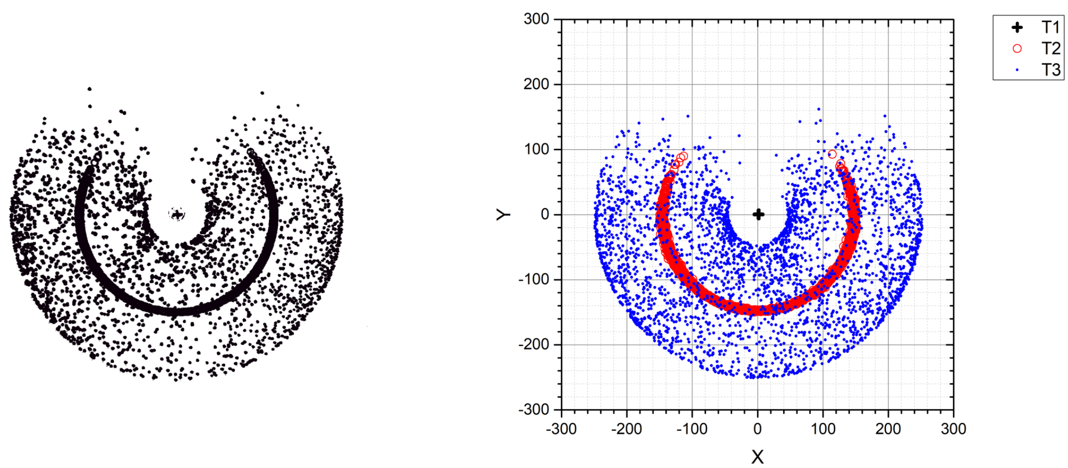

Figure 1: Digital photograph (negative view) composed from overlay of 2711 separate photographs for the irregular motion of both pendulums, $A = 4.10\,\mathrm{mm}$, $f = 4.50\,\mathrm{Hz}$; camera Canon 5D at 1920x1080 pixels, shutter released every excitation period – left and reconstructed from a movie taken at 45 fps by Phantom v711 high speed camera after choosing every 10-th frame from the record, irregular motion, 3069 frames collected, movie resolution 800x600 pixels – right

Keywords: angular position, rotary speed, video tracking, reconstruction, video analysis.

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HIGH-SPEED CAMERA ANALYSIS IN STRUCTURAL DYNAMICS AND VIBRATION FATIGUE—RECENT ADVANCES

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ABSTRACT

The digital image correlation technique were used in experimental mechanics for several decades; however, it is the last decade when the advancement in hardware and image processing have made the image approach feasible for structural dynamics research. In recent years, the simplified optical flow method enabled fast and accurate deflection shape analysis at full-field spatial resolution which has not been seen before (e.g. 100k measurement points) [1]. The digital image correlation (DIC) in general performs better than the simplified optical flow, but also requires orders of magnitude more computational effort. Motion magnification and other phase based techniques have also been researched in recent years. Recently, a single-camera experimental-modal-analysis (EMA) for full-field 3D deflection shape reconstruction was proposed [2, 3], see Fig. 1.
Besides visual spectra based cameras, also the thermal cameras are researched for applications in structural dynamics. Classical experimental procedures (*e.g.*, strain-gauge) allow to perform local damage evaluation, while thermoelasticity, as a non-contact and full-field measurement technique, permits to obtain also spatial information. Recently introduced thermoelasticity-based methods enable the time- and frequency-domain identification of full-field damage intensity; here the potential to relate the damage intensity to the underlying dynamics of the structure is discussed.

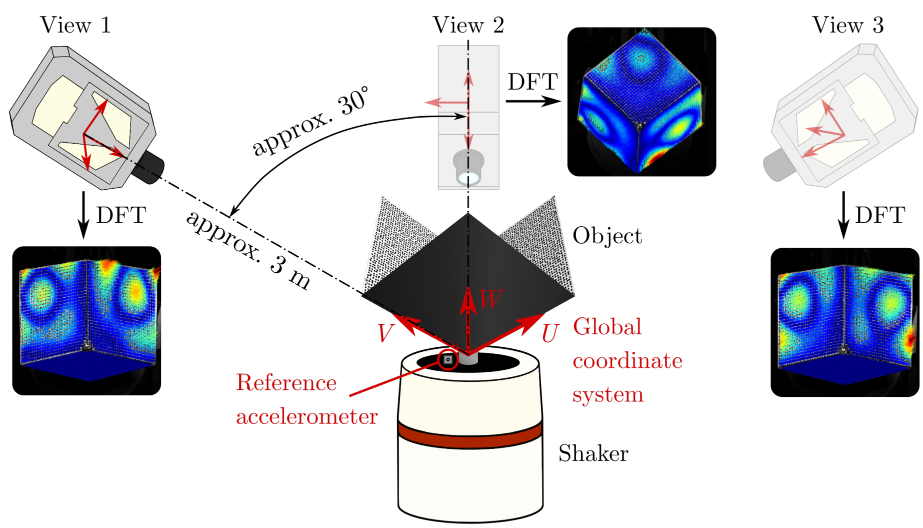

Figure 1: Full-field 3D deflection shape reconstruction using frequency domain triangulation.

Keywords: high-speed camera, experimental modal analysis, thermoelasticity.

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