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WHEEL SHIMMY AND DELAYED TYRE MODELS

Gabor Stepan

Budapest University of Technology and Economics, Department of Applied Mechanics
Műegyetem rkp 5, Budapest H-1111, Hungary
stepan@mm.bme.hu

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Abstract. *Brief description of low degree-of-freedom models of shimmying wheels is presented. When the wheels are rigid relative to other parts of the supporting structure, the complex shimmy phenomena can be described by single-contact-point models. These models can predict the loss of stability of stationary rolling, and the simple geometric nonlinearities of these systems lead to Hopf bifurcations, which are typically subcritical. Even isolated unstable periodic motions may exist for certain realistic parameters. The prediction of the isolated unstable self-excited vibrations is difficult at the design stage, and even extensive experiments may fail to identify them. This becomes even worse when Coulomb friction exists at the king pin of these structures, which can be modelled as another strong nonlinearity in the system. Analytical studies extended with AUTO-07p numerical bifurcation analysis of such systems are carried out to construct bifurcation charts while the rolling condition is also checked for the contact force between the wheel and ground. Parameter domains are identified, where bistability occurs, that is, where stable stationary rolling and violent shimmy may coexist.*

The wheel is modelled to be soft relative to the supporting structure if there is a pneumatic tyre on the wheel and the classical creep force theory usually gives satisfactory results regarding possible shimmy. In these cases, however, a certain time delay effect becomes relevant in the corresponding stretched-string-like tyre models. The stability charts obtained by linear stability analysis present various bifurcation phenomena. These are checked by experiments on a test rig and also by numerical simulation that involves the partial sliding of the tyre in the contact region as a nonlinear effect. The sense of the Hopf bifurcations is compared to various shimmy models including the classical single-contact-point ones. Double Hopf bifurcations leading to quasi-periodic oscillations are also investigated. The idea of micro-shimmy is also identified as a phenomenon where the small-amplitude tyre oscillations lead to increased heat, noise and as a consequence, to increased fuel consumption.

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1 INTRODUCTION

Shimmy is still a difficult, hard-to-predict lateral vibration phenomenon of towed rolling wheels. Although it usually appears with low probability, it may cause serious accidents on aircrafts, articulated buses, trailers, motorcycles, bicycles, even on strollers, shopping carts, wheel-chairs.

While it appears as a serious problem for design engineers and also as a hard task for experimentalists, it also provides an excellent research field to study peculiar vibration phenomena and also to improve the mechanical modelling of wheel and ground contacts.

The phenomenon of wheel shimmy became a central topic of dynamic studies when the speed of vehicles reached that limit where, more and more often, the lateral vibration of elastically supported wheels appeared. This was after World War I, when the dance "shimmy" was a favourite one all around in the Western world – this was the time when that peculiar wheel motion was named by the researchers (see [1] and [2]).

In the subsequent sections, the development of the mechanical models is presented together with the relevant nonlinear vibration phenomena that can be explored and studied by them. These include simple periodic self-excited vibrations including stable and unstable ones, then quasi-periodic oscillations and also chaotic and transient chaotic oscillations. However, still many open questions arise that represent that the dynamic tyre-ground contact problem is a multi-scale one, where the dynamics within the tiny contact region has an essential effect on the global dynamics of the whole vehicle, let it be a motorcycle or an airplane.

2 SINGLE CONTACT POINT MODELS

The simplest possible mechanical models are studied with the lowest possible mechanical degrees of freedom that still exhibit the shimmy phenomenon. In these models, the plane of the wheel is always perpendicular to the horizontal ground. It is towed by a caster attached to a vertical king-pin, and the king-pin has a prescribed constant velocity.

Shimmy cannot occur if all these three mechanical parts are rigid: if the wheel, the caster and the king-pin are all rigid, the wheel can perform straight rolling only with the prescribed constant speed. Thus, when the wheel is considered to be rigid together with the caster, the king-pin must be modelled as an elastic element. This case is shown in Fig. 1.

Without constraints between the wheel and the ground, the system would have 3 degrees-of-freedom described uniquely by the following 3 general coordinates: the caster angle ψ, the king-pin lateral deformation q, and the wheel rotation angle φ. The rolling condition between the wheel and the ground requires zero velocity for the contact point P, which is a kinematic constraint. Since the corresponding mechanical system is a non-holonomic one, the equations of motion can be obtained in the densest form of differential equations if the Appell representation is applied:

$$\begin{aligned} J_A\ddot{\psi} &= f(\psi,\dot{\psi},q;v,l) \\ \dot{q} &= v\tan\psi + l\frac{\dot{\psi}}{\cos\psi} \\ \dot{\varphi} &= \frac{v + l\dot{\psi}\sin\psi}{R\cos\psi} \end{aligned} \quad ,$$

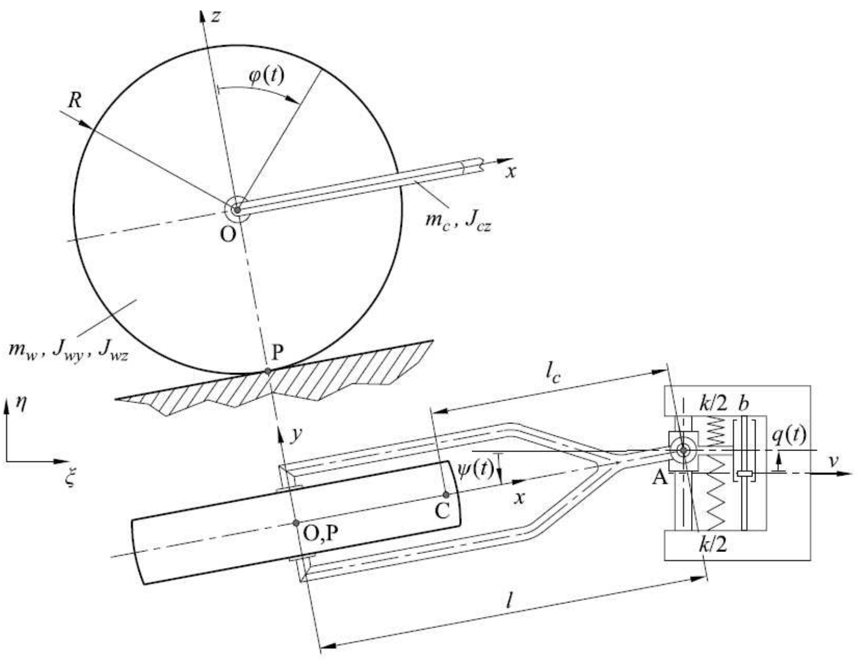

Figure 1: Single contact point model of shimmying wheel with elastic king-pin

where J_A is the mass moment of inertia w.r.t. the vertical axis at the king-pin A, l is the length of the caster beam, R is the radius of the wheel disc and v is the constant towing speed. The lengthy expression of the nonlinear function f is not presented here, it can be found in [3] with many further details including its dependence on the stiffness parameter k and the damping parameter b. Still, its dependence on the towing speed v and caster length l is denoted after the semi-colon since these physically relevant quantities are chosen as bifurcation parameters.

Clearly, the coordinate φ is a cyclic one, so the system can uniquely be represented in the 3 dimensional phase space of the caster angle ψ, caster angular velocity $\dot{\psi}$ and king-pin lateral deformation q.

The results of the linear stability analysis and the corresponding Hopf bifurcation calculations are presented in Fig. 2 for a set of realistic mechanical parameters [3]. The figure clearly shows that the longer the caster is, the more stable the steady rolling is. Even for the idealistic undamped case, there is a critical caster length $l_{\mathrm{cr}} = R\sqrt{3m_w/2m_c}$ above which the system is stable. For positive damping increasing towing speed results in instability, while increasing the speed further may lead to re-stabilization. The Hopf bifurcations are typically subcritical, that is, unstable periodic motions are born at these stability limits. The stable vibrations outside these limit cycles may have very large amplitudes with respect to the caster angle ψ, which may get close to the theoretical limit of 90 degrees. This explains the existence of the relatively large bi-stable region in the bifurcation chart in Fig. 2.

As shown in the upper panel of Fig. 2, even isola can occur deep in this bi-stable region: this is the worst case for an engineer, since linear stability analysis or even local bifurcation analysis are unable to detect the unstable and stable periodic motions co-existing with the stable steady rolling. In these cases large amplitude oscillations may occur with relatively low probability since special initial conditions are needed to initiate the large amplitude self-excited vibrations.

On the other hand, the existence of large amplitude oscillations with rolling usually requires large coefficient of friction. If the coefficient of friction is small, the wheel starts sliding and chaotic or even transient chaotic motion may occur (see [4]).

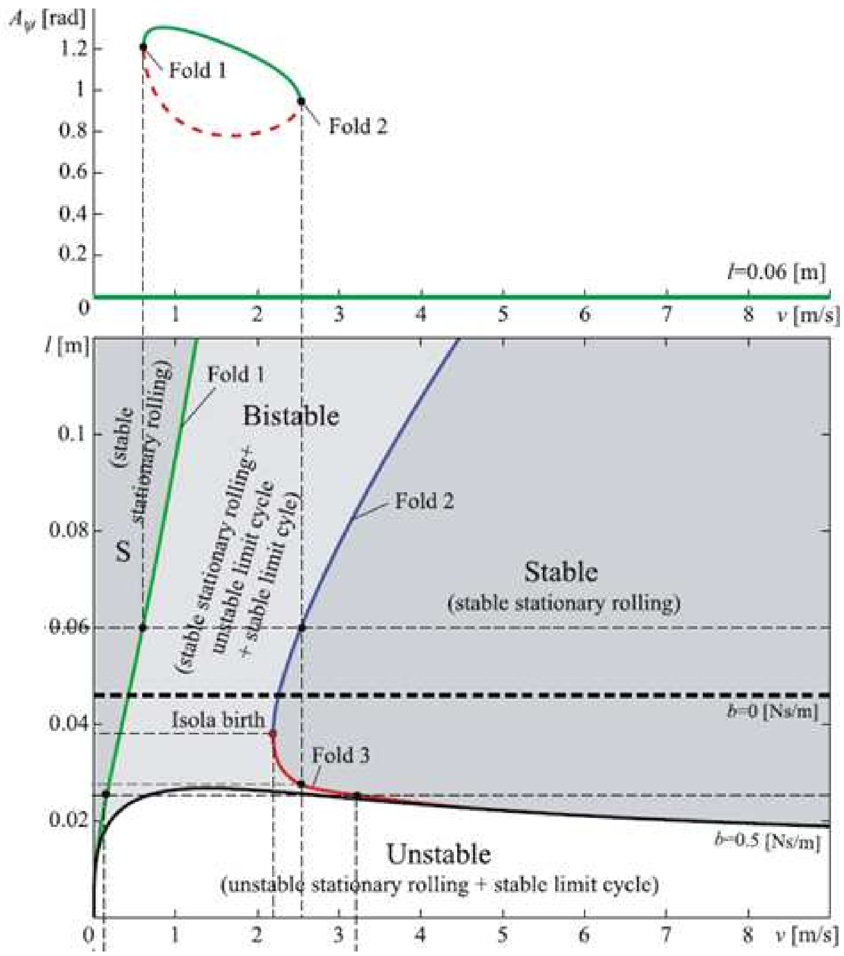

Figure 2: Two-parameter bifurcation chart of single contact point model for wheel shimmy and the existence of an isola for parameters of linearly stable steady rolling

3 CREEP FORCE MODELS

While the single contact point models provide reliable nonlinear models for strollers, shopping carts, wheel chairs, where the wheels are relatively rigid, the pneumatic tyres of motorcycles, cars, trucks and airplanes have much larger lateral elasticity than the vehicle structure has, including their king-pins or casters. In these cases, the lateral deformations in the finite contact region between the elastic tyre and the rigid ground are to be modelled.

The quasi-stationary creep force model of Pacejka [5] was introduced in the 1960's and has become widely accepted and used in the automotive industry [6,7]. The basic concept of the model is that the resulting lateral forces acting on the wheel from the ground are calculated for constant values of the caster angle ψ: the resultant force F_y and torque M_{0z} are calculated as functions of the lateral deformation q_0 of the leading edge point L of the contact region. The results of these analytical approximations are then improved by the results of extensive experiments leading to the so-called "Pacejka's Magic Formula" represented by the small panel in the upper left part of Fig. 3. Two new and relevant parameters of the model are the half contact length $a \approx \overline{\mathrm{LE}}/2$, and the relaxation length σ of the tyre.

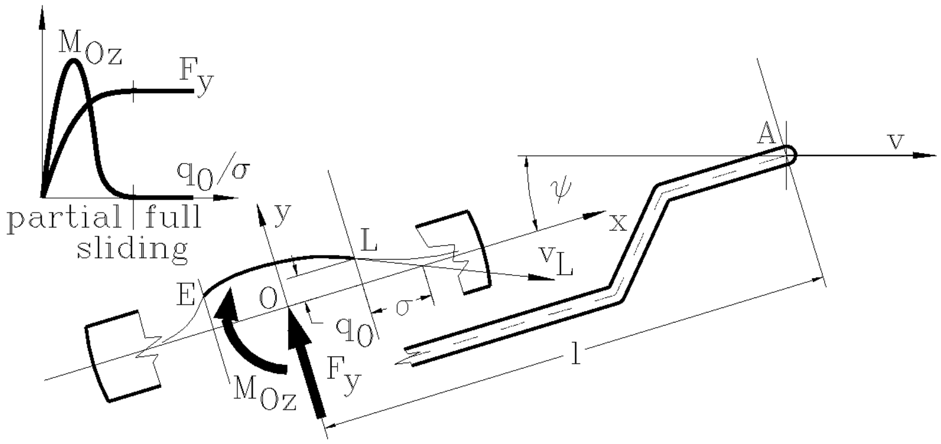

Figure 3: Creep force model for wheel shimmy. The upper panel shows the resultant force and torque induced by the lateral deformation of the tyre in case of stationary caster angle ψ

While these new parameters and functions refer to the essential difference between the single contact point models and the creep force models, the structure of the equations of motion remain the same:

$$J_A\ddot{\psi} = -lF_y(q_0) - M_{0z}(q_0)$$
$$\dot{q}_0 = v\sin\psi + (l-a)\psi - \frac{v}{\sigma}q_0\cos\psi \quad ,$$

that is, the dynamics of the system is described again in a 3 dimensional phase space. The stability charts and the Hopf bifurcations show the same qualitative properties as the single contact point models: increasing caster length improves stability properties and the bifurcations remain subcritical leading to the appearance of unstable limit cycles, again. The actual numerical values of the stability limits and the vibration amplitudes in unstable cases depend, of course, on the creep force functions and the additional tyre parameters; for example, the value of the critical caster length is $l_{\mathrm{cr}} = a + \sigma$ for zero damping.

There is one qualitative difference between the two models: the creep force model provides a smooth transition between rolling and sliding since it takes into account that some points in the contact region slide while others stick to the ground, and full sliding occurs only after a critical lateral deformation of the leading edge when all the points in the contact region slide. Since the nonlinearity is not as ‘strong’ as it is in case of sticking-or-sliding of the single contact point, chaotic oscillations and transient chaos can hardly be observed in those systems where elasticity appears at the wheel.

4 DELAYED TYRE MODELS

The creep force models introduced by Pacejka give a very reliable way of predicting shimmy in engineering applications, although it requires the experimental identification of some of its parameters. However, there are some critical cases when this model also has its limitations. These cases are, for example, when the dynamic behaviour of the tyre is to be modelled at low towing speeds, or when self-excited quasi-periodic oscillations occur, or when noise and heat is generated by the tyre-ground contact at caster lengths larger than the conservative estimate of its critical value l_{cr}.

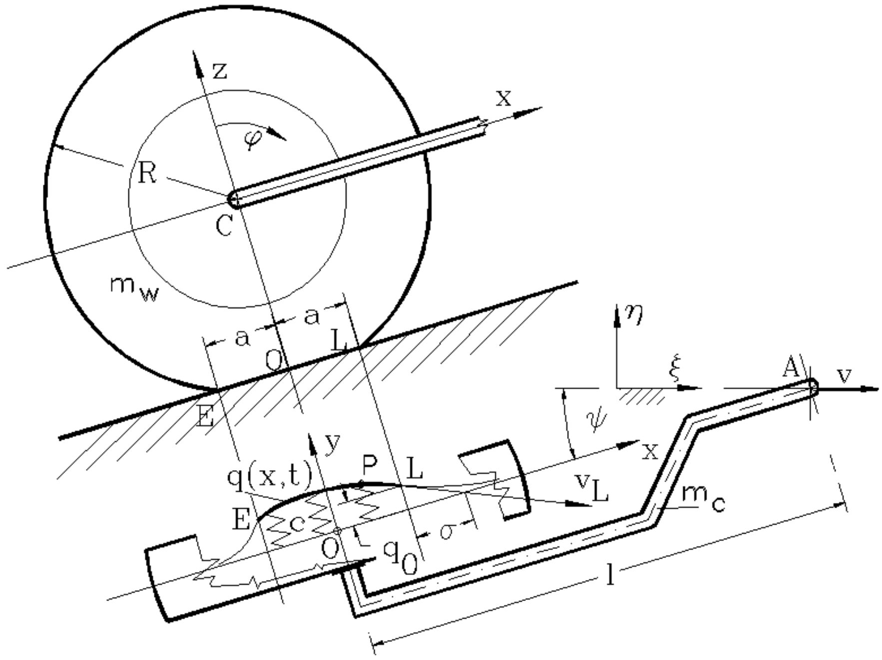

Figure 4: Delayed tyre model for wheel shimmy.

To improve the creep force model, it has to be taken into account that the deformed lateral shape $q(x,t)$ of the tyre is time dependent and it cannot be approximated always by the stationary deformed shape determined by the single scalar coordinate q_0 of the leading edge L.

With the dynamically varying lateral deformation of the contact shape, the linearized equation of motion will assume the form

$$\begin{aligned}
&J_A\ddot{\psi}(t) + c\int_{-a}^{a}(l-x)q(x,t)\mathrm{d}x = 0 && \text{(IDE)}\\
&\dot{q}(x,t) = v\psi(t) + (l-x)\dot{\psi}(t) + vq'(x,t) + \text{h.o.t.} && \text{(PDE)}\\
&x \in [-a,a], \quad t \in [t_0,\infty), \quad \text{and} \quad q(a,t) = 0
\end{aligned} ,$$

where c is the specific lateral stiffness of the linearly elastic tyre in the Newtonian integro-differential equation (IDE), while the first order partial differential equation (PDE) presents the kinematic constraint prescribing zero velocity for the contact points (see [8]). Strictly speaking, it still involves some approximation since the tyre points will always start sliding at the rear part of the contact region due to the diminishing normal contact force close to the end-point E.

The PDE part of equations of motion has a travelling wave-like solution

$$q(x,t) = (a-x)\psi(t) + (l-a)\left(\psi(t) - \psi\left(t - \tfrac{a-x}{v}\right)\right) + \text{h.o.t.}$$

that represents the memory effect of the tyre: a tyre element touches the ground at L, travels backward relative to the contact region and reaches the position at the local coordinate x with a time delay $\tau \approx (a-x)/v$; thus, the actual lateral forces depend on the previous positions of the contact point. The basic idea of this delay effect can already be traced in the early paper [9] from 1941. If the above travelling wave-like solution is substituted back to the IDE, the past values of the caster angle are integrated to affect the actual angular acceleration of the caster. The resulting linearized governing equation is a linear autonomous delay-differential equation (DDE):

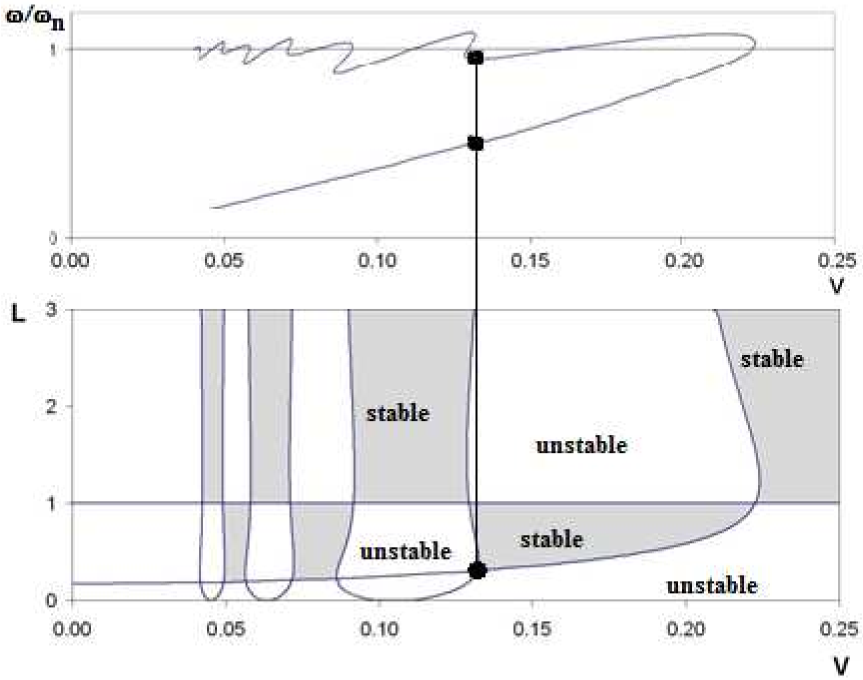

Figure 5: Stability chart of shimmy in case of delayed tyre model with zero damping. The upper panel presents the vibration frequencies of the self-excited vibrations at the stability boundaries. Black dot refers to quasi-periodic oscillation with two independent frequencies.

$$V^2\ddot{\psi}(t)+\psi(t)-\frac{L-1}{L^2+1/3}\int_{-1}^{0}(L-1-2\vartheta)\psi(t+\vartheta)\,\mathrm{d}\vartheta=0 \qquad \text{(DDE)}$$

where the dimensionless caster length L and the dimensionless towing speed V are defined as

$$L=\frac{l}{a},\quad V=\frac{v}{2a\omega_n},\quad \omega_{\mathrm{n}}=\frac{2ca(l^2+a^2/3)}{J_A}$$

with the angular natural frequency ω_{n} of the wheel at zero towing speed. The details of the stability investigation of the DDE model can be found in [8] or in [10]. The resulting stability chart in the parameter plane of the towing speed and the caster length in Fig. 5 shows a much more intricate structure than those of the traditional models.

There are three important phenomena that are described by the stability chart of the improved model. The intersection points of two Hopf bifurcation stability limits refer to possible quasi-periodic oscillations. The one denoted by a black dot in Fig. 5 survives even in the presence of large viscous damping. The corresponding 2 frequency components were also identified experimentally in [10] – classical tyre models cannot describe these oscillations.

Another important qualitative property of the delayed tyre model is that it presents a relatively large unstable region for $L>1$ and $0.15<V<0.2$ (see Fig. 5). This can hardly be found by traditional vibration measurement methods since the vibration amplitudes within the contact region are small due to the presence of micro-slips, and their effect on the vehicle structure is almost undetectable in the noisy environment. However, the increased tyre temperature was identified in these parameter domains in [11]. This means that the fuel consumption of the vehicles and also the noise of the tyres is larger in these parameter domains than it is in the stable regions.

Finally, it can also be mentioned that the dynamics of wheels at low speeds, especially before the vehicle stops, cannot be described properly by the creep force model. The delayed tyre model has the potential to be used in these parameter domains, too.

The experimental observations in [10] and [11] indicate that the subcritical sense of the Hopf bifurcation is inherited by the delayed tyre model, too. This means that shimmy may occur even in those parameter regions where the steady rolling of the wheel is stable: large enough perturbations may often lead to self-excited lateral oscillations.

5 CONCLUSION

The historical review of wheel models presented the classical single contact point model for rigid wheels, the creep force model for wheels of elastic tyres and the recently introduced delayed tyre model that combines the small-scale dynamics within the tyre/ground contact region with large-scale dynamics of the wheel itself.

Since wheel shimmy involves many intricate nonlinear vibration phenomena, it provides a perfect methodology to study and to improve the mechanical models of elastic tyres. These also have relevance in other applications like the development of ABS systems.

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