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A GLOBAL DYNAMICS PERSPECTIVE FOR THE ANALYSIS, CONTROL AND DESIGN OF MECHANICAL AND STRUCTURAL SYSTEMS

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Abstract. *The achievements occurred in nonlinear dynamics over the last thirty years entail a substantial change of perspective when dealing with vibration problems, since they are now deemed ready to meaningfully affect the analysis, control and design of mechanical and structural systems. The paper aims at highlighting and discussing the important, yet still overlooked, role that some relevant concepts and tools may play in engineering applications. Upon dwelling on such topical concepts as local and global dynamics, bifurcation and complexity, theoretical and practical stability, attractor robustness, basin erosion, dynamic integrity, recent results obtained by the authors and collaborators for a variety of systems and models of interest in mechanics and structural dynamics are overviewed in terms of analysis of nonlinear phenomena and their control. The global dynamics perspective permits to explain partial discrepancies between experimental and theoretical/numerical results based on merely local analyses, and to implement effective dedicated control procedures. This is discussed for discrete systems and reduced order models of continuous systems, for applications ranging from macro to micro/nano mechanics. Fundamental understanding of nonlinear physical phenomena producing bifurcations and complex response has now reached a critical mass, so it is time to exploit their potential to enhance performance, effectiveness, reliability and safety of technological systems, and to develop novel design criteria.*

1 INTRODUCTION

Nonlinear dynamics in mechanics has a recent history dating back to the Eighties, when the relevant international community realized the importance of nonlinear phenomena, initially addressed within the mathematics/physics community, in view of technical applications.

Since then, the area has been evolving in a revolutionary way, with applications to a wide variety of mechanical/structural systems made possible by the use of sophisticated analytical/geometrical/computational techniques employing powerful concepts/tools of dynamical systems, bifurcation, and chaos theory, properly updated and complemented with a view to engineering aims and with meaningful experimental verifications.

The achievements occurred over the last thirty years entail a substantial change of perspective when dealing with vibration problems, since they are now deemed ready to meaningfully affect the analysis, control and design of mechanical and structural systems. This paper aims at highlighting and discussing the important, yet still overlooked, role that some relevant concepts and tools may play as regards the load carrying capacity of engineering systems.

Attention is paid, in particular, to the evolution and update of the old concept of stability, as ensuing from consideration of global dynamic effects. Upon dwelling on such topical concepts as local and global dynamics, bifurcation and complexity, theoretical and practical stability, recent results obtained for a variety of systems of interest in applied mechanics and structural dynamics are overviewed in terms of analysis of nonlinear phenomena and their control.

Local and global stability of systems are discussed by also considering the effects of imperfections or small, but finite, dynamic perturbations, along with variations of some control parameters. All of them may arise in technical applications and experiments, and are to be properly considered in the design stage in order to secure the system capability to sustain changes without modifying the desired outcome.

Robustness of solutions against variations of initial conditions or control parameters, and system dynamic integrity, are fundamental concepts in analysis and design. They have to be addressed in view of global phenomena occurring in the system, which may indeed entail the existence of merely residual levels of robustness and integrity definitely unacceptable in technical applications. It is just this item that makes the concept of practical stability, and the associated global analysis, necessary.

The overall transition from a local stability perspective to a global safety concept has also major implications as regards the feasibility and effectiveness of techniques aimed at controlling nonlinear dynamics. In fact, these may drastically change according to whether the goal is increasing the overall dynamic integrity of the system, or merely realizing and/or stabilizing a specific kind of response.

In this paper, all these issues, which also permit to explain partial discrepancies between experimental and theoretical/numerical results based on merely local analyses, are discussed for a number of systems of interest in applications, ranging from macro to micro/nano mechanics. Archetypal discrete systems and reduced order models of continuous systems are addressed. The former consist of an inverted pendulum asymmetrically or symmetrically stiffened, the latter representing a simplified model of guyed cantilever tower, and to a parametrically excited experimental pendulum. The latter refer to minimal order approximations of infinite-dimensional systems at a micro-scale, such as beams for applications in micro-electro-mechanical systems or in atomic force microscopy.

Specific phenomenological aspects are discussed, however paying attention to the common or distinguishing nonlinear dynamic features which are expected to play a meaningful role in the analysis and design for engineering applications.

2 FROM THEORETICAL TO PRACTICAL STABILITY OF SYSTEMS

The problem of "practical stability" of attractors of dynamical systems originates from Thompson's idea [1] that classical (Lyapunov) local stability [2] is not enough for practical uses, an issue which bridges the gap between theoretical investigations and practical applications, as recently reconsidered by the authors [3-6]. The very basic idea is that, since in nature/technology there are always imperfections of static or dynamic nature, the system must be able to sustain changes both in control parameters and in initial conditions (i.c.), without changing the desired outcome.

In the mechanics community, the effects of changes of parameters were deeply studied by Koiter [7] and followers, whereas in the applied mathematics community their analysis led to the development of the structural stability theory. Yet this theory, when applied to classical mechanics engineering problems (such as buckling), has still a substantially local character, with the main abstract result being that pitchfork and transcritical bifurcations [8] perturb to a saddle-node bifurcation; thus, they cannot be observed in real world, unless some constraints (e.g., symmetry) force their occurrence. Hopf and period doubling bifurcations, on the other hand, are structurally stable [8], and this agrees with the fact that they are observed experimentally in both laboratories and practical applications.

Historically, the persistence of system response with respect to changes in i.c. was analyzed earlier, and led to the concept of Lyapunov local stability, which played a major role in the solutions of engineering problems ensuing from technological developments. It is so important that, contrary to structural stability, to date it is taught in any engineering program. The mathematical definition of stability roughly says that under infinitesimal changes in i.c. the system must keep the reference response.

Recently, without overlooking the fundamental value of the classical concept of stability, the need to properly update it for practical purposes has become apparent. In fact, considering only infinitesimal changes in i.c., which makes sense from the mathematical viewpoint, appears weak from a mechanical/practical viewpoint, especially close to bifurcation points where small (but finite) perturbations may significantly change the system response. This is the theoretical justification behind the fact that in experiments it is not possible to exactly detect the bifurcation points. Indeed, finite, though small, changes of i.c. have to be taken into account, this leading to the concept of "practical stability."

In spite of its conceptual simplicity, this is a paramount enhancement, full of consequences and problems to be accurately addressed. The most important one is concerned with the need to definitely abandon a merely local perspective and pass to a global one, in both time-independent and time-dependent problems, by considering the whole dynamic behaviour of the system even if being actually interested in only a small (but finite) neighbourhood of the considered solution. This has both theoretical and practical implications. In fact, in an actually dynamic environment, it requires on one side to analyze the effects of non-classical solutions such as homoclinic/heteroclinic orbits, quasi-periodic responses, chaotic attractors, etc. [9], while, on the other side, calling for the development of tools able to perform the global analysis of systems with large number of degrees of freedom (d.o.f.) in a reasonable computational time.

3 THE GENERALIZED CONCEPT OF DYNAMIC INTEGRITY

Basin erosion in phase space. The dynamic integrity concept was first formulated [1] and used [10, 11] in a strictly dynamic environment, with reference to the fractality of the boundary of the basin of attraction of a given dynamic solution. Such an extended fractality, owed to the occurrence of homoclinic or heteroclinic tangles of the manifolds of some saddle points

[9], causes penetration of eroding tongues of a competing outer basin inside the reference one, and leads to the well-known ‘butterfly effect’ [12]. Even small changes in i.c., i.e. small perturbations, give rise to time histories that approach different attractors. The system dynamics is definitely unsafe if anyone of these is unwanted. The topological process leading to this unsafe situation is named ‘erosion’ of the basin.

The starting point for an accurate analysis of system dynamic integrity is the correct definition of a “safe basin” [1, 3, 11], which is the set of i.c. sharing a common dynamical property. Safe basins are usually considered to coincide with classical basins of attraction, i.e. with sets of i.c. leading to given attractors. This is also the point of view necessary for studying their “practical stability”. However, nowadays, it is apparent that this “common property” may be not only the convergence toward an attractor, but also, e.g., the non-escape from a potential well (irrespective of what happens within the well), the behaviour in transient instead of in steady regime, or even the non-sensitivity to i.c. Also this point involves theoretical and practical issues. In fact, for certain definitions of safe basin (essentially when they are basins of attraction or union of them) the basin boundaries, which play a major role, are stable manifolds of given saddles, and have a well-defined behaviour and properties, not shared by other safe basin definitions. From a practical point of view the issue is that, apart from few cases, safe basins can be determined only numerically, and to date this is still computationally onerous for systems with more than, say, two mechanical d.o.f.

Once the safe basin has been properly defined, one can consider an attractor as “practically stable” if its safe basin is large enough. This correlation is correct, but the meaning of “large enough” is not trivial and needs a proper definition of “magnitude”. The most natural one refers to the geometrical hyper-volume of the safe basin and is technically formalized by the so-called Global Integrity Measure (GIM) [3, 10], which is a dimensionless measure suitable for parametric studies. Although GIM is sufficient in various cases, there are many others, mainly related to possible fractality of the basin of attraction, where it is inadequate. Fractal basin boundaries may spread the basin all over the phase space, destroying – or strongly reducing – the compact subset around the attractor, which is the sole one allowing for safe, finite changes of i.c.

Thus, one must introduce and use measures of the integer (or compact) part of the basin like, e.g., the Integrity Factor (IF) [13] or the Local Integrity Measure (LIM) [10], which are the normalized radii of the largest hyper-sphere belonging to the safe basin and of the one also centred on the attractor, respectively. They allow to rule out the fractal part of the basin from the measure of its magnitude, thus being required when fractality is a drawback (as in most mechanical/structural applications). The advantages and disadvantages of distinct measures are discussed, e.g., in [14], where one further measure (AGIM, Actual Global Integrity Measure) ruling out fractality in an effective, and more reliable, way is also introduced.

Solution/attractor robustness in phase space. Yet, the classical concept of dynamic integrity can be generalized by also considering proximity to a local or some global bifurcation event determining the disappearance of a static or dynamic solution, even in the absence of fractality of the boundary between competing safe basins. In fact, also in a static case, as a driving system parameter approaches the (local) bifurcation value, the basin of the reference solution shrinks to zero [5], so that in the neighborhood of the bifurcation threshold it becomes unsafely small, although the solution is still stable in the Lyapunov sense. In this case, though being its basin integer, the pursued system response is ‘non-robust’ with respect to finite dynamic perturbations. The same occurs in the case of a dynamic solution existing and possibly competing with another one, either belonging to the same potential well or being out-of-the-well, its robustness solely depending on the magnitude of its (certainly integer) safe basin. Of course, in the dynamic case, robustness also depends on compactness of its basin, which is thus an essential ingredient of the very definition of safe basin, as mentioned above. Thus, in a generalized sense, one can embed within the synthetic concept of dynamic integrity the two notions of solution (attractor) robustness and basin compactness. It is apparent that the combination of basin smallness and fractality entails the worst situation in terms of reduction of dynamic integrity.

Robustness/erosion in parameter space. Still having in mind the "practical stability" concept, it is necessary to investigate how attractor robustness and basin erosion evolve with a varying control parameter. This is done via the so called robustness and erosion profiles, which are useful graphical tools allowing for an immediate feeling of the safe basin size reduction (or even increase), to be possibly explained also in terms of global phenomena (homo/heteroclinic bifurcations, crises, etc.) occurring in system dynamics. A sample 3D diagram is reported in Figure 1, which refers to an inverted pendulum with an asymmetric elastic constraint, subjected to static axial load p and transverse dynamic excitation q_1 [5]. GIM is seen to provide measures either of the reduction of attractor robustness (for varying p) or of the extent of basin erosion (for varying q_1) [6]. The figure highlights the coupling effects entailed on the system load carrying capacity by the coexistence of axial load and lateral dynamic excitation, and provides a clear view of how integrity decreases up to vanishing when approaching a limit curve in the (q_1,p) parameters plane. This limit curve highlights the strong reduction both of the (Koiter) practical critical load owed to the presence of a dynamic excitation and of the (Thompson) escape dynamic excitation owed to the presence of a static axial load. Overall, one important result is that robustness and integrity may become residual well before the disappearance of the attractor by local bifurcations: it is just this item to make the concept of "practical stability," and the associated global analysis, necessary in applications.

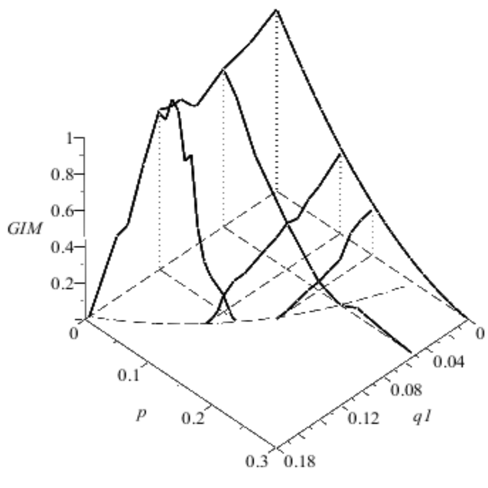

Figure 1. Robustness (varying p) and erosion (varying q_1) profiles for an asymmetrically constrained inverted pendulum: coupling effects due to coexistence of axial load and dynamic excitation.

A further unsafe and potentially dangerous situation is related to other global bifurcations, like, for example, the boundary crisis [9] governing the disappearance of an in-well chaotic attractor. Here, in fact, the basin of attraction may remain uncorrupted up to the bifurcation point, where the attractor suddenly disappears and its former basin is captured by another coexisting attractor (e.g., a cross-well chaotic attractor). Of course, strictly speaking, this situation does not fall within the realm of dynamic integrity, because the dangerous situation occurs for (Koiter-like) perturbations of the structure, by means of small changes of the relevant parameters, and not by (Lyapunov-like) perturbations of i.c. In fact, below the bifurcation threshold, if it is guaranteed that the system parameters do not change, the state of the system is definitely safe.

Yet, thinking of the small imperfections always present in the real world, one has to consider their effects in some more general terms, by referring to perturbations either of i.c. in phase space or of system parameters in control space. The former may be directly sufficient to drive the response out of the safe basin, towards a different, more robust, attractor, when the basin itself is too small or too eroded; the latter may indirectly affect the possibility to get the expected response, owing to the associated meaningful, and possibly sudden, reduction of dynamic integrity entailed by an also small parameter variation.

Local vs global control. Finally, the overall transition from a local stability perspective to a global safety concept has also major implications as regards the feasibility and effectiveness of controlling the nonlinear dynamics of engineering systems [15]. In fact, control approach and techniques may drastically change according to whether the goal is concerned with the system "local" dynamics (e.g., stabilizing a single trajectory, as pursued by classical chaos control techniques [16], or keeping a reference periodic response which is suitable in a given practical application [17]), or with its "overall" dynamics, where some global (homoclinic or heteroclinic) bifurcation event – triggering response fractalization – has to be exploited to possibly increase the system dynamic integrity [15].

4 DYNAMIC INTEGRITY FROM MACRO TO NANOMECHANICS

For the sake of readability, theoretical and practical use of the dynamic integrity concept is addressed in the following by grouping some associated topics under three main themes, namely, how dynamic integrity can be referred to (i) for analyzing and controlling global dynamics, (ii) for interpreting and predicting experimental behavior, (iii) for getting hints towards engineering design. To the aim of illustrating these three classes of problems, sample results of systematic investigations made on different discrete or continuous systems/models of interest in applied mechanics and structural dynamics, at either the macro or the micro/nano scale, are presented.

4.1 Analyzing and controlling global dynamics

Usefulness of dynamic integrity in analyzing global behaviour of single-d.o.f. systems [3,4] has been discussed in recent papers dealing with a variety of mechanical, dynamical and control issues such as hardening versus softening behavior, symmetry versus asymmetry, smoothness versus non-smoothness, homoclinic versus heteroclinic bifurcations, uncontrolled versus controlled dynamics. Among smooth systems, besides the classical twin-well Duffing oscillator exhibiting escape to a neighboring well [18] and also representing a minimal order model of buckled beam, Helmholtz-like oscillators [13] representing in turn minimal order models of different complex/continuous systems (ship capsize [19], electrodynamically actuated microbeams [20-22], AFM cantilevers [23]) and exhibiting features of escape to infinity of variable origin, have been addressed. Also discrete (planar parametrically excited pendulum [24]) and possibly piece-wise smooth systems (two-well inverted impacting pendulum [25], rigid block undergoing overturning [3], model of a suspension bridge [26]) have been considered. Moving to two-d.o.f. systems representing more reliable reduced order models of complex structures (cylindrical shells [27], general slender structures liable to interacting buckling phenomena [28]), analysis and control of global dynamics become considerably more involved. Given the richness and variety of systems/problems, the reader is referred to the general/specific literature for in-depth description and understanding.

Here, to the sake of a synthetic illustration of global dynamic effects, reference is made just to a two-d.o.f. model of a guyed tower [29]. As other slender structures, this one may fail at a load level well below the theoretical buckling load, due to complex nonlinear phenomena that affect the structure safety and dynamic integrity; this occurs as a consequence of the erosion of basins of attraction of the safe pre-buckling solutions and of the occurrence of mechanical imperfections. So, it is of paramount practical importance to increase the safety of the structure in a dynamic environment. This can be achieved by increasing the integrity of the safe basin, a goal attainable by a control method exploiting features of global dynamics [30].

The GIM integrity profiles in Figure 2 are obtained with increasing values of the horizontal harmonic excitation F applied at the basis of the structure in a direction allowing for its reduction to a single-d.o.f. model. They allow to shortly dwell on various aspects of interest.

(i) In all profiles, basin erosion is very poor within a large range of relatively low values of excitation amplitude, but it becomes suddenly sharp above a certain excitation threshold, due to the homoclinic intersection of the stable and unstable manifolds of the hilltop saddle bounding the potential well, which entails strong fractalization of the basin of attraction. The overall system dynamics ends up with final escape (practically corresponding to structural collapse) after a further smooth decrease of integrity down to values which in the present case are not yet vanishing, contrary to what happens for other systems. Yet, the already occurred dramatic decrease of integrity down to practically unacceptable values corresponds to a substantially unsafe behavior of the system.

(ii) Application of a control technique properly optimized to the aim of eliminating/delaying the occurrence of the homoclinic bifurcation which triggers fractal dynamics allows to shift the sudden occurrence of sharp integrity decrease towards a relatively higher value of excitation amplitude, thus resulting very beneficial for system safety.

(iii) The presence of system geometrical imperfections reduces the size – and thus the robustness – of the initially un-eroded safe basin, and shifts the fall down of all integrity profiles to meaningfully lower values of excitation amplitude, thus accelerating the dramatic process of loss of global safety which marks the actual system un-serviceability.

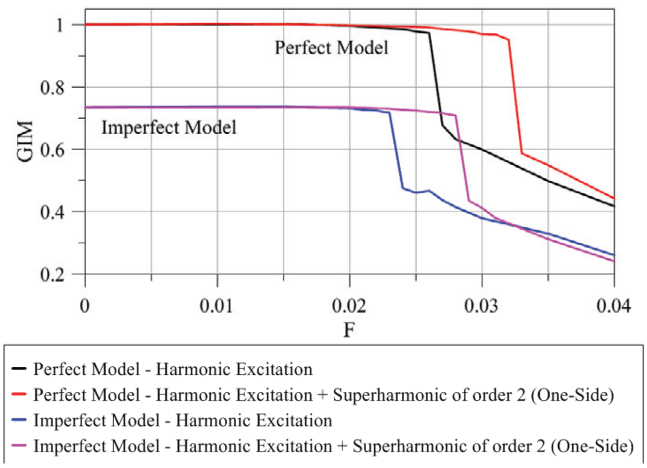

Figure 2. Comparison of GIM for perfect and imperfect, original and controlled, guyed pendulum models.

4.2 Interpreting and predicting experimental behavior

Dynamic integrity helps to understand experimental results; in particular, it helps to illustrating why solutions which are theoretically stable cannot be actually detected in experimental situations. In fact, when their basins of attraction are too small or too eroded, just small perturbations of either i.c. or system parameters, always present in the real world, are sufficient to directly or indirectly get the response out of its safe basin towards a different, more robust, attractor [14].

To illustrate the matter, two cases in which dynamic integrity is very useful to interpret experimental results are presented, by considering a macro-experiment (a rotating pendulum [31]) and a micro-experiment (a MEMS device [21, 22]). Attention is paid, in the former case, to robustness of a special class of solutions, the rotating ones, against competing oscillations; in the latter case, to the overall escape from system potential well – irrespective of the associated solutions – leading to the so-called dynamic pull-in. In both cases, the theoretical limit of existence of the considered system state cannot be reached in practice. The experimental limit, on the other hand, is associated with an acceptable level of integrity measure, different from case to case but fixed for a specific mechanical system; this provides evidence of the practical role played by dynamic integrity.

The two considered cases differ also for their dynamic characteristics and the underlying technical interest. In fact, in the pendulum, this consists of obtaining rotating solutions to be possibly exploited in view of, e.g., green energy production from see-wave excitation; they are a class of responses which occur outside the potential well bounded by a heteroclinic orbit and, in general, are not robust against erosion from the in-well attractors, corresponding to oscillating solutions and/or to rest position. By contrast, in MEMS, the technical interest consists of avoiding dynamic pull-in, where the microcantilever tip gets into contact with the charged substrate, and prevents the system from being used as a micro-resonator or micro-sensor. Here, there is a unique potential well surrounded by a homoclinic orbit, and the main loss of integrity is due to its overall erosion from the unwanted out-of-well attractor, which leads to system final escape (practically corresponding to pull-in). It is thus worth noting that, also from the nonlinear dynamics viewpoint, the two examples are complementary, and implicitly confirm the general value of the dynamic integrity analysis.

Parametrically excited pendulum. In the underlying theoretical model [24], determining several bifurcation diagrams for increasing excitation amplitude p and different values of excitation frequency ω via computer simulations of the system equation of motion, allows to obtain the region of existence of period 1 rotating solutions in parameters space. It is bounded by a saddle-node (SN) threshold below and by a period doubling (PD) threshold (followed by a boundary crisis (BC) curve) above (Figure 3).

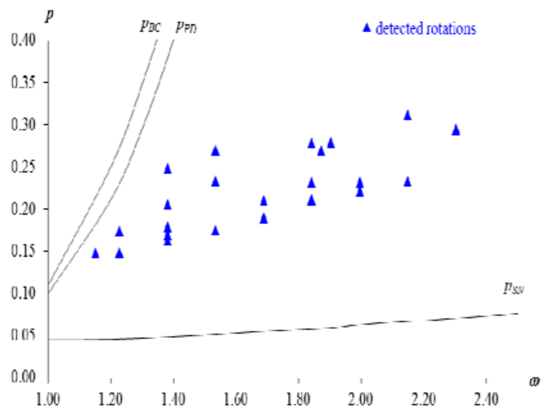

Figure 3. Numerical (lines) and experimental (dots) regions of existence of rotations in excitation parameters plane.

In the experiments [31], practical rotations, identified by the blue triangles in Figure 3, cover just a strip of finite magnitude, which belongs to the central part of the region of theoretical existence and shrinks for low frequencies.

The difference between experimental and theoretical regions of existence is too large to be attributed only to experimental uncertainties. A dynamic integrity analysis performed by constructing curves which, for each value of the excitation frequency and for increasing excitation amplitude, provide the value of an integrity measure, allows to interpret the discrepancy between experimental results and theoretical predictions. Experimental rotations are found only in the central strip of the excitation amplitude range because only in this range the attractor corresponding to the period 1 rotation is relatively robust and its basin is substantially uneroded, thus being characterized by high integrity. It is also possible to understand why rotations are not observed for excitation amplitudes just above the SN bifurcation.

Constructing the contour plot of the considered IF measure allows to show that the experimental points lie approximately along the main ridge of the IF(p,ω) surface [31]. This is a confirmation that only rotations with a very large safe basin, i.e. dynamic integrity, can be detected in practice, the bottom line of the experimental points in Figure 3 corresponding to the minimal dynamic integrity necessary for practical observability.

Micro-electro-mechanical system. While in the pendulum problem attention is paid to a special class of solutions, in the capacitive accelerometer experimentally tested in [21] attention is dedicated to the overall escape phenomenon (pull-in) which represents the most dangerous practical situation and the most interesting dynamic phenomenon. Indeed, it is related to subsequent bifurcations and strong erosion, thus having a double shrinking effect on the safe basin, i.e. its reduction in extent due to approaching the bifurcation, and its erosion due to fractal tongues. Depending on i.c., escape can occur also in the presence of bounded attractors; yet, attention is focused here on the so-called inevitable escape, i.e. on those situations where pull-in is the unique attractor in phase space.

Dynamic pull-in data come from a frequency-sweeping process, where the voltage V_{AC} of the electrodynamic excitation is kept fixed and the frequency is increased or decreased quasi-statically around primary resonance of system natural frequency. For intermediate values of excitation amplitude, the frequency-response curve exhibits the classical range of coexistence of resonant and non-resonant attractors [32], between which the response jumps for varying frequency in a possibly un-desired, yet no-destructive, manner. Building several response curves for different values of the excitation amplitude allows to identify the experimental and numerical/theoretical escape regions in parameters space (Figure 4). The latter has the classical V-shape of softening mechanical systems ([13, 33], see also Subsection 4.3), bounded on the left by the SN where the non-resonant solution disappears for increasing frequency and on the right by the BC at the end of the existence region of the resonant oscillation for decreasing frequency. The underlying Δ-shaped region where resonant and non-resonant oscillations coexist also occurs. Here, however, the interest is solely in the erosion of the overall in-well dynamics, with no care about which one is the dominant bounded attractor. As in the pendulum case, the difference between experimental and theoretical regions of escape is too large to be attributed only to experimental uncertainties. Again, it can be explained via a dynamic integrity analysis.

Erosion profiles are obtained by analyzing the evolution of the safe basin, which is assumed as the basin of the dominant bounded attractor, i.e. the non-resonant (resonant) one for low (high) values of excitation frequency. Systematically building them for different values of the AC voltage, it is possible to obtain also the contour plot of the IF measure, which provides an overall understanding of system dynamic integrity.

Below the theoretical escape region and depending on the excitation frequency, dynamic integrity can be so low that there is no hope to observe the associated theoretical attractor in practice. This is particularly apparent right of the Δ-shaped region, where the dynamic integrity of the nominally unique bounded attractor is indeed very small and, correspondingly, the practical escape threshold is much lower than the theoretical one. In practice, for a given excitation amplitude, the theoretically dominant bounded attractor is solely observed when its integrity is large enough, namely for lower (higher) excitation frequencies away from the primary resonance for the non-resonant (resonant) attractor.

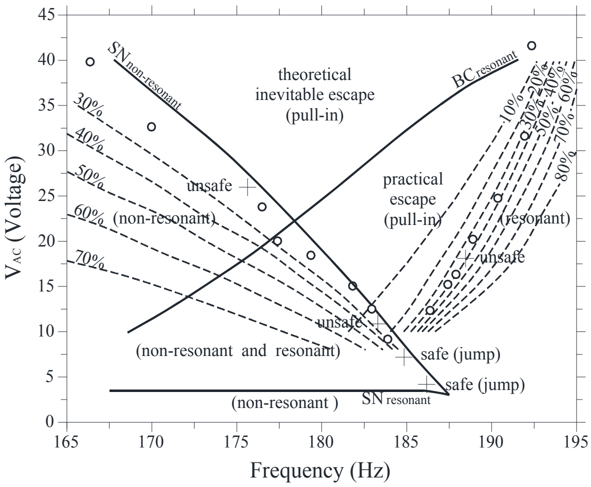

Figure 4. Theoretical (solid) regions of existence of non-resonant/resonant attractors, and of inevitable escape; thresholds of experimental escape (dots) against level curves (dashed) of IF.

Once more, dynamic integrity provides the justification for the non-coincidence between theoretical and experimental results, confirming that a theoretical attractor can be observed experimentally only if its basin of attraction is enough large and compact. Actually, dynamic integrity analysis also says that the experimental threshold of escape stands approximately along a level curve of IF (which in this case is about 30%). Thus, the curves of constant percentage of integrity may have a meaningful predicting role as to the effects of disturbances in the experiments, with the theoretical escape thresholds holding only in the abstract limit case of no perturbations.

4.3 Getting hints for engineering design

Global dynamic effects raise the problem of properly accounting for robustness and erosion phenomena in the design stage of whatever mechanical/structural system, a problem earlier addressed as regards ship capsizing in beam seas [34]. In this respect, the integrity concept can play an even major role than the solely, though fundamental, interpretative one of seemingly strange experimental results. Depending on the specific application and on the pursued system dynamics, the important practical lowering of theoretical escape threshold ensuing from residual integrity evaluations is entitled to meaningfully affect the design process. In fact, in a global dynamics perspective, load carrying capacity and system safety are guaranteed only if a minimal residual integrity is attained. The matter is dealt with by referring to one more mechanical system, also addressing some further relevant issues.

Atomic force microscopes (AFMs) are of considerable importance to characterize properties of whatever material at the nano-scale. For a vertically vibrating noncontact AFM cantilever designed to perform horizontal scan of a sample surface, it is essential to avoid any jump-to-contact phenomenon potentially entailing dangerous damage of the sample.

Yet, even correctly referring to the escape boundary of the corresponding single-mode dynamic model (bd) (Figure 5), obtained by mapping local bifurcation diagrams, the major problem in a safety assessment perspective ensues from the associated lack of information concerning features of the basin erosion which paves the way to final escape. In practical applications, integrity evaluations allow us to evaluate the possibility to realize an a priori defined safe design target depending on the occurrence of, e.g., meaningful external/parametric resonances. Figure 5 shows also four level curves of GIM in the frequency-amplitude space of the scan parametric excitation; they correspond to possible design target values in the neighborhood of the principal resonance region ($\omega_U = 2\omega = 1.67$). Again, the relative variation of the level curves with varying frequency ensues from the variable underlying erosion process, which is sharper (smoother) on the right (left) side of the down vertex corresponding to nonlinear resonance, similar to what already highlighted for the MEMS under vertical excitation at primary resonance.

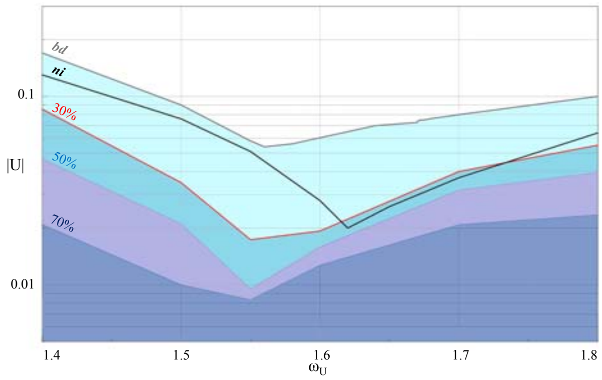

Figure 5. Comparison between theoretical, global (bd) and local (ni), escape threshold, with detection of level curves of residual integrity, close to principal resonance of scan parametric excitation.

One more escape curve (solid black) is also plotted in the chart, i.e., the one obtained by looking at the divergence of system response obtained through numerical integration (ni) with fixed i.c., herein corresponding to the equilibrium position of the system in the undamped, unforced, case. Depending on the outcome of a trajectory starting from a given point in system phase space, this escape threshold plays a "local" role, contrary to the "global" threshold. The latter corresponds to safe basin annihilation and is the upper bound of all diverging trajectories ensuing from whatever pair of i.c. in the basin, the former provides theoretically acceptable excitation amplitudes which are of course everywhere lower than those of the global escape, in the present case up to twenty-five times. So, in principle, if one is interested in avoiding the jump-to-contact of a motion ensuing from a specific i.c., the associated local threshold is the one to be properly considered in design for it is more conservative than the global threshold. Yet, it does not provide any information on the robustness of response against variations of i.c., which could indeed become very low in the case of a too small basin of the equilibrium solution. This information is instead embedded in the GIM level curves, which are those to be actually taken into account in the system design stage, in order to guarantee an acceptable response target. Selecting for instance an acceptable target of 30% residual integrity (the red threshold), left of nonlinear resonance the local threshold is definitely unacceptable because of corresponding to very low values of residual GIM (0÷30%). Moreover, even being possibly acceptable in an ideally perfect environment with very limited variations of i.c., it is definitely unacceptable owing to its un-homogeneous conservativeness with respect to variations of excitation frequency: see, e.g., the strongly variable residual integrity occurring in the narrow range around nonlinear resonance, where the excitation frequency can indeed undergo small variations in an actual technical problem. Also, local minimum occurs at principal resonance, while global minima – with a variable residual integrity – are shifted towards left (i.e., to the corresponding nonlinear resonance), due to the softening behavior of the system. Overall, local threshold corresponds to a poor safety reserve just where this is more needed, namely where there are the lowest values of escape excitation. In summary, consideration of the outcome of a single trajectory furnishes misleading and too conservative information in terms of overall system safety, unless being specifically interested in the response ensuing from that particular set of i.c. and reliably confident that escape will not come into play.

One more issue highlighting the potentially dangerous consequences of a merely local dynamical perspective is concerned with control of nonlinear dynamics. In Sect. 4.1, for the illustrative case of the inverted pendulum, it has been shown how a control based just on the underlying global features may succeed in modifying a global dynamic event entailing a dangerous system outcome. Here, referring to the AFM, an external feedback control technique [17] aimed at keeping the system dynamics to a reference periodic response – to be practically selected as a good one for reliable system operation –, is considered [35]. In fact, being the technique well-conceived and well-implemented, it succeeds in attaining the specific reference “local” target, at least in some ranges of values of system/control parameters. Yet, looking at its global effects in terms of escape, a dramatic picture occurs in the frequency-amplitude plane of Figure 6, where the global escape thresholds of the controlled and the uncontrolled systems are compared with each other for the same case of parametric excitation as considered above. The external feedback control strongly reduces the escape value of the forcing amplitude, up to about 99,9% around fundamental resonance, 99,4% around principal resonance and 94% around superharmonic resonance. This effect is connected with the close proximity to the resonance frequencies, which leads to a substantial increase of the response amplitude that the feedback control barely dominates and which manifests itself in a strong unexpected decrease of the escape threshold.

Of course, if the technique works well for the "local" purpose for which it is designed, i.e. solely keeping the system dynamics to a reference period 1 response, it cannot be discredited for the associated "globally" unfavorable effects. Yet, these are very important in the overall dynamics, and should govern the system design process unless being strictly confident that, in practice, those unfavorable effects would never be activated.

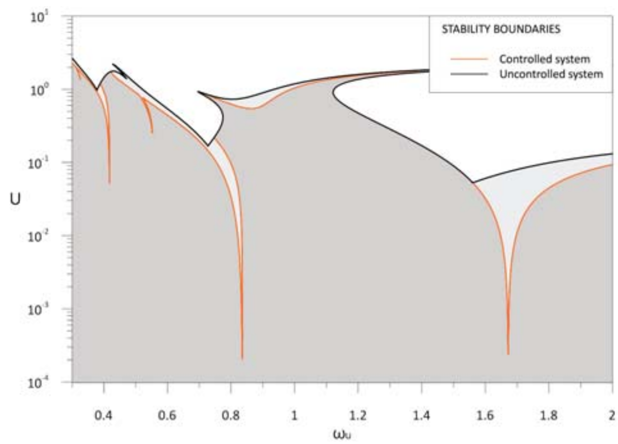

Figure 6. Global escape thresholds for controlled (orange) and uncontrolled (black) AFM under scan excitation. Dark (light) gray area is the stability region of controlled and uncontrolled (solely uncontrolled) systems.

5 CONCLUSIONS AND FUTURE PERSPECTIVES

The concept of practical stability implies definitely abandoning the merely local perspective traditionally assumed in the analysis and design of systems and structures, and moving to a global one where the whole dynamic behaviour of the system is considered, even if being actually interested in only a small (but finite) neighbourhood of a given solution. In spite of its conceptual simplicity, this is a paramount enhancement, full of theoretical and practical implications. Among them, the need to analyse the effects of non-classical solutions (stable and unstable manifolds of a given saddle, chaotic attractors, escape, etc.) and to develop new numerical tools able to perform the global analysis of systems with large number of degrees of freedom are mentioned.

Yet, although enjoying interesting developments also in technical areas, the great potential of nonlinear dynamics to significantly enhance performance, effectiveness, reliability and safety of systems has not yet been exploited to the aim of conceiving/developing novel design criteria and related technologies. Fundamental understanding of various nonlinear physical phenomena producing bifurcations and complex response has now reached a critical mass, so it is time to develop basic technologies to take advantage of the natural richness of behavior offered by nonlinear systems.

Two main issues characterize the present framework: (i) the need to overcome limitations inherent to archetypal single/few-d.o.f. models and deal with real systems from macro to nano mechanics; (ii) the increased interest towards exploiting nonlinear and global dynamics modeling and analysis for designing and controlling engineering systems. Of course, passing from simple models to actual engineered systems is quite involved.

Yet, a novel design philosophy should stand in investigating conditions that optimize the behavior of naturally nonlinear systems in such a way to possibly generate favorable operation. Nonlinearities may arise as inherent characteristics of the system, or may be artificially created. Properly taking them into account should radically influence current design, control and exploitation paradigms of technological systems, within a magnitude of contexts. In this respect, bridging the gap between theory and applications is a really hot topic.

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