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INDUCED ENERGY TRANSFER BETWEEN VIBRATION MODES USING TIME-PERIODICITY

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Abstract. *Introducing time-periodicity in system parameters may lead, in general, to a dangerous and well-known parametric resonance. In contrast to such a resonance, a properly tuned time-periodicity is capable of transferring energy between vibration modes, of extracting vibrational energy from the system and of increasing the effective damping of transient vibrations. At this specific time-periodicity the system operation is tuned at a parametric anti-resonance. The basic principle of this concept has been studied theoretically and was proven in different experiments. The physical interpretation of this concept was proposed in "Damping by Parametric Stiffness Excitation: Resonance and Anti-Resonance", Journal of Vibration and Control, 2008, for a multiple degrees of freedom system. The present contribution highlights those findings on a multiple degrees of freedom system. It is highlighted that a parametric anti-resonance induces an energy transfer between two of the many vibration modes of the underlying system with constant coefficients. The physical interpretation of this concept is outlined in detail. The induced energy transfer can be utilized to transfer the vibration energy from low frequency to high frequency or vice versa. The consequence of a cascaded switching of the parametric anti-resonance frequency is discussed.*

1 INTRODUCTION

Systems of differential equations with time-periodic coefficients, also termed parametrically excited systems, have been the focus of scientific research in mechanical engineering since many decades. Parametrically excited vibrations occur if one or more parameters of the differential equations are not constant but are described explicitly by a function of time; periodic and independent of the system motion. Classical example are the pendulum with periodically varying length or periodically moving pivot point leading to the famous Mathieu, Hill or Meissner equation. Parametrically excited systems and structures have been studied extensively in the past because of the interesting phenomena that occur in such systems. A parametrically excited system may exhibit a destabilising parametric resonance if at least one system parameter is varied close to a parametric excitation frequency, see e.g. [1, 2],

$$\nu^{kl,n} = \frac{|\omega_k \mp \omega_l|}{n}, \qquad k, l = 1, 2, \ldots . \tag{1}$$

Herein ω_k and ω_l denote the k-th and l-th natural frequency of the underlying undamped system with constant coefficients (Hamiltonian system). The denominator n represents the order of the parametric resonance. This frequency is called a principal parametric resonance for $k = l$, and a parametric combination resonance for $k \neq l$. Mostly only first order resonances $n = 1$ are significant, see e.g. [1, 3].

Almost all investigations on the dynamics of a single or of coupled differential equations having time-periodic coefficients are focusing on the resonant behaviour of parametric excitation, see e.g. [1–7]. The main focus there was to investigate the destabilising effect of parametric excitation, i.e. the instability boundary curves in the domain of system parameters. The non-resonant cases were not considered since they do not compromise the operation of a machine or do not amplify the system vibration in sensor applications. The benefit of introducing a non-resonant parametric excitation in an unstable coupled system was first highlighted in the work [8]. It was shown that an unstable self-excited system can be stabilised by introducing a time-harmonic stiffness coefficient, a parametric excitation, at a specific parametric combination resonance frequency. Since this occurs at the frequency of a parametric resonance, the mechanism was named parametric *anti*-resonance.

This work initiated theoretical studies of several *self-excited two-mass systems* with time-harmonic stiffness, damping and/or inertia. For a long time it had been believed that parametric anti-resonance can occur only in combination with self-excitation, see the review article [10,20]. However, the stabilising mechanism of a parametric anti-resonance was identified as the coupling of eigenvalues (vibration modes) of the underlying system with constant coefficients, see [11, 23]. This qualitative interpretation drastically enlarged the applicability of the method, since a properly chosen parametric anti-resonance not only stabilises an already unstable system (stabilisation by parametric excitation) but is also capable of enhancing the already existing damping; independent of self-excitation being present or not. Furthermore, introducing a parametric anti-resonance in a system with multiple degrees of freedom offers the unique possibility, at least in first order approximation [11], of coupling only two of the many vibration modes of the original system and induce an energy transfer between these selected vibration modes while the remaining vibration modes stay decoupled.

In recent years several attempts were undertaken to verify the existence of parametric anti-resonances experimentally. Starting with analog computer calculations [8], followed by discrete two-mass systems of an artificial nature [12, 13], the method of damping by parametric excitation was confirmed for simple continuous flexible cantilevers [14, 15], on flexible rotor

supported by two active magnetic bearings [16, 17] or a piezo-actuated journal bearing [18]. Most of the experiments are benchmarked in [19] based on an analytically derived performance index, enabling an objective comparison between the effectiveness of different experimental implementations of a parametric anti-resonance.

The physical interpretation of a parametric anti-resonance as *modal coupling*, which incorporates a modal energy transfer, was proposed in [11] and the energy flow was considered in [9, 21, 24]. The physical interpretation of modal interaction in this context was rediscovered in [22] and outlined in detail on an exemplary three degrees of freedom system. In the following, the basic working principle of a parametric anti-resonance is discussed for a system with four degrees of freedom for which different parametric anti-resonances are realised by a harmonic stiffness variation. The physical interpretation proposed in [11] of coupling two of the many vibration modes of the underlying constant system is discussed in means of physical and modal time histories according to [21]. This interpretation leads intuitively to the calculation of the energy of each vibration mode as proposed in [22] and leads to clear physical insight of how parametric anti-resonances work. A recent review on the parametric anti-resonance can be found in [23].

2 PROBLEM DEFINITION

A system with n degrees of freedom linearised at an operation point can be described by a set of coupled linear differential equations with time-periodic coefficients. For the case of a harmonic stiffness variation, the mass matrix is constant while some components of the stiffness matrix are varied harmonically at a fixed frequency and at a fixed amplitude,

$$\mathbf{M}_0\ddot{\mathbf{x}}(t) + \mathbf{K}_0\mathbf{x}(t) = -\sum_{m=1}^{M} \varepsilon_m \mathbf{K}_m\, \mathbf{x}(t) \cos \nu_m t, \tag{2}$$

with the time t, the parametric excitation frequencies ν_m, the position vector $\mathbf{x}$, the scalar parametric amplitudes ε_m and a nonsingular matrix $\mathbf{M}_0$. The matrices $\mathbf{M}$ and $\mathbf{K}$ correspond to the matrices of mass/inertia and stiffness coefficients. The index 0 denotes constant matrices and the index m the time-dependent parts of the stiffness matrix. Since $\mathbf{K}_m$ are matrices, more than one stiffness parameter of the system can be varied. Each index m corresponds to a parametric stiffness excitation at the frequency ν_m. The present study is restricted to systems with symmetric mass/inertia and stiffness matrices, having distinct natural frequencies.

Applying a modal transformation, $\mathbf{x} = \mathbf{T}\mathbf{p}$, the physical system in Eq. (2) is transformed to its modal space,

$$\ddot{\mathbf{p}}(t) + \boldsymbol{\omega}^2\, \mathbf{p}\,(t) = -\sum_{m=1}^{M} \varepsilon_m \mathbf{Q}_m\, \mathbf{p}(t) \cos \nu_m t, \tag{3}$$

where

$$\mathbf{T}^T\mathbf{M}_0\mathbf{T} = \mathbf{I}, \quad \mathbf{T}^T\mathbf{K}_0\mathbf{T} = \boldsymbol{\omega}^2 = \mathrm{diag}(\omega_1^2, \omega_2^2, \ldots), \quad \mathbf{T}^T\mathbf{K}_m\mathbf{T} = \mathbf{Q}_m = [Q_{m,ij}]\,. \tag{4}$$

The transformation matrix $\mathbf{T}$ consists of the eigenvectors of the underlying constant and undamped system ($\varepsilon_j = 0$ in Eq. (3)) and diagonalises the constant mass/inertia and stiffness system matrices. The ω_i are the undamped natural frequencies of the underlying Hamiltonian system.

For the sake of simplicity we consider a simple, linear chain mass system with four degrees as shown in Figure 1. The dimensional system matrices of the equations of motion in Eq. (2)

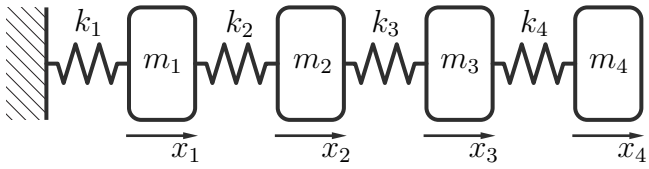


Figure 1: Linear example system possessing four degrees of freedom x_1, x_2, x_3, x_4.

can be written as

$$\mathbf{M}_0 = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{bmatrix}, \tag{5}$$

$$\mathbf{K}_0 = \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & -k_3 & k_3+k_4 & -k_4 \\ & & -k_4 & k_4 \end{bmatrix}, \quad \mathbf{K}_c = \begin{bmatrix} k_1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}.$$

corresponding to the position vector $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$. Note that only the stiffness parameter k_1 is varied harmonically. The physical parameters of the example system are

$$\begin{gathered} m_1 = m_2 = m_3 = m_4 = 0.1 \text{ kg}, \\ k_1 = 5 \text{ kN/m}, \ k_2 = 1 \text{ kN/m}, \ k_3 = 2 \text{ kN/m}, \ k_4 = 4 \text{ kN/m}. \end{gathered} \tag{6}$$

The natural frequencies of the underlying conservative system with constant coefficients (Hamiltonian system) are

$$\begin{aligned} \omega_1 &= 47.4 \text{ 1/s}, \quad \omega_2 = 172.7 \text{ 1/s}, \\ \omega_3 &= 250.2 \text{ 1/s}, \quad \omega_4 = 308.7 \text{ 1/s}. \end{aligned} \tag{7}$$

Some of the corresponding anti-resonant parametric combination frequencies are

$$\begin{aligned} \nu_a^{12} &= \omega_2 - \omega_1 = 125.3 \text{ 1/s}, \quad \nu_a^{13} = \omega_3 - \omega_1 = 202.8 \text{ 1/s}, \\ \nu_a^{23} &= \omega_3 - \omega_2 = \ \ 77.4 \text{ 1/s}. \end{aligned} \tag{8}$$

Some of the resonant parametric combination frequencies and principal parametric resonance frequencies are

$$\begin{aligned} &\omega_1 + \omega_2 = 220.1 \text{ 1/s}, \quad \omega_1 + \omega_3 = 297.6 \text{ 1/s}, \\ &\omega_2 + \omega_3 = 422.9 \text{ 1/s}, \\ &2\omega_1 = 94.8 \text{ 1/s}, \quad 2\omega_2 = 345.4 \text{ 1/s}, \\ &2\omega_3 = 500.4 \text{ 1/s}. \end{aligned} \tag{9}$$

For the present system with symmetric system matrices according to Eq. (5), the parametric anti-resonance frequencies are located at the parametric excitation frequency of the difference type only, see [11, 23] for more details. Only parametric resonances of first order ($n = 1$ in Eq. (1)) are investigated since higher orders affect the system in a very narrow frequency interval of ν and only at much higher excitation amplitudes ε, see e.g. [3].

For the system in Eq. (2), the total energy can be written as

$$\begin{aligned} & W(t) - W(0) \\ & = \int_0^t \dot{\mathbf{x}}^T \Big(\mathbf{M}_0 \ddot{\mathbf{x}} + \mathbf{K}_0 \mathbf{x} + \sum_{m=1}^{M} \varepsilon_m \mathbf{K}_m \mathbf{x} \cos \nu_m \tau \Big) \, d\tau \\ & = \Big[T(\tau) + U(\tau) + \sum_{m=1}^{M} W_m(\tau) \Big]_{\tau=0}^{t} = \text{const.} \end{aligned} \tag{10}$$

Herein, W_m denotes the work of the time-periodic elastic forces acting on the system. The kinetic and potential energies T and U of the underlying Hamiltonian system are simply

$$T = \frac{1}{2} \dot{\mathbf{x}}^T \mathbf{M}_0 \dot{\mathbf{x}}, \qquad U = \frac{1}{2} \mathbf{x}^T \mathbf{K}_0 \mathbf{x}. \tag{11}$$

Individual amounts of energy that are accumulated during a time interval $[0, t]$ are calculated by integration of the energy flow over the time t. The total amount of work dissipated and work transferred into the system remains constant. For given initial conditions, the initial values of the works and energies are

$$\begin{aligned} T(0) &= \frac{1}{2} \dot{\mathbf{x}}(0)^T \mathbf{M}_0 \dot{\mathbf{x}}(0), \qquad W_m(0) = 0, \\ U(0) &= \frac{1}{2} \mathbf{x}(0)^T \mathbf{K}_0 \mathbf{x}(0), \qquad W(0) = T(0) + U(0). \end{aligned} \tag{12}$$

In terms of the modal displacements $\mathbf{p}$ introduced in Eqs. (3) and (4), the kinetic and potential energies can be rewritten as

$$T + U = \sum_n \frac{\dot{p}_n^2}{2} + \frac{\omega_n^2 p_n^2}{2} = \sum_n W_{p_n}. \tag{13}$$

3 INDUCED ENERGY TRANSFER

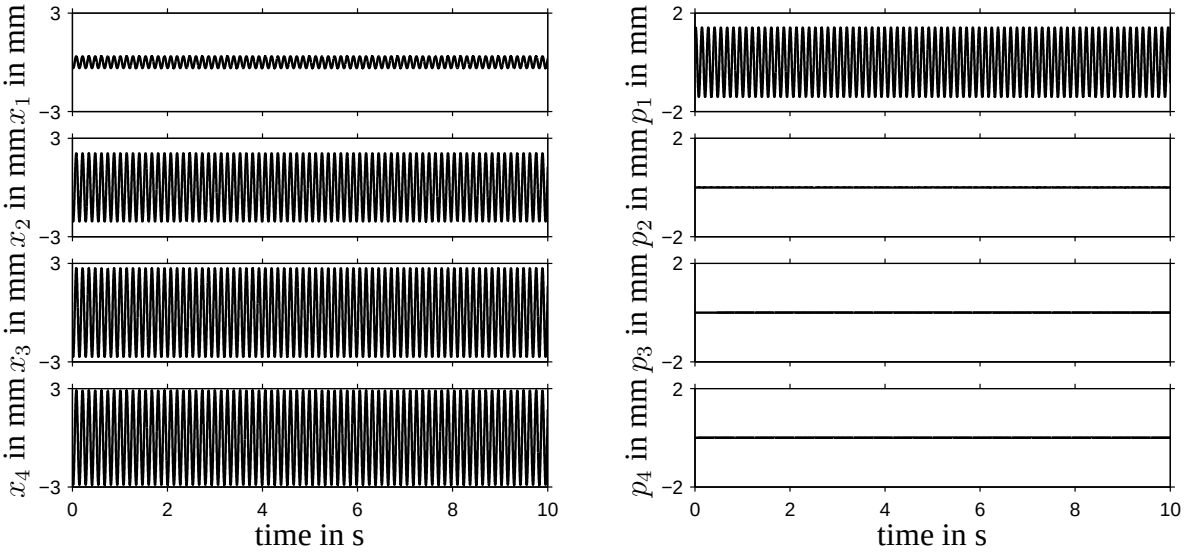
The aim of the present study is highlighting that a parametric anti-resonance introduces a coupling of only two of the many vibration modes of the system presented in Figure 1 while the remaining vibration modes stay decoupled. Therefore, the initial conditions are chosen to correspond to the first vibration mode

$$\mathbf{x}(0) = \mathbf{T}_1, \qquad \dot{\mathbf{x}}(0) = \omega_1 \mathbf{T}_1, \tag{14}$$

where $\mathbf{T}_1$ is the eigenvector of the first mode taken from the modal transformation matrix $\mathbf{T}$ in Eq. (4). Direct numerical time-integration of the equations of motion in Eqs. (2), (5) with the initial conditions taken in Eq. (14) results in the time histories of the physical and modal displacements.

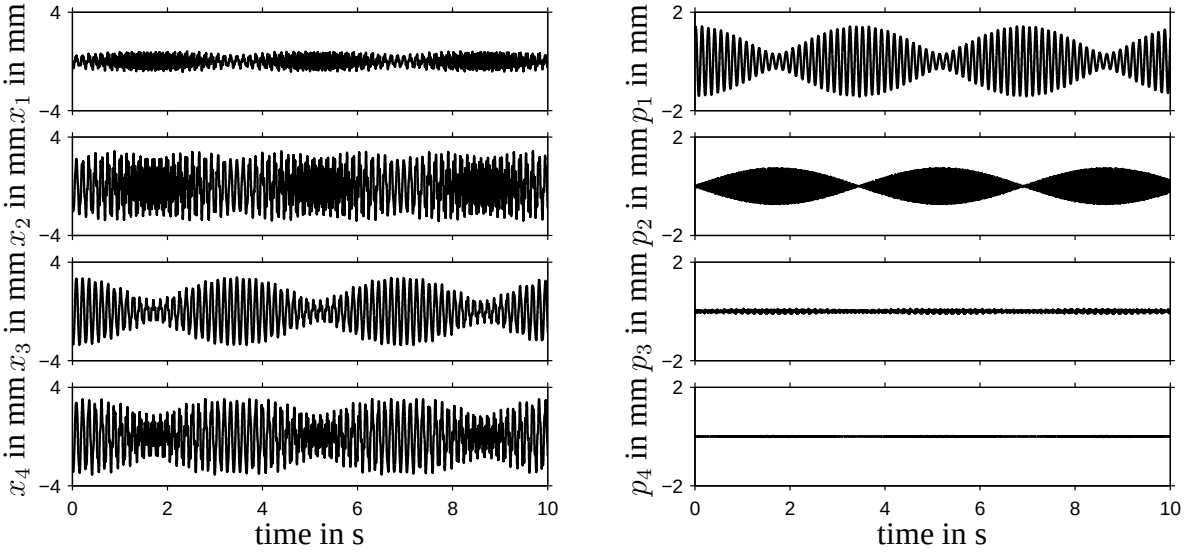
First, no parametric excitation is considered, $\mathbf{K}_m = \mathbf{0}$. Starting with the deflections and velocities of the first vibration mode in Eq. (14), the system vibrates in its first mode with the frequency ω_1, see Figures 2(a) and 2(b) for the physical x_i and modal displacements p_i of the system in Figure 1.

Secondly, we consider the same system and the same initial condition and activated parametric anti-resonance frequency $\nu_a^{12} = \omega_2 - \omega_1$ with an excitation amplitude of $\varepsilon = 30\%$. The



(a) Physical displacements. (b) Modal displacements.

Figure 2: Time series for the undamped system without parametric excitation for the initial condition in Eq. (14).

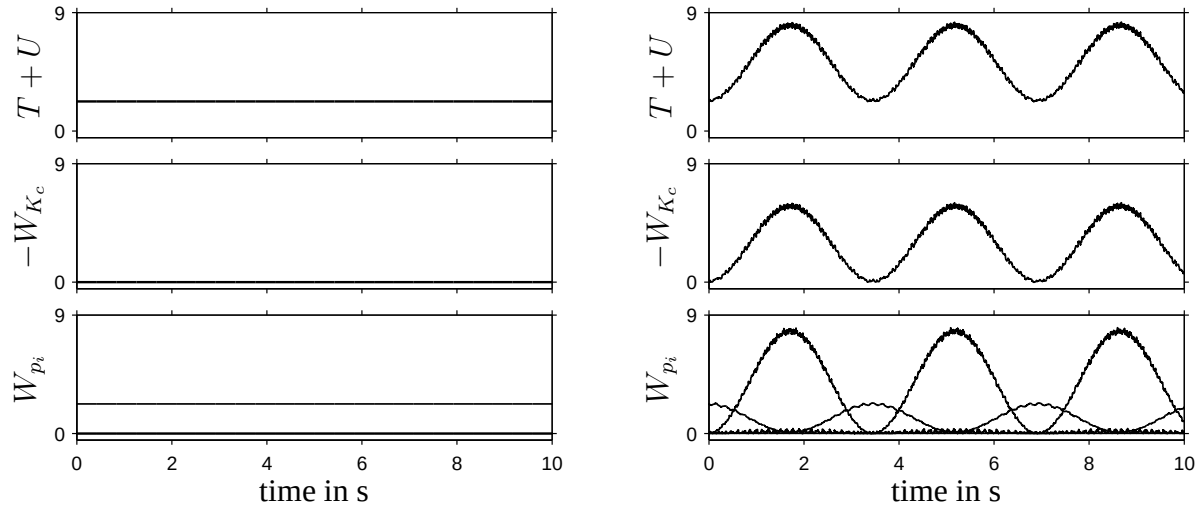


(a) Physical displacements. (b) Modal displacements.

Figure 3: Time series for the undamped system with parametric excitation (parametric anti-resonance frequency $\nu_a^{12} = \omega_2 - \omega_1$ and at an excitation amplitude of $\varepsilon = 30\%$) for the initial condition in Eq. (14).

physical and the modal displacements are displayed in Figure 3. Inspection of the modal displacements reveals that the first modal displacement p_1 vibrates at the first natural frequency ω_1 while (at the same time) the second modal displacement p_2 vibrates at the second natural frequency ω_2. For an undamped system and no parametric excitation, the modal displacements are uncoupled and their amplitudes are constant. Now, with a parametric anti-resonance acting in the system, the modes are coupled, strongly, and the conserved total system energy is transferred from mode 1 to mode 2 and vice versa. The remaining vibration modes 3 and 4 stay decoupled and are not involved in the energy transfer. This is a visualization of the physical interpretation proposed in [11].

The physical displacements in Figure 3(a) are the corresponding linear combination of the modal displacements in Figure 3(b). Consequently, the physical displacements x_1, x_2 possess two frequency components: ω_1 and ω_2. At times when a physical displacement is high the corresponding frequency component dominates in the vibration signal. For example, at times when x_1 dominates (at 0 and 3.5 s), the x_1 vibrates at the frequency ω_1 but also x_2 has a significant frequency component at ω_1. Similarly, at times when x_2 dominates (at 1.8 and 5.3 s), x_2 vibrates at the frequency ω_2 but x_1 has a significant frequency component at ω_2, too.



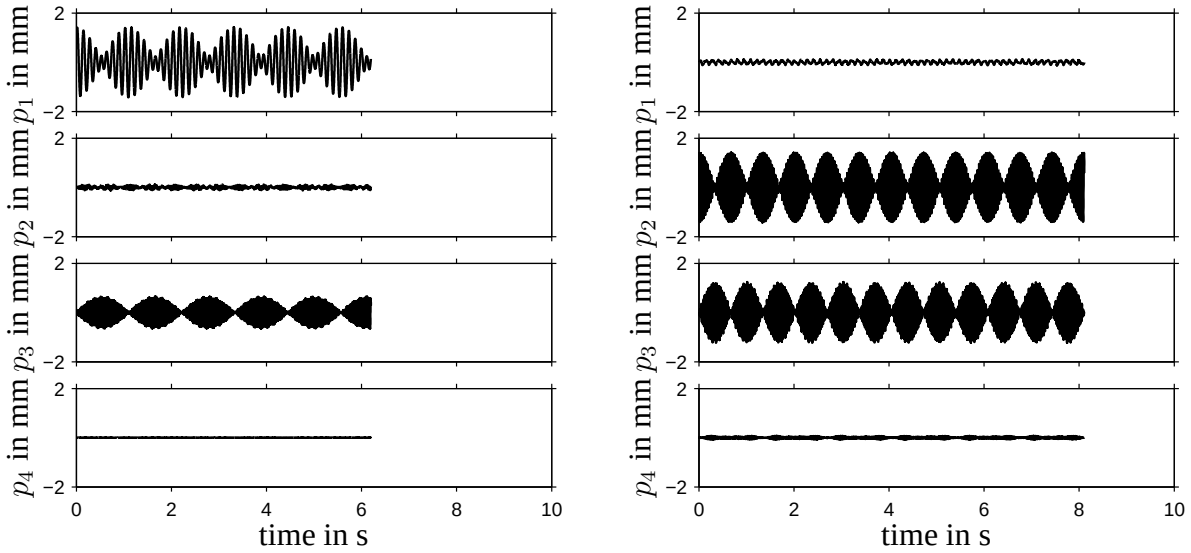
(a) Deactivated parametric anti-resonance.

(b) Activated parametric anti-resonance.

Figure 4: Time series for the undamped system in Figure 3 of (top) total kinetic and potential energy $T+U$, (centre) modal energies W_{p_i} with $\sum W_{p_i} = T+U$, and (bottom) work of time-periodic forces W_{kc}.

Now, we investigate the energy transfer within the system and between the system and its environment according to [9, 21] utilising the relation stated in Eq. (10). The different energy portions are grouped into the total vibrational energy $T+U$ and the work of the time-periodic force W_{kc} and are plotted in Figure 4. In order to highlight the modal coupling achieved at parametric anti-resonance, the total vibration energy is decomposed by its modal energy components W_{p_n} according to Eq. (13).

For the undamped system and deactivated parametric anti-resonance, the vibration energy is constant and is equal to the initial vibrational energy. Due to the choice of the initial condition, the system starts its vibration in its first mode and remains there. For the system with parametric anti-resonance activated, we observe a large periodical increase and decrease of the vibrational energy on top of the initial vibrational energy in the system. The parametric anti-resonance feeds the system with energy and dissipates vibration energy in the subsequent time interval. This induces an energy transfer from the first mode to the second, non-vibrating mode. At 1.8 s the total vibration energy is transferred to the second mode. This energy is transferred back to the first mode within the next 1.8 s. At 3.5 s the displacement of the second mode vanishes. This process is repeated since no damping is present in this system configuration. Summarising, vibrational energy is indeed transferred from one mode to the other and vice versa and the term *modal coupling* is fully justified, keeping in mind that modes of the underlying undamped system with constant parameters are meant (Hamiltonian system).



(a) For initial condition in Eq. (14) and $\nu_a = \omega_3 - \omega_1$. (b) For initial condition in Eq. (15) and $\nu_a = \omega_3 - \omega_2$.

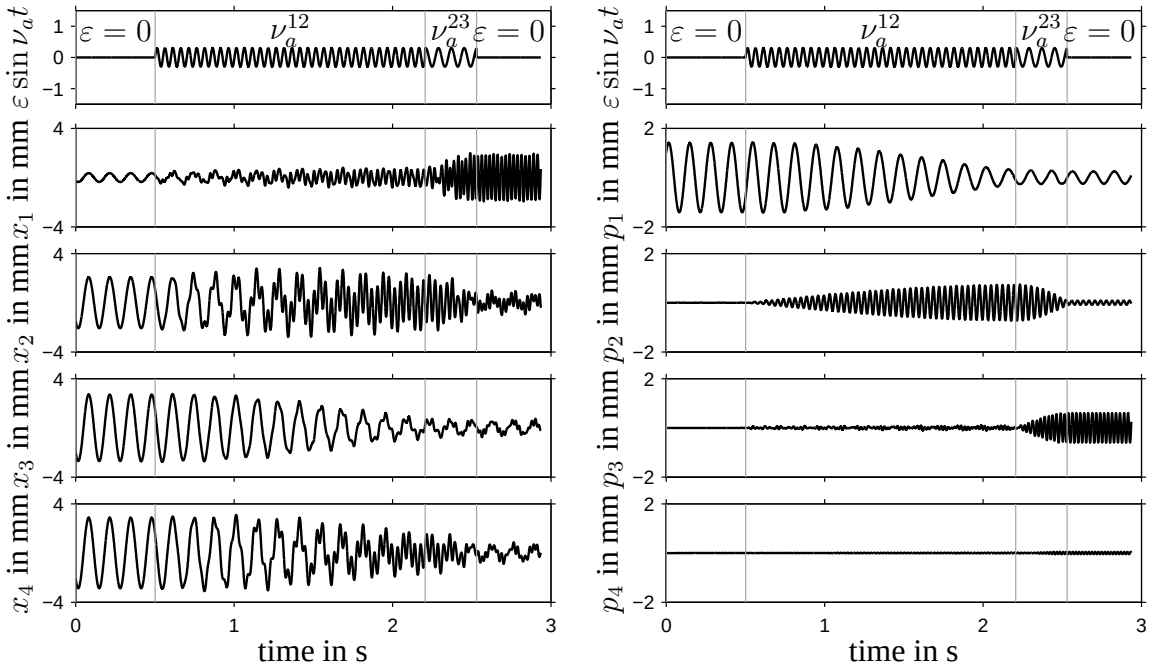
Figure 5: Time series of modal displacements for the undamped system at parametric anti-resonance.

Up to now, only the energy transfer between the first two modes has been discussed. Which modes are involved in the energy transfer induced by time-periodicity is purely defined by the parametric anti-resonance frequency. Choosing the parametric anti-resonance as $\nu_a = \omega_3 - \omega_1 = 202.8$ 1/s instead of $\nu_a = \omega_2 - \omega_1 = 125.3$ 1/s in Eq. (8), results in the modal displacements shown Figure 5(a). All remaining parameters are kept the same, especially the initial condition in Eq. (14) and the excitation amplitude of $\varepsilon = 30\%$. Choosing the second vibration mode as the initial condition,

$$\mathbf{x}(0) = \mathbf{T}_2, \qquad \dot{\mathbf{x}}(0) = \omega_2 \mathbf{T}_2, \tag{15}$$

and the parametric anti-resonance as $\nu = \omega_3 - \omega_2 = 77.4$ 1/s, induces an energy transfer between the vibration modes 2 and 3. The resulting modal displacements are shown in Figure 5(b). These results highlight clearly that now the vibration modes 1 and 3 exchange energy while vibration mode 2 does not participate in this interaction. Summarising, a parametric anti-resonance induces a coupling the vibration modes of the underlying system with constant coefficients and enables *an energy transfer between only two of the many vibration modes*.

It was shown that by choosing different parametric anti-resonance frequencies induces an energy transfer between different modes. This knowledge is exploited further by introducing a cascaded change of the stiffness element k_1 in Figure 1 as shown in Figure 6 on the top. Starting with the system without parametric excitation and the initial condition in Eq. (14) (first mode), the stiffness coefficient is kept constant in the first 0.5 s. Then, the parametric anti-resonance frequency ν_a^{12} in Eq. (8) is activated with an amplitude ε of 30%. This induces an energy transfer between the first and the second vibration mode. At the time instant of 2.2 s most of the vibration energy of the first mode has been transferred to the second mode. Now, the parametric anti-resonance is instantly switched to the frequency ν_a^{23} in Eq. (8). This induces an enegy transfer between the second and the third mode as clearly visualised in the modal displacements in Figure 6(b). At the time instant of 2.5 s most of the vibration energy has been transferred to the third mode and the parametric excitation is switched off again. Such a cascade of parametric anti-resonances enables the transfer of low-frequency vibration energy to high frequency vibration energy.



(a) Physical displacement. (b) Modal displacement.

Figure 6: Time series for the undamped system with parametric excitation for the initial condition in Eq. (14) and cascaded switching of the parametric anti-resonance frequency.

4 CONCLUSIONS

This study visualizes the physical interpretation of the working principle of a parametric anti-resonance. Introducing a specific time-periodicity in a system with constant coefficients (Hamiltonian), leads to a strong coupling of only two of the many vibration modes which induces an energy transfer between these two modes. It was highlighted that a parametric anti-resonance periodically pumps and extracts vibration energy into and from the system. An induced, cascaded energy transfer was proposed that enables to transfer low-frequency vibrational energy to high-frequency vibrational energy.

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