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FINITE ELEMEMT ANALYSIS OF THE DYNAMIC BEHAVIOUR OF TRANSMISSION LINE CONDUCTORS USING MATLAB

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Abstract. *The dynamic behaviour of power line cables have been a source of interest to researchers ever since the phenomenon was first noticed in the 1920s. Conductor oscillation is mostly caused by the dynamic forces of nature such as wind loading [1]. This imposes a periodic force on the conductors which is highly undesirable. It is therefore important for engineers to account for the possible effect of the wind loading when designing the power line.*

Investigations have shown that modeling the exact dynamic behaviour of a conductor is very difficult. Based on this fact, getting the exact analytical solution to conductor vibration is difficult, which is almost impossible, hence the numerical approximation becomes an option. This paper presents the developed finite element method used to analyse the dynamic behaviour of transmission line conductors. The developed finite element method (FEM) is implemented on MATLAB.

The numerical analysis using MATLAB that is presented in this paper is used to simulate the response of the conductor when subjected to external loading in time domain. The simulation is used to analyse the transverse vibration of the conductor. The formulation of the stiffness and load vector is done and the results obtained is used to evaluate the conductor" internal energy dissipation. This finite element solution is compared with the results documented by C. Hardy [2]. This numerical simulation is used to investigate the effects of varying the axial tension on energy dissipation within the strands. Hence, this evolved in physically appropriate energy characterization process that can be used to evaluate the conductor self-damping with respect to line contact.

1. Theoretical Background

The investigation of the mechanics of power line conductors have been a subject of both academic and industrial research ever since the phenomenon was first noticed. Most of early research in this field, investigated the global behaviour of the conductor, in which the conductor was treated as a solid, homogeneous continuous system. Several of these models developed for conductors, model the conductor as a beam or taut string and are available in literatures. These developed models are used to predict the global response of the conductors as a means of determining its natural frequencies, modes shapes and damping.

R. Claren and G. Diana [3] developed the mathematical model for the conductor transverse vibration using the concept of the conductor's principal modes. The authors [4-6] investigated this phenomenon in the aspect of fluid-solid dynamic excitation, by conducting experiments in wind tunnel. They investigated how and the amount of force impacted that causes the conductor oscillation. The findings from these wind tunnel experiments have been used to develop emperical formulae used to determine the wind input power to the conductor. Also the authors [7-11] have all come up with various models to determine the conductor damping and the number of vibration absorbers needed on the line to curtail the effect of mechanical oscillation on the line conductors.

Unlike regarding a conductor as a solid system, a more realistic approach would be to consider a conductor as a composite consisting of an assembly of a number of helical strands. In regarding it as a composite structure, help to explain why the dynamics of cables tends to exhibits a non-linearity. This is because conductor dynamics is a function of parameters that has to do with its geometry and this proffers the reason why it is impossible to obtain an exert analytical solution. Therefore, the approach of modeling the conductor as a single, homogenous system is incapable of addressing local phenomenon within the strands like contact, strand slippage and frictional effects when under the influence of tension and bending.

Some recent investigations have employed the discretized approach in analyzing cables. In [12] the author carried out the analytical analysis of rope. In the work by R. E Hobbs and M. Raoof [13], the various forms of contacts within the cable geometry were explained. These forms of contact in a cable can be used to identify where frictional effects which can be used to analyse stick and slip process within the Cables. R. H. Knapp [14] analysed the cables helically using the contact deformation and friction effects to calculate stresses. On the bases of stick and slip conditions of strands, in reference [15], the author presented a novel and more realistic model to determine bending stiffness of a conductor under both axial and bending loads. He developed the secant bending stiffness model that can be used to determine the variable bending stiffness under the conditions of inter-layer friction and slip as a function of displacement and curvature. This model established the fact that the bending stiffness value at any point along the conductor varies non-linearly with the curvature. C. Hardy [2] explained how to evaluate energy dissipation along line contacts. This paper presents, first in the series of investigations into the concept of conductor self-damping. A FEM for addressing the conductor dynamic behaviour as a function of its inter-strand line contact is developed and implemented on Matlab software. This numerical method focuses mainly on the work documented by C. Hardy [2] and this entails treating the conductor as composite structure formed by the assembly of helical strands layers over the core with the analysis of the inter-strand line contact areas, the effects of friction and slip are analyzed during the periodic motion. In the quest for developing this finite element simulation for the conductor, it is first of all imperative to present the geometric description of the conductor and concepts that will be used for the formulation.

2 Stranded Bare Conductor

Transmission line conductor is made up of single or multiple layer(s) of strands. The strand is the basic structure of the conductor; whose arrangements comprises of the central core and the helically curved strand. The stranded conductor is treated as composite structure of a collection of strands, arranged in layers and each layer is constituted by helically wound profile with a lay angle which is alternated in successive layers. For a single-layered conductor as shown in figure 1, comprise of a centre core strand and a layer of 6-strand.

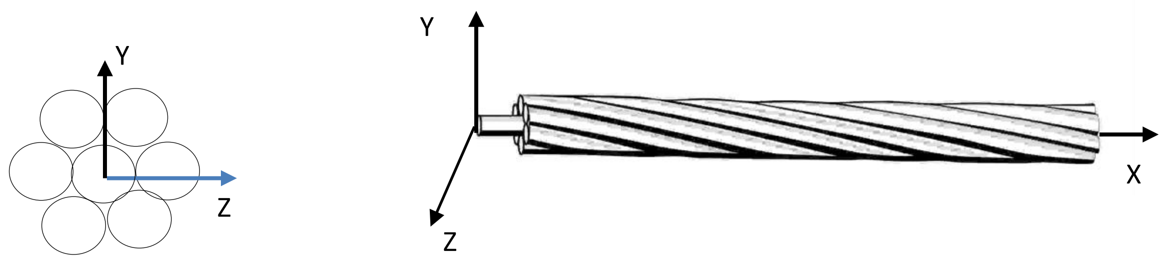


Figure 1: single layer conductor

2.1 Description of a Stranded Bare Conductor

Consider figure 2, a stranded bare conductor with cross-section A_T; consist of i-layers $(i=1, 2,...., N)$, each layer made of a number of strands of radius r_i with lay angle β_i, wrapped over the centrally located core strand, of radius r_0. Each particular layer consists of m number of strands and the position of each strand is defined as $m(i, j)$, where $j\,(j=1, 2,...., m)$ defines the strand position from the z-axis in the anti-clockwise direction. The distances of each layer of strand at origin and the distance along the curvilinear axis of helix from the conductor neutral axis are:

$$h_{i\,j} = R_{i\,j}\sin(\varphi_{i\,j}) \quad \text{and} \quad h_i(X) = R_i \sin\left(\frac{2\pi X}{\lambda} \pm \varphi_{i\,j}\right) \qquad \text{.....................(1)}$$

Where $\varphi_{i\,j}$ is the angular position from the z-axis in the anti-clockwise direction, $R_{i\,j}$ is the radius between layers and R_i radius of the circular path that passes through the centre of strands located at i-layer and both can be calculated by

$$R_{i\,j} = r_0 + (k)\sum_{i=1}^{N-1} d_i \qquad \text{Where } k = \{ {}^{0\ \ i=0}_{1\ \ i=1,2,...}$$

$$R_i = r_0 + \sum_{i=1}^{N-1} d_i + r_N \qquad \text{..................... (2)}$$

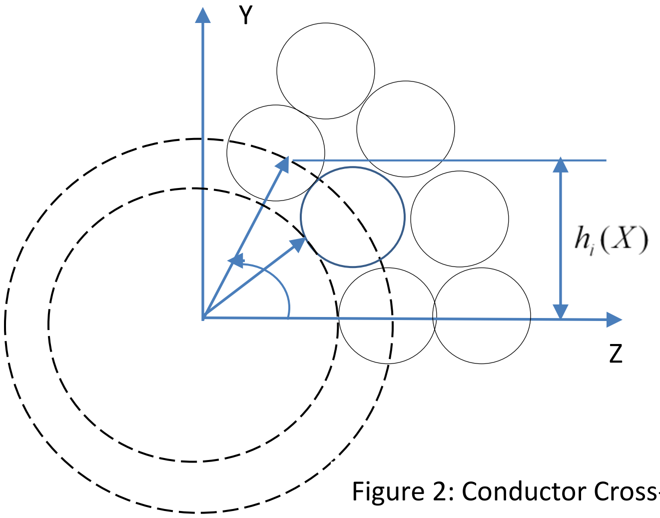


Figure 2: Conductor Cross-section

2.2 Inter-strand Contacts

The power line cables when strung on the towers, the ends are subjected to tensile force and when allowed to rest under the influence of gravity forms a centenary profile. This condition creates a compact form of the conductor giving rise to some form of contacts between strands and with the core. As mentioned before, in [13] the various forms of contacts within the cable geometry were identified. The authors identified two forms of contacts: the line and the point contacts.

The line contact occurs between the parallel layered helical strands of the same layer. This form of contact also exists between the first layer and the core. Thus, this form of wire-wire contact, exist between wires in the same layer are, and between first layers with the core. The point or trellis contact occurs due the helical arrangement of strands. Due to the opposite lay angle, alternate layer strands crosses each other producing a point or trellis contact i.e. a layer-layer contact where the strand in the one layer touches only those in adjacent layers at a point. The knowledge of the form of force, deformation and stress in the area contact is very important because due to friction.
Under the stick conditions both strands exert equal and opposite force at the region of contacts. During bending resulting from dynamic loading, the frictional force is overcome resulting in relative sliding between the strands in the area of contacts. This sliding can have effect on the dynamic behaviour of the conductor because energy dissipation does occur in the contact areas.

1 Conductor Damping

Transmission lines when exposed to the dynamic forces of nature the cables experiences transverse vibrations. The energy on the cable is dissipated internally and external by vibration absorber if present on the line. Conductor self-damping (internally dissipation) can be attributed to two main sources.

Conductors can damp out this energy by internal energy losses at microscopic (molecular) level within the core and individual strands of the conductor and this known as metallurgical or material damping [1]. Also when the conductor flexes, the strands of the conductor slip against each other; this relative motion generates frictional forces that provide damping [2]. The combination of these energy dissipative processes by a conductor is known as conductor self-damping. During bending, the energy dissipation due to frictional effects around the area of contacts induced by the sinusoidal forcing function coupled with the material damping tends to limit the amplitude of vibration. Reference [2] inferred that energy dissipation during the slip regime mainly account for the conductor self-damping.

2 Conductor's Stress and Strain

Conductors are strung at both ends with axial tensioned which is a percentage of its ultimate tensile strength. This axial loading constrains the strands to have one form of contacts, where resistive frictional force tends to prevents displacements. This axial force result to some form of stresses along the contacts areas. Based on the fact that energy dissipation is caused by sliding along the areas of contacts where the frictional forces is overcome, in this paper the evaluation of damping is focused on flexure during the slip regime a similar approach adopted in [2] .

Hence, for the evaluation of energy dissipation, the stresses due to line contacts are considered and for now application is for singled-layered conductors.

The strands in an axially tensioned conductor are subjected to tensile, bending and torsional stresses but mainly to tensile stress [16]. Neglecting bending and torsional effects, the tensile stress acting across the section of the conductor can be evaluated as

$$\sigma_x = \frac{S}{A_T} \qquad \text{……………………. (3)}$$

Where A_T is the conductor cross section and can calculated as $A_T = \pi\left(r_0 + \sum_{i=1}^{N} d_i\right)$

Each individual strand is assumed to be subject to the same tensile force given by [15]

$$Z_{i\,j} = \frac{E_i A_i \cos^2 \alpha_i}{\sum_{all\ strands} E_i A_i \cos^3 \alpha_i} .S \qquad \text{……………………. (4)}$$

Where E is the Young modulus, $A_{i\,j}$ is the strand cross section area, α_i is the layer lay angle, S is the axial tension

The tensile stress acting on the individual strands

$$\sigma_{i\,j} = \frac{Z_{i\,j}}{A_{i\,j}} \qquad \text{……………………. (5)}$$

Neglecting the effect of Poisson ratio, the strain acting along the line contact between the strand and the core [2]

$$\varepsilon = -h_{i\,j} \frac{d^2 Y}{dX^2} = E_i \sigma_{i\,j} \qquad \text{……………………. (6)}$$

For a laminar wind flow across, the loading causes the power line cables experiences transverse sinusoidal displacement which is known as Aeolian vibration. The displacement of this sinusoidal function as can be defined as

$$Y = Y_{max} \sin\left(\frac{2\pi X}{\lambda}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The axial strain due to the axial loading along the line contacts as function curve helical path can be determine as

$$\varepsilon_A = \frac{T(s)}{EA_i} = \int_0^s \mu(\xi).q\, d\xi \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The pressure q can be evaluated by [17]

$$q = \frac{Z_{ij} \sin^2 \alpha_i}{R_{ij}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

For sinusoidal displacement according to [2] can the friction coefficient is evaluated as

$$\mu(x) = \frac{R_c Y(EA)\cos^3\alpha}{q}\left(\frac{2\pi}{\lambda}\right)^3\left[\frac{\lambda}{p}\cos\left(\frac{2\pi X}{p}+\varphi_{ij}\right)\sin\left(\frac{2\pi X}{\lambda}\right)+\sin\left(\frac{2\pi X}{P}+\varphi_{ij}\right)\cos\left(\frac{2\pi X}{P}\right)\right] \ldots (10)$$

3 Finite Element Model

The objective of this study is to develop a finite element simulation to evaluate the parameters such as the displacement field. In the formulation, only the axial strain is considered and this is similar to the finite element for a bar element. Using the bar element as shown in figure 3, taking into account the sinusoidal axial displacement for the two-node bar element.

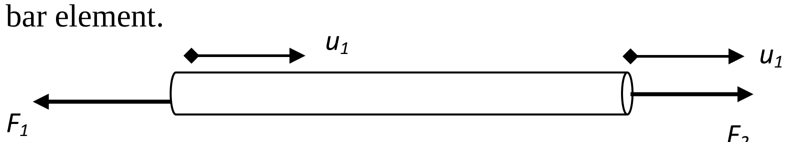


Figure 3: The Bar Element

The strain ε_x can related to the axial displacement u as

$$\varepsilon_x = \frac{du}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The axial displacement is interpolated by

$$u = N_1(\xi)u_1 + N_2(\xi)u_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Where the shape functions are defined as

$$N_1(\xi) = \frac{1}{2}(1-\xi)\,; \qquad N_2(\xi) = \frac{1}{2}(1+\xi)$$

The resultant tensile for on the strands can be evaluated as

$$f_2 - f_1 = EA\frac{\Delta l}{l} = \frac{EA}{l}(u_2 - u_1)$$

The above equation can be written in matrix for as follows

$$\begin{bmatrix} \frac{EA}{l} & -\frac{EA}{l} \\ -\frac{EA}{l} & \frac{EA}{l} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f \end{bmatrix}$$

From equation (8) , the axial force acting the strand can be determined by

$$Z_{i\,j}(s) = EA\int_0^s \mu(\xi).q\; d\xi \qquad \text{……………………. (13)}$$

Hence,

$$\frac{EA\cos^2\alpha}{l}\left[u_2 - u_1\right] = qEA\mu(X)\cos^3\alpha$$

$$EA\cos^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = qEA\mu(X)\cos^3\alpha\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \text{……………………. (14)}$$

The finite element formulation is expressed on the finite element equilibrium and the force-displacement relation is given by

$$[K]\{u\} = \{f\} \qquad \text{……………………. (15)}$$

Where [K] the stiffness matrix, {u} is the vector displacements and {f} is the force vector and displacements {u} are interpolated over the whole conductor strand as $\{u\} = [N]^T\{d\}$

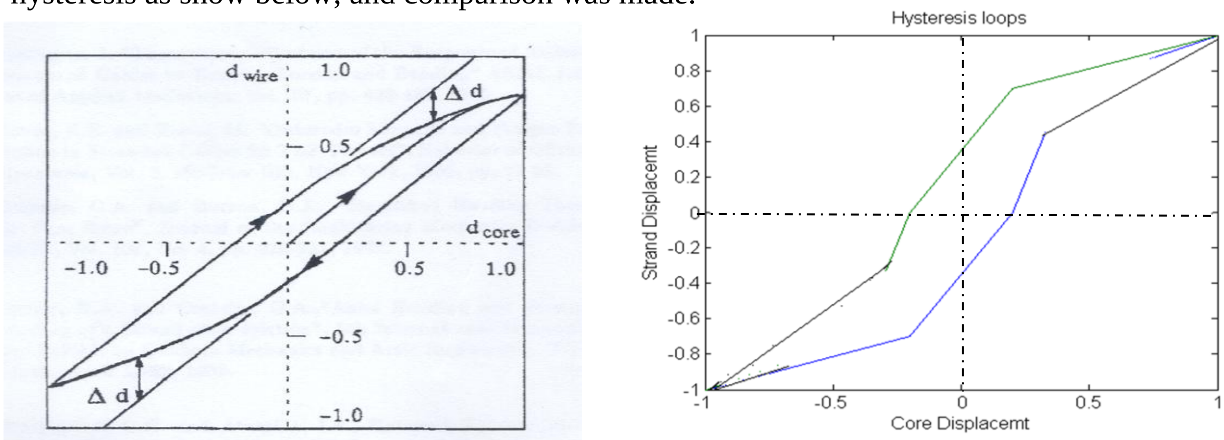
4 Matlab Implementation

A MATLAB code [18] was used to simulate above finite element equation for the bar element. The code was modified to include the sinusoidal forcing function. The implementation was used to simulate this numerical problem developed above for the vibrating conductor. The implementation was done for single layer conductors. This model was formed by the assembly of the finite element equation into a global matrix in the form of stiffness and force vector. In assembling all the finite elements equations requires the satisfaction of the boundary conditions in which displacements are constrained at both end of the strand. The code was simulated to find the solution to the displacement field developed from FEM.

Another Matlab code was developed in which the displacement field severs as input variable to generate the hysteresis loop. This computer program simulation was used to generate loops for three different axial tensions. Using the MATLAB results, the work that is documented in [19] was verified in which the author inferred that the conductor damping decreases with increase in axial tension.

5 Results

The numerical technique was used to solve the assembled FEM equation. The system equation [K]{d} = {f} was solved for the system displacements {d}. In [2] the author draw the inference that the non-linearity in strand axial movement result in closed hysteresis loops. He gave a classsical explanation on how the loop can be use to determermine energy per cylce. This concept was varied by using Matlab code generat the hysteresis as show below, and comparison was made.



A *B*

Figure 4: A-Hysteresis loop from [2] and B- hysteresis loop from Matlab

The data for the single layer conductor; the family of AAAC- 6201 (Akron, Alton, Ames) whose data are in table 1 were used to for the implementation.

Code Word	Stranding	Diameter (mm)		Cross-Sectional Area (mm^2)	Rated Tensile Strength (N)
		Individual Strands	Complete Conductor		
Akron,	6/1	I.679	5.029	15.484	4937.526
Alton	6/1	2.118	6.350	24.645	7828.870
Ames	6/1	2.672	8.026	39.226	12455.020

Table 1.Single layer conductor AAAC- 6201[20]

This numerical simulation was used to investigate the effects of varying the axial tension on energy dissipation using the conductors' data in the table 1 above. The implementation was done for these conductors with their axial tensions set at 15, 20, 25 and 30% of ultimate tensile strength. The results are shown in table 2.

Axial Tension (% UTS)	Energy Dissipation per cycle		
	Akron,	Alton	Ames
15	0.1468	0.1459	0.1375
20	0.1224	0.1107	0.1001
25	0.0879	0.0723	0.0684
30	0.0452	0.0357	0.0296

TABLE 2: The energy dissipation at various axial tensions

6 Conclusion

The evaluation of conductor damping is a complex problem. In this paper, finite element implementation of an axially displaced bar model when subjected to bending was presented. This was used to simulate the axial displacement of conductor when subjected to both tensile and bending. This model was used to determine the amount of energy dissipated along the line contact. Using the Matlab finite element results a comparison was made with that documented [2] and this shows a good agreement.

This was extended to determine the energy dissipation at three different conductor stringing tensions this done with respect to the percentage of its UTS. The results of varying the axial tension established the fact that damping decreases with the increase in tension .The parameters obtained from the above, to some degree accuracy can be used to evaluate conductors damping due to line contacts.

Presently, this work is being extended to include an implementation of the damping due to point contacts which the analysis of this form of contact can be found in a more recent paper by C.B. Rawlins [21]. This will include the formulation that will be applicable to multilayer conductors. To enhance the reliability of the finite element model that will be developed, the conductor will be treated as composite beam which is more realistic with respect to the conductor structure as compared to solid beam which is mostly found in several literatures.

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