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MULTI-ACTIVE-LAYER LAMINATES AS ACTUATORS FOR VIBRATION CONTROL

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Abstract. *Piezoelectric materials are frequently used as actuators for vibration control of laminate structures. These active laminates may have several piezoelectric laminas, possibly with different actuation characteristics and, due to the inherent complexity of the design problem, numerical analysis tools are usually applied for optimization. However, while focusing on obtaining the optimal result, these tools give the designer very little insight on the actuation capabilities of a multi-active-layer laminate. For instance, whenever directional piezoelectric actuators are used, a whole range of in-plane force vectors can be generated within the laminate after fabrication, depending on the number of active laminas, lamina orientations, and the independently applied electric fields. However, information about this range is most likely to be difficult to obtain from the optimization algorithm. This work presents a different approach to the understanding of multi-active-layer laminates as induced strain actuators. The actuation capabilities are studied by using an analogy between the generated in-plane forces and the Mohr circle for plane stress. While limited in scope by the initial assumptions, this approach can provide the designer with plenty of insight on what can be expected in terms of actuation forces, from the combination of several active laminas with either similar, or different, piezoelectric characteristics. Also, from this study a simple set of equations emerges as a closed form solution for the design of an active laminate with two active laminas, that is intended to generate a specified in-plane force state while minimizing the applied electric field. An example application to the selective excitation of vibrations of a plate is presented.*

1 INTRODUCTION

Research in induced strain actuation has made available a number of solutions that are applicable to the design of an active laminate. In particular, a number of piezoelectric actuators has been developed in recent years, with different characteristics and actuation possibilities. The basic monolithic PZT plate, with surface electrodes and polarized through the thickness, has in-plane isotropic actuation and isotropic elastic characteristics. It is also very brittle, and therefore difficult to conform to curved surfaces. One of the first developments of this basic actuator appears in [1] where the monolithic PZT plate is encapsulated in an electrically insulating material to allow the insertion of the actuator in carbon fiber laminates. In [2] the limitation of in-plane isotropic actuation is overcome by attaching the basic monolithic actuators to the host structure through a special shaped bonding layer. This form of non-isotropic actuation is known as *Directionally Attached Piezoceramics* (DAP). In [3] an electrode design named *InterDigitated Electrodes* (IDE) is used, that allows for the pooling of the piezoceramic material to be made along a direction belonging to the plane of the lamina. This design results in directional (non-isotropic) in-plane actuation, and also allowed for a considerable increase in available actuating force. In all the above mentioned cases the active element is the basic monolithic ceramic plate. In [4] a composite actuator with piezoceramic fibers is presented. This actuator is made of cylindrical fibers of piezoceramic material in epoxy matrix, packaged in insulating film with an etched IDE pattern. This design has non-isotropic elastic characteristics, but it is also much less brittle than the previous designs, and easily allows the actuator to be bonded to slightly curved surfaces. These composite actuators with cylindrical fibers are known as *Active Fiber Composites* (AFC). A further development of this concept, named *Macro Fiber Composite* (MFC), can be found in [5] where the cylindrical fibers are replaced by rectangular cross section fibers, that allowed for an increase in efficiency due to the increase of active material density. In [6] an MFC actuator with special fibers made of a single piezoelectric crystal is presented. The single-crystal fibers that were used resulted in a decreased force induction capability when compared to the original fibers, however the free strain of the actuator was increased.

The variety of actuator types available includes characteristics of isotropic actuation and isotropic elasticity, non-isotropic actuation and isotropic elasticity, and both non-isotropic actuation and non-isotropic elasticity. Active laminates with several piezoelectric laminas are usually optimized with resource to numerical tools, and so the choice of active layer types and orientations can be left to the optimization algorithm. However, this method is focused on the end result for the given design specifications, and provides the designer very little insight over the general actuation capabilities of a multi-active-layer laminate. This paper presents a study of the actuation capabilities of an active laminate using an analogy between the generated in-plane forces and the Mohr circle for plane stress. The analysis is straightforward for a single active lamina, however, by taking advantage of the geometric interpretation, the analysis is extended to a pair of active laminas. From this analysis a simple set of equations emerges as a closed solution to the design of an active laminate that is intended to generate a specified in-plane force state with minimum electric field being applied to the active layers. It must be made clear that the proposed analysis and design method only address the force generation problem, and not the elastic problem of the active laminate. An example application of this design method is presented, where the directionality of actuation is explored in order to maximize or minimize the excitation of selected modes of vibration of a plate.

2 PIEZOELECTRIC ACTUATION

The linear constitutive relationship describing piezoelectric material coupled behavior is

$$\mathbf{D} = \mathbf{e}\,\varepsilon + \epsilon\,\mathbf{E} \tag{1}$$

$$\sigma = \mathbf{C}\,\varepsilon - \mathbf{e}^T\mathbf{E} \tag{2}$$

where **D** is the electric displacement vector, ε the strain vector, σ the mechanical stress vector and **E** the electrical field vector. The material constants **C**, **e** and ϵ represent the stiffness matrix, the piezoelectrically induced stress, and the electrical permittivity respectively. Eq. (1) describes the direct piezoelectric effect that allows the use of piezoelectric materials as sensors, while Eq. (2) describes the inverse piezoelectric effect that allows the use of these materials as actuators.

According to [4], under conditions of plane stress and considering the Kirchoff assumptions for laminate structures, the piezoelectrically induced in-plane forces per unit length ($\mathbf{N}_p$) and moments per unit length ($\mathbf{M}_p$) for the case of active laminas with constant properties through the thickness are given by

$$\begin{bmatrix} \mathbf{N}_p \\ \mathbf{M}_p \end{bmatrix} = \sum_{k=1}^{n_p} h_{p\_k} \begin{bmatrix} \mathbf{I} \\ z_{m\_k}\mathbf{I} \end{bmatrix} \mathbf{e}_k^* E_k \tag{3}$$

where n_p is the number of active laminas, **I** is the 3×3 identity matrix and h_{p_k} and z_{m_k} represent the thickness and the mean coordinate of active lamina k respectively. Also E_k is the scalar value of applied electric field in the direction of polarization, and $\mathbf{e}_k^* = [e_{31}\ e_{32}\ 0]_k^T$ is the vector of piezoelectrically induced stress constants of lamina k, after considering the plane stress assumptions. Subscripts 1 and 2 refer to the in plane coordinates of the lamina, while subscript 3 refers to the coordinate perpendicular to that plane. Poling direction of the actuator is assumed to be the standard direction 3 also according to [4], and while this is not the case for actuators with IDE, it carries no loss of generality for the analysis since e_{31} is never assumed to be equal to e_{32}.

If we consider the case where each lamina is allowed to have its own orientation θ_k relative to the referential of the generated forces, a rotation matrix $\mathbf{T}_k$ must be considered in Eq. (3) leading to

$$\begin{bmatrix} \mathbf{N}_p \\ \mathbf{M}_p \end{bmatrix} = \sum_{k=1}^{n_p} h_{p\_k} \begin{bmatrix} \mathbf{I} \\ z_{m\_k}\mathbf{I} \end{bmatrix} \mathbf{T}_k^{-1} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix}_k E_k \tag{4}$$

where

$$\mathbf{T}_k = \begin{bmatrix} \cos^2\theta_k & \sin^2\theta_k & 2\sin\theta_k\cos\theta_k \\ \sin^2\theta_k & \cos^2\theta_k & -2\sin\theta_k\cos\theta_k \\ -\sin\theta_k\cos\theta_k & \sin\theta_k\cos\theta_k & \cos^2\theta_k - \sin^2\theta_k \end{bmatrix}$$

3 ACTUATION DESIGN

3.1 Characterization of an actuator lamina

For convenience the analysis of directionality will focus on the in-plane force vector $\mathbf{N}_p$, since the only difference between the in-plane forces and moments is the product by the lamina coordinate z_{m_k} that plays no role in the directionality issue. The in-plane force vector can be written for a single lamina as

$$\mathbf{N}_p = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = h_p \mathbf{T}^{-1} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix} E \tag{5}$$

It is possible to recognize in Eq. (5) an analogy with Mohr's circle for plain stress. The resulting circle equation is

$$\left(N_x - h_p E \frac{e_{31} + e_{32}}{2}\right)^2 + N_{xy}^2 = \left(h_p E \frac{e_{31} - e_{32}}{2}\right)^2 \tag{6}$$

By considering $h_p E = 1$ one gets a circle that is characteristic of the actuator, depending only on the piezoelectrically induced stress constants e_{31} and e_{32}. Another characteristic of the actuator is the line passing through the origin of the force axis, defining the maximum possible shear force N_{xy}^{max} as a function of $h_p E$. This results from the assumption of linearity of the force response of the actuator to the applied electric field, the line slope being equal to $(e_{31} - e_{32})/(e_{31} + e_{32})$.

3.2 Actuation with a single lamina

From Eqs. (5) and (6) one concludes that for any single lamina the force vector $\mathbf{N}_p$ will have a representation that lies on a circle with center position and radius both proportional to $h_p E$. For a given angle θ this representation is given by points $X = (N_x, N_{xy})$ and $Y = (N_y, -N_{xy})$ like the example shown on the right side of the axis in Fig. 1.

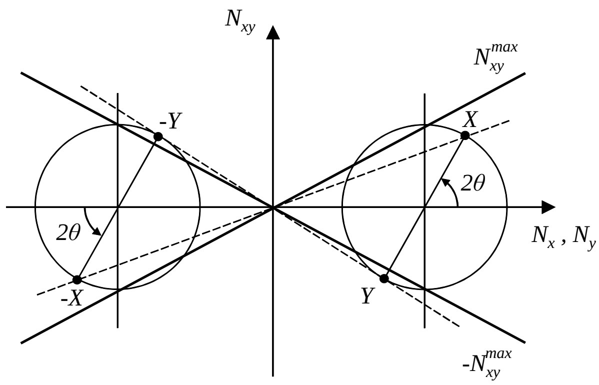

Figure 1: Mohr circle representation of specific force states.

Any change in the applied electric field E will result in a displacement of points X and Y along the corresponding dotted lines. In particular, if the sign of the applied electric filed is switched, then all force components in $\mathbf{N}_p$ also change sign. This case is represented by points $-X$ and $-Y$ on the left side of the axis in Fig. 1.

3.3 Actuation with two laminas

Consider two in-plane force vectors represented by the pairs (X_1, Y_1) and (X_2, Y_2). For simplicity the following analysis will focus on points X_1 and X_2 only. Consider two position vectors $\overrightarrow{p_1}$ e $\overrightarrow{p_2}$, defining points X_1 and X_2 belonging to circles C_1 e C_2 respectively, as represented

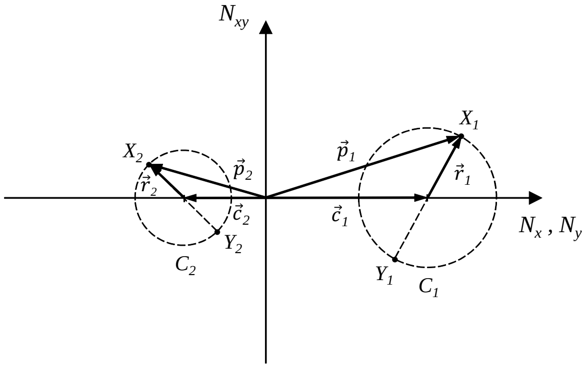

Figure 2: Decomposition of position vectors $\overrightarrow{p_i}$ as the sum of vectors $\overrightarrow{c_i}$ and $\overrightarrow{r_i}$.

in Fig. 2. Both these vectors can be decomposed as the sum of a vector $\overrightarrow{c_i}$ defining the center position of circle C_i, and a vector $\overrightarrow{r_i}$ defining the position of X_i relative to the center of C_i. The position of point X, of the in-plane force state (X, Y) that results from the superposition of (X_1, Y_1) and (X_2, Y_2), is therefore given by $\overrightarrow{p_1} + \overrightarrow{p_2} = \overrightarrow{c_1} + \overrightarrow{c_2} + \overrightarrow{r_1} + \overrightarrow{r_2}$, as represented in Fig. 3. A similar reasoning could have been made to determine point Y from Y_1 and Y_2. It can be shown that if point X_1 is allowed to be any point on circle C_1, and point X_2 is allowed to be any point on circle C_2, then point X would lie within the annular surface centered in $\overrightarrow{c_1} + \overrightarrow{c_2}$ with outer radius equal to $|\overrightarrow{r_1}| + |\overrightarrow{r_2}|$ and inner radius $||\overrightarrow{r_1}| - |\overrightarrow{r_2}||$. This annular surface is also represented in Fig. 3 as $C_1 + C_2$.

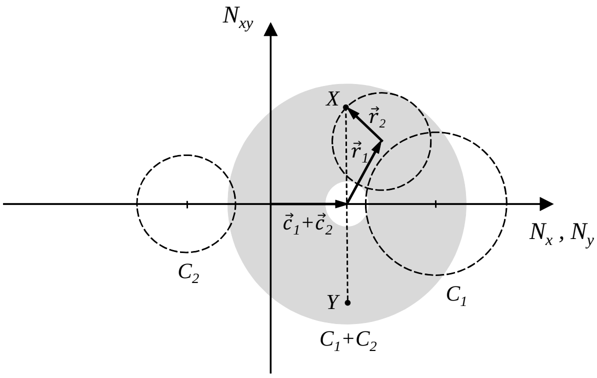

Figure 3: Force state (X, Y) resulting from the superposition of the force states (X_1, Y_1) and (X_2, Y_2).

3.4 Design of an active laminate for a specified force vector

The problem of designing the actuation to generate a specific force state is the inverse of the reasoning developed in section 3.3. Given a force state (X, Y) one would wish to find the values of the angles θ_1, θ_2 and the electric fields E_1 and E_2 so that each lamina would produce (X_1, Y_1) and (X_2, Y_2) respectively. For a laminate with two active laminas with the same characteristics and $e_{31} \neq e_{32}$, Eq. (4) is simplified to

$$\mathbf{N}_p = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = h_p \left(E_1 \mathbf{T}_1^{-1} + E_2 \mathbf{T}_2^{-1} \right) \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix}$$

For a given type of active lamina, the values of h_p and $\mathbf{e}_k^*$ are fixed. The design of the laminate has four unknown values θ_1, θ_2, E_1 and E_2, and only three equations are available to determine these values. Let the force state $\mathbf{N}_p$ be described by the center c and radius r of the circle where the pair (X, Y) lies, and the principal direction ψ (similar to θ in Fig. 1) that uniquely defines points X and Y on that circle

$$\begin{aligned} c &= (N_x + N_y)/2 \\ r &= \sqrt{N_{xy}^2 + ((N_x - N_y)/2)^2} \\ \tan 2\psi &= 2N_{xy}/(N_x - N_y) \end{aligned} \tag{7}$$

The center c_p and radius r_p of the characteristic circle of the active laminas is given by

$$\begin{aligned} c_p &= (e_{31} + e_{32})/2 \\ r_p &= |e_{31} - e_{32}|/2 \end{aligned} \tag{8}$$

It is possible to show that if the relation $r/|c| > r_p/|c_p|$ holds, then choosing $\theta_2 = \theta_1 + \pi/2$ as fourth constraint leads to the minimization of $|E_1| + |E_2|$. The solution of the design problem is

$$\begin{aligned} \theta_1 &= \psi & E_1 &= \tfrac{1}{2h_p}\left(\tfrac{c}{c_p} + \tfrac{r}{r_p}\right) \\ \theta_2 &= \psi \pm \tfrac{\pi}{2} & E_2 &= \tfrac{1}{2h_p}\left(\tfrac{c}{c_p} - \tfrac{r}{r_p}\right) \end{aligned} \tag{9}$$

When the case is such that $r/|c| \leq r_p/|c_p|$ one finds the value of $|E_1| + |E_2|$ to be constrained by the value of c. Considering $E_1 = E_2$ to be the fourth constraint the solution is

$$\begin{aligned} \theta_1 &= \psi + \arccos\left(\tfrac{r\, c_p}{c\, r_p}\right)/2 & E_1 = E_2 &= \tfrac{1}{2h_p}\tfrac{c}{c_p} \\ \theta_2 &= \psi - \arccos\left(\tfrac{r\, c_p}{c\, r_p}\right)/2 \end{aligned} \tag{10}$$

4 APPLICATION TO A VIBRATION PROBLEM

In this section this design method is applied to the excitation of modes of vibration of a laminated plate with embedded piezoelectric actuators. The goal is to take advantage of the directionality properties of the actuation, instead of the actuator placement, in order to achieve the maximization or minimization of the amplitude of specific modes of vibration.

The work per unit area performed by the in-plane forces and moments in a laminate under the previously stated assumptions is

$$\overline{u}\, h = \frac{1}{2}\left[\mathbf{N}^T\ \mathbf{M}^T\right]\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \tag{11}$$

where $\overline{u}$ is the mean energy density along the total thickness h of the laminate, ε^0 is the vector of the mid-surface strains, and κ is the vector of curvatures of the area element under consideration. As result of a modal analysis, the vector of local strains and curvatures may be decomposed into an infinite series of modal components with modal amplitude ν_j resulting in

$$\overline{u}\, h = \frac{1}{2}\left[\mathbf{N}^T\ \mathbf{M}^T\right]\left(\sum_{j=1}^{\infty}\nu_j\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}_j\right) \tag{12}$$

	Mode 1	Mode 2	Mode 3	Mode 4
Freq. (Hz)	0.95	13.6	28.6	56.2
$\bar{\varepsilon}_I$	3.90×10^{-3}	-4.32×10^{-2}	-5.92×10^{-2}	1.46×10^{-1}
$\bar{\varepsilon}_{II}/\bar{\varepsilon}_I$	-0.282	-0.579	-0.709	-0.865
$\bar{\theta}_I(^\circ)$	-7.37	33.9	-43.4	-39.6

Table 1: Values for natural frequencies, mean principal top surface strains, and resulting principal angles over the active area of the plate.

Assuming that the in-plane forces and moments are induced exclusively by the piezoelectric laminas the work performed by the actuators on mode j is given by

$$\overline{u}_j\, h = \frac{\nu_j}{2}\left[\mathbf{N}_p^T\ \mathbf{M}_p^T\right]\left[\begin{array}{c}\varepsilon^0\\ \kappa\end{array}\right]_j \tag{13}$$

It is easy to see how it is possible, for instance, not to perform work on mode j, if the vector of applied forces and moments can be made orthogonal to the vector of strains and curvatures within the active area element.

4.1 Plate model

As a case study, the model of a laminated plate clamped along one of the edges is considered. The laminated structure is made of four active laminas and a passive isotropic substrate made of aluminium with the following ply sequence: $[\theta_1^{PZT}/\theta_2^{PZT}/iso^{Al}/\theta_1^{PZT}/\theta_2^{PZT}]$. The active layers have properties similar to the actuators in [3], meaning that they are elastically isotropic, but generate in-plane directional actuation due to the IDE design. This choice of actuators allows the angle of actuation to be selected in the design stage without changing the elastic characteristics of the laminate, that could otherwise also modify the mode shape solutions of the vibration modes. Assuming small displacements and $\varepsilon^0 \approx 0$ the top surface strains are related to local curvature by $\varepsilon = z\,\kappa$, where z is the coordinate of the top surface relative to the mid-surface. This relation allows the active laminate to be designed based on the principal directions of the top surface strain for each mode of vibration. The choice of layer sequence allows for the generation of pure bending moments $\mathbf{M}_p$, by applying electric fields of opposite signal to each of the two layers that share the same orientation angle, meaning that $\mathbf{N}_p = 0$. Therefore, in this example, Eq. (13) is reduced to

$$\overline{u}_j\, h = \frac{\nu_j}{2}\mathbf{M}_p^T \kappa_j \tag{14}$$

Although the piezoelectric layers cover the entire plate, the applied electric fields are limited to a small area portion near the clamped edge. Also, a small tip mass is located at the lower free vertex of the plate, so that the mode shape solution will show more interesting strain angles according to the vibration mode considered. In Fig. 4 the mode shapes of the first four modes of vibration resulting from a finite element modal analysis are shown, along with the corresponding top surface strains of the finite elements that model the actuated area of the plate. The frequency values for each mode of vibration are presented in Table 1, along with the mean principal strains for the top surface of the actuated area of the plate, and the corresponding principal strain angle.

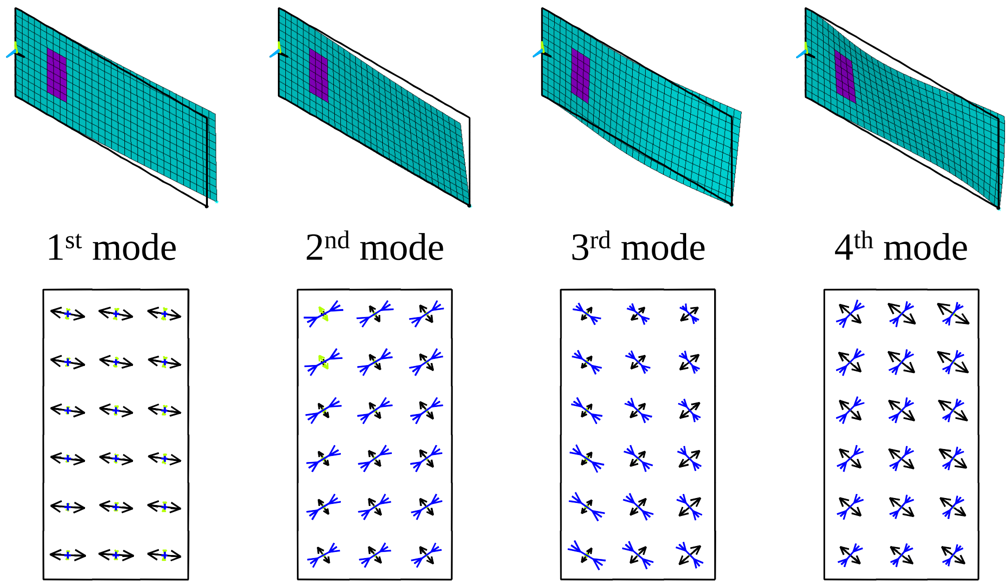

Figure 4: Mode shapes and surface strains for the first four modes of vibration.

4.2 Results

Eight different harmonic simulations were performed, with eight different plate models. The type of actuators chosen required the values of the actuation angles θ_1, θ_2 to be calculated from Eq. (10) only. In accordance with this solution, a fixed value was used for $E1 = E2$, that was assumed to be the maximum value of electric field allowed by the actuators. The eight plate models were designed so that the amplitude of the first four modes of vibration of the plate was first maximized, and then minimized according to Eq. (14) over the actuated area. In practice, since all finite elements have the same dimensions, the mean of the top surface principal strain vectors was used to determine the direction of actuation for each mode. A damping coefficient of 0.01 was assumed for all four modes to prevent numerical problems in the simulation.

The results of the eight harmonic simulations are presented in Fig. 5. The curves represent the magnitude of the off-plane displacement of the upper free vertex of the plate. Results show a clear difference between the amplitudes when the actuation is designed for maximization or minimization of each mode. In particular, the value of the displacement magnitude at the frequency of the targeted modes shows no significant resonant peak when the actuation is designed for the minimization of that particular mode.

5 CONCLUSIONS

A study of the actuation capabilities of an active laminate is presented, using an analogy between the generated in-plane forces and the Mohr circle for plane stress. From this analysis, a simple set of equations emerges as a closed form solution to the problem of designing the laminate actuation with two active laminas to generate a specific force state. An example application of this design method is also presented, where it is suggested that, with appropriate actuation design, it is possible to prevent the excitation of a selected mode of vibration without changing the actuator placement on the structure. The simulation results of the example application show no significant resonant peaks at the targeted modes of vibration. These results were achieved by selecting appropriate actuation angles and electric fields on the design stage of the piezoelectric actuation, and without choosing a specific (nodal) position for the actuated area of the plate.

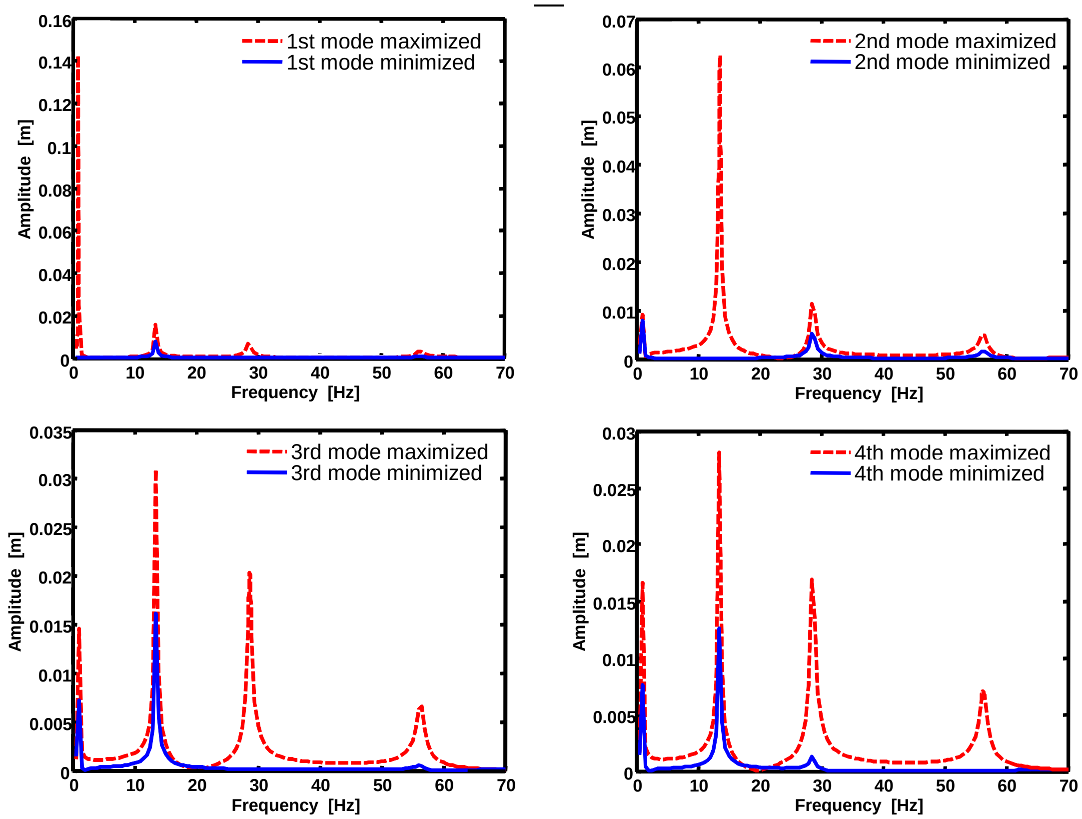

Figure 5: Simulation results of the harmonic analysis for the eight plate models.

These results also support the proposed analysis and design method, although such support is limited to the specific conditions of this case study.

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REFERENCES

[1] E.F. Crawley, J. de Luis, Use of Piezoelectric Actuators as Elements of Intelligent Structures. *AIAA Journal*, vol.25, n.10, pp. 1373-1385, (1987)

[2] Barrett, R.M., Modeling techniques and design principles of a low-aspect-ratio active aeroservoelastic wing, *Proc. SPIE Smart Structures and Materials 1993: Smart Structures and Intelligent Systems*, Albuquerque, NM, USA, (1993)

[3] Hagood, N.W. and Kindel, R. and Ghandi, K. and Gaudenzi, P., Improving transverse actuation of piezoceramics using interdigitated surface electrodes, *Proc. SPIE Smart Structures and Materials 1993: Smart Structures and Intelligent Systems* , Albuquerque, NM, USA, (1993)

[4] Hagood, N.W. and Bent, A.A., Development of Piezoelectric Fiber Composites for Structural Actuation, *34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, pp.3625-3638, LaJolla, California; USA, (1993)

[5] Wilkie, W.K. and Bryant, G.R. and High, J.W. and Fox, R.L. and Hellbaum, R.F. and Jalink, A. and Little, B.D. and Mirick, P.H., Low-Cost Piezocomposite Actuator for Structural Control Applications, *Proceedings, SPIE 7th Annual International Symposium on Smart Structures and Materials*, pp.323-334, Newport Beach, CA, (2000)

[6] Wilkie, W. Keats and Inman, Daniel J. and Lloyd, Justin M. and High, James W., Anisotropic Piezocomposite Actuator Incorporating Machined PMN-PT Single Crystal Fibers, *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Palm Springs, CA, U.S.A., (2004)