11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

NATURAL VIBRATIONS OF A CYLINDRICAL SHELL CONTAINING FLUID AND RODS BUNDLES

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Keywords: Vibration, Rods bundle, Cylindrical shell, Dynamic properties, Complex dynamic density.

Abstract. *The results of theoretical investigations of bending vibrations of cylindrical shells containing a liquid and bundle of elastic rods are presented. Fluid and rods bundle are considered as an effective medium with dynamic inertial and viscous properties, depending on density of the material and volume concentration of rods, density and viscosity of fluid, as well as the natural frequencies of bending vibrations of rods. The theoretical dependence and the result of experimental investigation of dynamic properties of effective medium are presented. Resonant dependence of the complex dynamic density of effective medium on the oscillation frequency is established. The dependencies for the natural frequencies and damping coefficient of the shell considering the inertias and viscous properties of effective medium are given. These results will be useful in the analysis and diagnostics of vibration state of heat-exchangers apparatus and steam generators.*

1 INTRODUCTION

The cylindrical shell and the bundles of rods or tubes are the main elements of designs of reactors, heat exchangers and steam generators. Practically always such systems are subject to the dynamic impact of the flow of the heat-carrier and to vibrations affecting their reliability and safety. Research of vibration characteristics of such systems has been the subject of many theoretical and experimental works.

Modeling the vibrational dynamics of bundles of elastic rods and shells of heat-exchange equipment is quite a difficult task, since all the elements of structures form complex spatially distributed systems. A lot of design elements of heat-exchange devices are dynamically linked through the liquid coolant, carrying out the transmission of vibrations from some elements to others. In a numerical simulation of vibrational dynamics of such systems direct discretization of complex multielement rod and tube systems is not practical, as it leads to models of very large dimensions and are excessively cumbersome for calculations.

To solve tasks of this kind use can be made of continuous approach, based on the introduction of effective dynamic properties of heterogeneous media, formed by a fluid and a bundle of elastic rods. The main idea of this approach is that the bundle of rods or tubes in a fluid is considered as a pseudo-homogeneous medium with effective properties - dynamic density and translational viscosity, which determine the high-viscous effect of a heterogeneous medium on the vibrations of the contacting elastic structural elements with distributed parameters (rods, plates, shells and etc.). The effective properties of heterogeneous media depend on the properties of the fluid and inclusions as well as on the frequency of vibration impacts.

In general, the effective dynamic properties of heterogeneous media with inclusions, whose role the elastic rods can perform, are not constant values but are described by the resonant dependencies.

In the theoretical analysis of effective dynamic properties of heterogeneous media we consider oscillations of the fluid and its subsequent vibrations inclusions, in particular, those of elastic rods. Resonance develops at a frequency of vibration impacts on heterogeneous medium, which coincides with the natural frequency of inclusions. The vibrational density and translatoinal viscosity take maximum resonance values.

Formulas for dynamic density and translational viscosity of a fluid with a bundle of elastic rods are given in this work on the basis of dependence common for effective dynamic properties of heterogeneous media with inclusions. These formulas are then used in the calculation of natural frequencies and amplitude-frequency characteristics of vibrations of a cylindrical shell, containing a fluid and a bundle of elastic rods.

2 EFFECTIVE DYNAMIC PROPERTIES OF THE FLUID WITH INCLUSIONS

Consider the translational oscillations of the heterogeneous medium, the fluid formed and identical inclusions evenly distributed in it and elastically mounted in the fixed coordinate system. In the harmonically oscillating fluid, inclusion will also perform harmonic oscillations.

The equation of inclusions-oscillator motion under inertial, elastic and viscous forces takes the form [1]

$$(\rho-\rho_0)G_0\ddot{X} - m\left(\ddot{X}_0 - \ddot{X}\right) - \xi\left(\dot{X}_0 - \dot{X}\right) - KX_0 = 0\,, \tag{1}$$

where X, X_0 –are oscillatory displacements of the fluid and inclusions, ρ, ρ_0 – density of the fluid and material inclusions, G_0 – the volume of the inclusions, m – added mass, ξ –viscous resistance coefficient, K – stiffness of the inclusion elastic connection. (1)

Let us introduce the following notation: $\omega_0 = \left[K/(\rho_0 + \gamma\rho)G_0\right]^{1/2}$ - the natural frequency of inclusions in a fluid, $\Delta = \rho_0/\rho$ - relative density of inclusions, $\gamma = m/\rho G_0$ - the added mass coefficient, $h = \xi/2(\rho_0 + \gamma\rho)G_0$ - the inclusion attenuation coefficient. In this notation, the equation (1) takes the form

$$\ddot{X}_0 - \ddot{X}\frac{1+\gamma}{\Delta+\gamma} + 2h\left(\dot{X}_0 - \dot{X}\right) + \omega_0^2 X_0 = 0. \quad (2)$$

The ratio of displacement of inclusions oscillators and cell heterogeneous medium X/X_0 is found from the solution of the equation (2). The ratio is a complex value, whose real part gives the dispersion component of the displacement and the imaginary part gives dissipation component or translational viscosity $\eta^*(\omega)$.

$$\frac{X_0'}{X} = \frac{\frac{1+\gamma}{\Delta+\gamma}\left(1-\frac{\omega_0^2}{\omega^2}\right)+\frac{4h^2}{\omega^2}}{\left(1-\frac{\omega_0^2}{\omega^2}\right)^2+\frac{4h^2}{\omega^2}}, \quad (3)$$

$$\frac{X_0''}{X} = \frac{2h}{\omega}\left[\frac{\frac{\omega_0^2}{\omega^2}+\frac{1-\Delta}{\Delta+\gamma}}{\left(1-\frac{\omega_0^2}{\omega^2}\right)^2+\frac{4h^2}{\omega^2}}\right]. \quad (4)$$

Each of these components, as can be seen from the formulas, depends on parameters of a heterogeneous medium Δ, γ, ω_0, h and frequency of exposure ω.

3 DYNAMIC DENSITY

Define the complex heterogeneous medium density $\rho^*(i, \omega)$ as the coefficient of proportionality between the external pressure force acting on a unit volume of a heterogeneous medium and its instantaneous acceleration

$$F_s = \rho^*(i\omega)\ddot{X} = \rho^*(\omega) - \frac{1}{\omega}\eta^*(\omega). \quad (5)$$

With this definition the real part of the complex density is a dynamic density of the heterogeneous medium $\rho^*(\omega)$ as a measure of its inertia, and the imaginary part, without the factor $1/\omega$, is the viscous resistance coefficient or translational viscosity $\eta^*(\omega)$.

Such an approach means that we consider dynamic density as a measure of the medium inertia only in relation to external pressure forces; the reaction forces from the elastic connections are considered to be internal. From this point of view, it should be considered that the medium momentum arising under external forces contains some component, caused by the elastic ties inclusions reaction.

In fact, the rate of change of the medium momentum I, consisting of the fluid momentum I_f and the inclusions momentum I_i, depends on the joint effect of the surface force F_s and the reaction force of the elastic ties inclusions $F_e = -KX_0$.

Thus, the rate of the momentum change of the fluid-inclusion system can be written as follows

$$\frac{d}{dt}\left(I_f + I_i\right) - F_e = F_s. \tag{6}$$

Considering further that the F_s changes in sinusoidal law, the left part of (6) can be written, respectively, in the form of the sum of the sine temporal component

$$\frac{d}{dt}\left[I(t) + \int KX_0' \sin\omega t dt\right] = -\frac{d}{dt}\left[I_0 - \frac{KX_0'}{\omega}\right]\cos\omega t, \tag{7}$$

where $I(t) = I_f + I_i$ - is the instantaneous value of the momentum of the fluid-inclusion systems, I_0 - a peak value of the momentum.

Thus, the value $I_0{}^* = I_0 - \frac{KX_0'}{\omega}$ can be considered as a peak value of the effective momentum element of the medium, with elastic fixed inclusions, which includes the effect of the reaction of the elastic connection of inclusions. The real momentum $I(t)$ can be defined as the sum of in-phase with velocity $U(t)$ of the components of the momentum of the fluid and inclusions

$$I(t) = I_f + I = U(t)\left[\rho\left(1 - \frac{V_0'}{U}\varphi\right) + \rho_0\frac{V_0'}{U}\varphi\right], \tag{8}$$

where φ - volume concentration ratio of the velocities of the inclusions and the cell V_0'/U, equal to the ratio of displacements X_0'/X is determined by the formula (3).

Further, expressing the stiffness coefficient of relations inclusions in unit volume of the medium through its natural frequency of the inclusions

$$K = n(m_0 + m)\omega_0^2 = \rho\varphi\omega_0^2(\Delta + \gamma), \tag{9}$$

where, n - is the number of starts per unit volume, $m_0=\rho_0 G_0$ – the mass of the inclusions, effective momentum medium have the form

$$I^*(t) = \rho\dot{X}(t)G\left(1 - \frac{X_0'}{X}\varphi + \varphi\Delta\frac{X_0'}{X} - \frac{\varphi\omega_0^2(\Delta+\gamma)}{\omega^2}\frac{X_0'}{X}\right). \tag{10}$$

By dividing the (10) on the velocity $\dot{X}(t)$ and using the relation (3), we obtain the formula for the dynamic density of the medium with elastic fixed of the inclusions. [1]

$$\frac{\rho^*}{\rho} = 1 + \left[\frac{\frac{1+\gamma}{\Delta+\gamma}\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{4h^2}{\omega^2}}{\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 + \frac{4h^2}{\omega^2}}\right]\left((\Delta - 1) - (\Delta+\gamma)\frac{\omega_0^2}{\omega^2}\right)\varphi. \tag{11}$$

As can be seen from (11), dynamic density depends on the geometrical characteristics of the inclusions (γ, φ), physical properties of the fluid and inclusions (ρ, ρ_0, h, ω_0), as well as on the frequency of vibration impacts ω.

Formula (11) can be used to determine the dynamic density of such a heterogeneous medium as a bundle of elastic rods in the fluid. If the ends of the rods are fixed in the still supports (in heat exchangers of - tube plates), in case of translational oscillations of a fluid in the perpendicular to their axes direction, the rods will perform bending oscillations simulator the inclusions.

In the case of a inviscid fluid (h=0) the formula (11) takes a simpler view

$$\frac{\rho^*}{\rho} = 1 - (1+\gamma)\left(\frac{\frac{\omega_0^2}{\omega^2} + \frac{1-\Delta}{\Delta+\gamma}}{1 - \frac{\omega_0^2}{\omega^2}}\right)\varphi, \tag{12}$$

where the coefficient of added mass for the rods in the bundle is expressed by the formula [2]

$$\gamma = (1+\varphi)/(1-\varphi), \tag{13}$$

and the natural frequency of flexural vibrations of the rods in the fluid is determined by the formula

$$\omega_0 = \frac{\pi^2}{L^2}\sqrt{\frac{EJ}{(\rho_0 + \rho\gamma)S}}\,. \tag{14}$$

Here L is the length, EJ - Flexural rigidity, $S=\pi a^2$ - cross sectional area of rods with a radius of a.

To illustrate Fig. 1 shows the dependence of the dynamical density of a heterogeneous medium with bundles of elastic rods on the relation of frequencies of vibration impacts ω to the natural frequency of oscillation. Solid lines in figure show theoretical dependence of (12) for the corresponding parameters of the rods bundles.

From figure 1 we can see that the dynamic density has resonance frequency dependence and in a certain range of frequencies takes negative values. Such a dependence of dynamic density from frequency is due to the resonant oscillations of elastic rods.

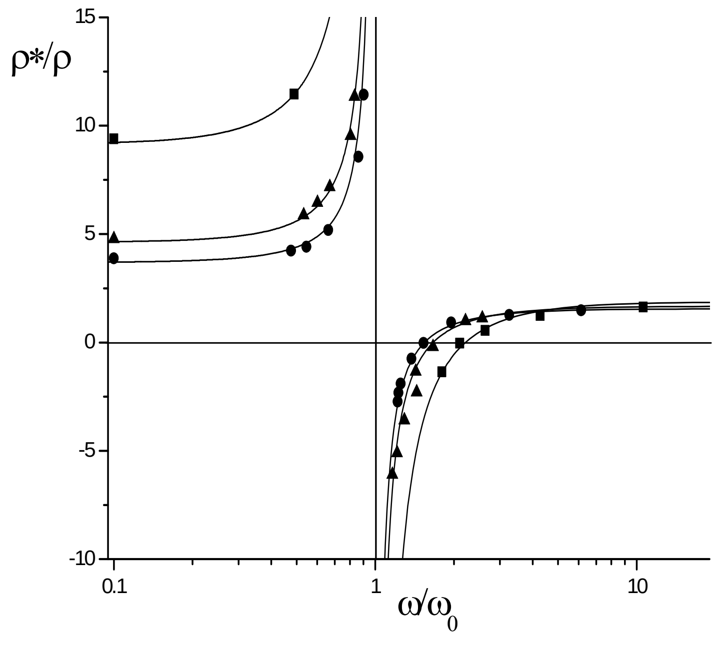

Figure1: The resonance dependence of the effective dynamic density. Curves - formula(12), ●, ▲, ■ - the experimental data [3-4] when $\Delta=\rho0/\rho=2{,}3$ and $\varphi=0{,}536;\ 0{,}630;\ 0{,}749$, respectively.

4 TRANSLATIONAL VISCOSITY

Under vibrations of a heterogeneous medium movement inclusions (rods) relative to the carrier fluid leads to the dissipative losses in the medium. In accordance with our definition of (5) and using the formulas (4,6), you can get a general formula for the translational of viscosity of an medium. [5]:

$$\eta^* = 2\rho\varphi(\Delta+\gamma)\frac{\left(\frac{\omega_0^2}{\omega^2}+\frac{1-\Delta}{\Delta+\gamma}\right)h}{\left[\left(1-\frac{\omega_0^2}{\omega^2}\right)^2+\frac{4h}{\omega^2}\right]}. \tag{15}$$

From (15) it is visible that translational viscosity depends on the same parameters of the heterogeneous medium, as dynamic density (11).

For a bundle of elastic round rods in low-viscous fluid when $h \ll \omega_0$ the formula (15) can be written in the following form:

$$\frac{\eta^* a\delta}{\eta} = \frac{4\varphi}{(1-\varphi)^2}\frac{\left(\frac{\omega_0^2}{\omega^2}+\frac{1-\Delta}{\Delta+\gamma}\right)^2}{\left(1-\frac{\omega_0^2}{\omega^2}\right)^2}. \tag{16}$$

where $\delta = \sqrt{2\eta/\rho\omega}$ - the boundary-layer thickness on the surface of the rods. The coefficient of the added mass of γ is determined by the formula (13). Fig. 2. shows the dependence of the translational viscosity on the frequency of impacts with the same parameters of the bundle rods, as in figure 1.

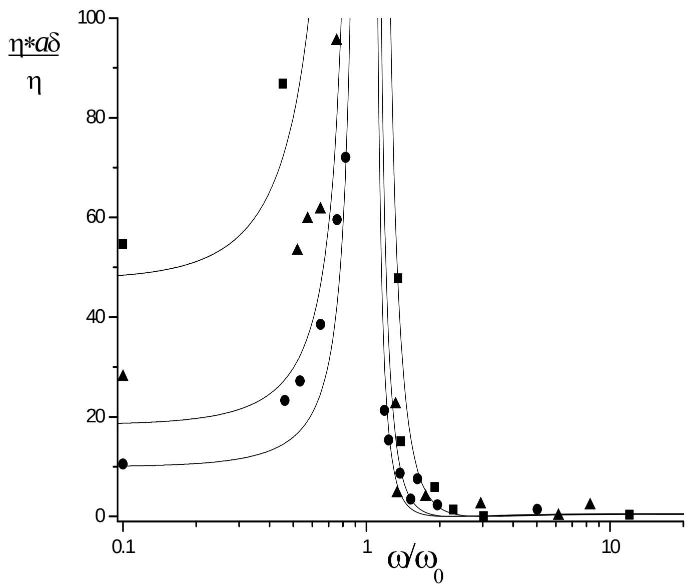

Figure 2: The resonance dependence of the translational viscosity (Legend: see figure 1).

5 VIBRATIONS OF A SHELL WITH A FLUID AND A BUNDLE OF RODS

It is known that the natural frequencies of oscillation of a shell filled with fluid (Fig. 3), depend on both the geometrical parameters and elastic properties of a material of the shell, and because of the density of the fluid. The inertial effect of the fluid on the vibrating shell is determined by the attached weight that depends on the form of vibrations of the shell and the density of the fluid.

Similarly fluid with a bundle of elastic rods, as heterogeneous medium with effective dynamic properties, has the inertial effects of the vibrating shell.

Consider the vibrations of a cylindrical shell length L, radius R and thickness of the shell h_s. The solution of the equations of vibrations of an empty shell (without fluid and a bundle of rods) leads to the following known formula for natural frequencies of vibrations [6]

$$\omega_{mn}^2 = \left[\left(k_1^2 + k_2^2\right)^2 + \frac{k_1^4 k_0^4}{\left(k_1^2 + k_2^2\right)^2}\right]\left(\frac{D}{\rho_s h_s}\right), \tag{17}$$

where $k_0 = \left(\frac{Eh_s}{DR^2}\right)^{1/4}$, $k_1 = \frac{m\pi}{L}$, $k_2 = \frac{n}{R}$ - the wave number, the relevant terms of the longitudinal and the circular direction, $D = Eh_s^{\,3}/12\left(1-\nu^2\right)$ – cylindrical rigidity of the shell, ρ_s – the density of the material of the shell, h_s – the thickness of the shell.

If the shell is filled of incompressible fluid and its vibrations occur on one of eigenform shapes with one half-wave on forming (m=1), the added mass of the fluid, related to unit the surface of the shell, is expressed by the following formula [7]

$$M = \rho R \frac{n}{n^2+1}. \tag{18}$$

The dependence of the natural frequencies of vibration of a shell with a fluid on the number of waves in the circumferential direction can be recorded through the attached mass in the form of

$$\Omega^2{}_n = \frac{\omega^2{}_{1n}}{1+\frac{M}{\rho_s h_s}} = \frac{\omega^2{}_n}{1+\frac{R}{h}\frac{\rho}{\rho_s}\left(\frac{n}{n^2+1}\right)}, \tag{19}$$

where ω_n – the natural frequency of an empty shell.

If the shell contains fluid and a bundle of rods, then the formula (19) instead of the density of the fluid ρ you must supply a dynamic density of a heterogeneous medium $\rho^*(\omega)$. As dynamic density depends on frequency, the formula (19) in this case becomes the equation for required frequency Ω_n. Substitution (11) in (19) gives the biquadratic equation

$$\Omega^4{}_n\left(1+\frac{R\rho^*{}_1}{\rho_s}\frac{n}{n^2+1}\right) - \Omega_n{}^2\omega_0^2\left(1+\frac{R\rho^*{}_2}{h_s\rho_s}\frac{n}{n^2+1}+\frac{\omega_n{}^2}{\omega_0^2}\right) + \omega_n{}^2\omega_0^2 = 0, \tag{20}$$

where ρ^*_1 and ρ^*_2 are determined by the formulas

$$\rho^*{}_1 = \rho\left[1+\frac{(1+\gamma)(\Delta-1)\varphi}{\Delta+\gamma}\right], \tag{21}$$

$$\rho^*{}_2 = \rho\left[1+(1+\gamma)\varphi\right]. \tag{22}$$

Formulas (21), (22) give values of dynamic density of the heterogeneous medium in two limit cases $\omega_0 = 0$ and $\omega_0 = \infty$.

The solution of the biquadratic equation (20) gives two natural frequencies for each value n. It is easy to be convinced that in limit cases $\omega_0 \to 0$ or $\omega_0 \to \infty$, i.e. when rods of a bundle represent freely weighed or motionless (not oscillating) inclusions, the solution of the equation (20) gives for each value *n* one frequency

$$\Omega_n = \frac{\omega_n}{\sqrt{1+\dfrac{R\rho^*_1}{h_s\rho_s}\dfrac{n}{n^2+1}}} \qquad \text{(when } \omega_0 \to 0\text{)} \tag{23}$$

or

$$\Omega_n = \frac{\omega_n}{\sqrt{1+\dfrac{R\rho^*_2}{h_s\rho_s}\dfrac{n}{n^2+1}}} \qquad \text{(when } \omega_0 \to \infty\text{)} \tag{24}$$

In turn, in the absence of rods in the fluid ($\varphi = 0$), these formulas provide their natural frequencies of vibration of a shell filled with a homogenous fluid, and when ρ=0 – the natural frequencies of an empty shell $\Omega_n = \omega_n$.

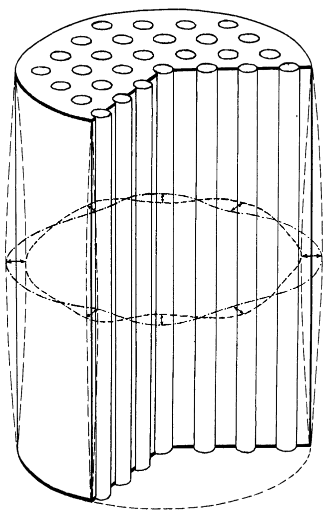

Figure 3: A shell with a fluid and a beam of elastic rods.

To illustrate the dependence of the natural frequencies of vibration of a shell with a bundle of elastic rods consider the numerical example.

Let the shell be characterized by the following parameters: L/R=4, R/h_s=100, ρ/ρ_s=0.15, ρ_1^*/ρ =2, ρ_2^*/ρ=3, γ=3, Δ=5, φ=0.5, and elastic rods of a bundle have natural frequency of flexural vibrations ω_0, equal to the minimum frequency of vibrations of an empty shell.

In this example, the minimum frequency of an empty shell corresponds to five waves in the circumferential direction.

Figure 4 shows the dependence of the natural frequencies of vibration of a shell with a bundle of rods on the number of waves in the circumferential direction, consisting of low-frequency and high-frequency branches. For comparison, the dashed line shows the dependence of (17) for an empty shell.

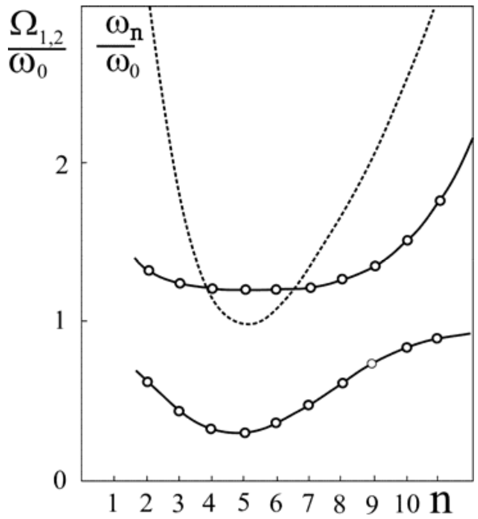

Figure 4: Natural frequencies of vibration of a shell with a fluid and a bundle of rods with different numbers of waves in a circumferential direction.

With the forced vibrations of a shell on their natural frequencies resonances arise. The amplitudes of resonance vibrations of the shell depend on the hydrodynamic damping or from the translational viscosity of a heterogeneous medium.

In particular, if the distributed power $F = F_0 \sin\frac{\pi x}{L}\cos n\theta\, e^{i\omega t}$ impact the shell it is easy to define its response. The coefficients the dynamic (q-factor) under appropriate natural frequencies of vibration of a shell $\Omega_{1,2}(n)$ is expressed by the formula

$$D(n) = \frac{\rho_s h_s + \rho^*(\Omega_{1,2})R\frac{n}{n^2+1}}{\eta^*(\Omega_{1,2})R\frac{n}{n^2+1}}\,\Omega_{1,2}(n). \qquad (25)$$

Calculated amplitude-frequency characteristics of the shell as the sum of the dynamic factors for all enveloped vibration forms $D(\omega/\omega_0)$ is presented in figure 5.

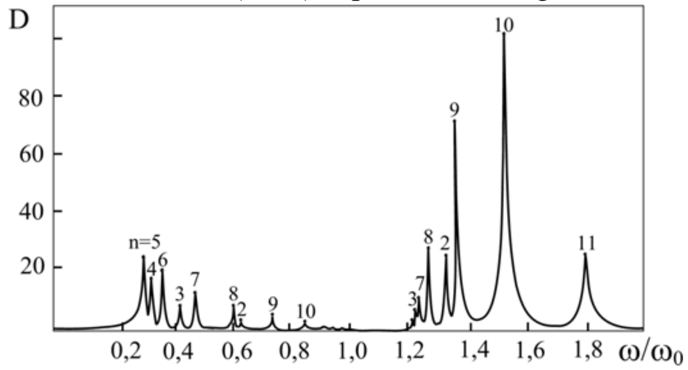

Figure 5: Amplitude-frequency characteristics of the shell (The figures above the resonance peaks denotes the number of waves in the circumferential direction).

The figure shows that the amplitude resonances (or q-factor of vibration of a shell on their own frequencies) in the low-frequency part of the spectrum increases with the increase of n from two to five, and then decrease. In the high-frequency part of the spectrum amplitude resonances first is reduced by changing n from two to five, and then increases, reaching a maximum value for n=10, and then again decreases.

6 CONCLUSION

For the decision of tasks on natural vibrations of a distributed elastic systems, such as elastic shell containing a fluid and a bundle of elastic rods, it is advisable to use continuous approach with the introduction of effective dynamic properties of dynamic density and the translational viscosity. The main effect resulting from the use of such an approach is that each form of shell vibrations corresponds to two of its own frequency, and the dispersion dependence of natural frequency of the number of waves has two branches. Low-frequency branch corresponds to the anti-phase vibrations of rods and shells, and high-frequency - phase. In addition, the use of translational viscosity of a fluid with a bundle of rods allows you to evaluate the amplitude of the resonance peaks on natural frequencies of vibration of a shell, with non-monotonicdepending on the number of waves in the circumferential direction of the shell.

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