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INCREASING THE MODAL OVERLAP FACTOR OF A BEAM USING ACOUSTIC BLACK HOLE EFFECT

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Abstract. *The flexural waves propagating in a beam can be efficiently absorbed if one extremity is tapered with a power law profile and covered by a very thin viscoelastic layer [1]. Such a termination induces an effect known as the "acoustic black hole effect" (ABH), which is a consequence of propagation characteristics of flexural wave in structures having non homogeneous thicknesses: if the thickness decreases locally, flexural waves slow down and the amplitude of the displacement field increases, leading to efficient energy dissipation if an absorbing layer is placed where the thickness is minimum [2]. An experimental investigation shows that the ABH effect significantly increases the modal overlap factor (MOF) of beams, and thus reduces the overall level of vibration. Experimental modal analysis of a ABH beam is performed using a High Resolution Technique called ESPRIT (Estimation of Signal Parameter Via Rotational Techniques [3]). It permits the estimation of modal density, modal loss factor and MOF. Further investigations, including a two-dimensions numerical model of the structure, show that this MOF can be explained by an increase of the modal density and a high damping of a number of modes of the structure due to the ABH. Emphasis is put on the modal loss factors, resulting from a high energy localization and inhomogeneous damping properties in the tapered region.*

1 INTRODUCTION

Vibration damping of mechanical structures as beams and plates is an essential designing aspect in many industrial applications. In order to reduce structural vibrations, classical methods consist in covering the structure by a heavy visco-elastic material. The efficiency of these methods has been shown and widely studied but they result in a strong added mass of the overall treated structure, which may be prohibitive in transportation industry for ecological reasons, for example. Then, the development of vibration damping methods without added mass becomes of a great interest.

One of these methods consists in taking advantage of an Acoustic Black Hole (ABH) effect [1, 2]. The ABH effect is related to bending wave properties in a beam of decreasing thickness : Mironov [4] shows that at the neighbourhood of the edge, if the thickness decreases smoothly to zero, the wave slows down and stops without being reflected at the edge. The condition of sufficient smoothness can be fulfilled by a power-law thickness profile $h(x)$ in the form :

$$h(x) = \epsilon x^n, \tag{1}$$

where $n \geq 2$. If the thickness is zero at the edge, it can be shown that the needed travel time for a wave to reach the edge becomes infinite. Thus, the reflection coefficient tends to zero. Despite this promising theory, manufacturing processes are such as the truncation of the tapered profile can never be small enough for the effect to be attractive.

However, by covering the tapered profile by a thin damping layer, Krylov [1] shows, in the framework of geometric acoustics, that the damping layer compensates the finite thickness at the edge of practical structures. He obtains an analytical expression of the reflection coefficient which is drastically reduced. A model that takes into account the effect of evanescent waves in 1D structures has been proposed by Georgiev et al. [2]. The results notably show that some rules can be specified for determining the optimal geometrical and material properties of the damping layer. These works also presents first experimental results on circular ABH, merely consisting in an axisymetrical development of the 1D-profile, on plates. Others experimental works demonstrate the damping efficiency of various ways of including circular ABH in plate-like structures practical cases.

This paper deals with the implementation of the ABH effect on beams and shows both experimentally and numerically that it can be seen as a way to increase the modal overlap factor of the structure, therefore reducing the level of vibration. The first section uses a high resolution modal identification technique to estimate experimental modal parameters of the ABH structure. The second section proposes a numerical model in order to explain the experimental results. The third section shows numerical results in terms of modal dampings and modal shapes of the eigenmodes and provides a good understanding of how the ABH makes the MOF increase.

2 EXPERIMENTAL MODAL ANALYSIS

2.1 Experimental setup and method

The experimental setup consists in two different beams to be tested, an impact hammer, an accelerometer and a acquisition card. The beams are vertically suspended with thin wires glued on their side ridges, in order to avoid unwanted additional damping from suspensions. Hammer and accelerometer are placed on the same arbitrary point on the centerline and acquire time signal of force and acceleration. Three measures are made on each arbitrary point. This is repeated on three points in order to obtain the maximum of information on the modes of the structures. These structure are a reference uniform beam and a beam with a ABH profile and a damping layer at one extremity. They are made of a polymer material (Objet™ VeroWhitePlus FullCure835) and measure $1\times$ 0.02 $\times$ 1.5.10^{-3} m^3; the tapered profile extends on a length of 0.12 m and the terminal thickness is about 6 μm. The excitation on the centerline allows to excite only the flexural modes of the beam.

While one can easily observe the effect of the ABH on the frequency response (see Figure 1), additional post-treatment is necessary to obtain the modal parameters of the two structures. The modal parameters that are to be observed are the modal density, the modal loss factors and the modal overlap factor (MOF). The MOF is usually estimated with the following expression :

$$MOF = n\eta f \tag{2}$$

where n is the modal density, η is the modal loss factor and f is the frequency. The MOF actually quantify the reduction of the resonant behaviour of the structure.

ABH structure have been shown to be highly damped and classical modal analysis technique may not be sufficient to estimate the modal parameters. However, so-called High Resolution techniques have been developed that overcome the limitations of classical methods on damped system or in the high frequency domain (high MOF) [3]. The method used here is called ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique) : it is a subspace method that considers the signal as a sum of damped exponentials and estimate the parameters of these. ESPRIT is used in combination with a criterion called ESTER (ESTimation of ERror) that estimate the order of the model.

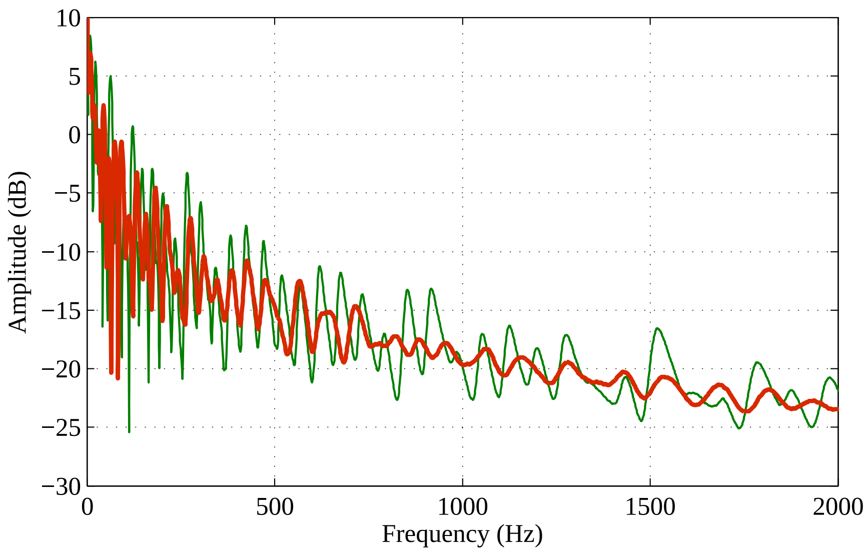


Figure 1: Frequency response of the uniform beam (thin line) and the acoustic black hole beam (thick line).

2.2 Experimental results

Figure 1 shows the frequency response of the two structures on the frequency band 0–2000 Hz. While the two curves are not very different below 400 Hz, the resonant behaviour for the ABH case is visually reduced above this frequency, even if the reduction is not as strong as with the aluminium structures [2].

ESPRIT/ESTER allows to estimated the modal density, the modal loss factor and the modal overlap factor, that are plotted on Figure 2. Remind that the method is not perfect and may miss some component in the modal identification, especially in the ABH case. The first observation is that the modal density does not differ very much between the two structures; plus the modal density of the uniform beam respects the theory for flexural modes of a beam. Second thing is the augmentation of the loss factor : from a quite constant value of 3.5% in the reference case (which is the material loss factor), they reach 6.5% in the ABH case. The same observation can be made on the MOF which is also more or less doubled, since it is the product of the last two quantities.

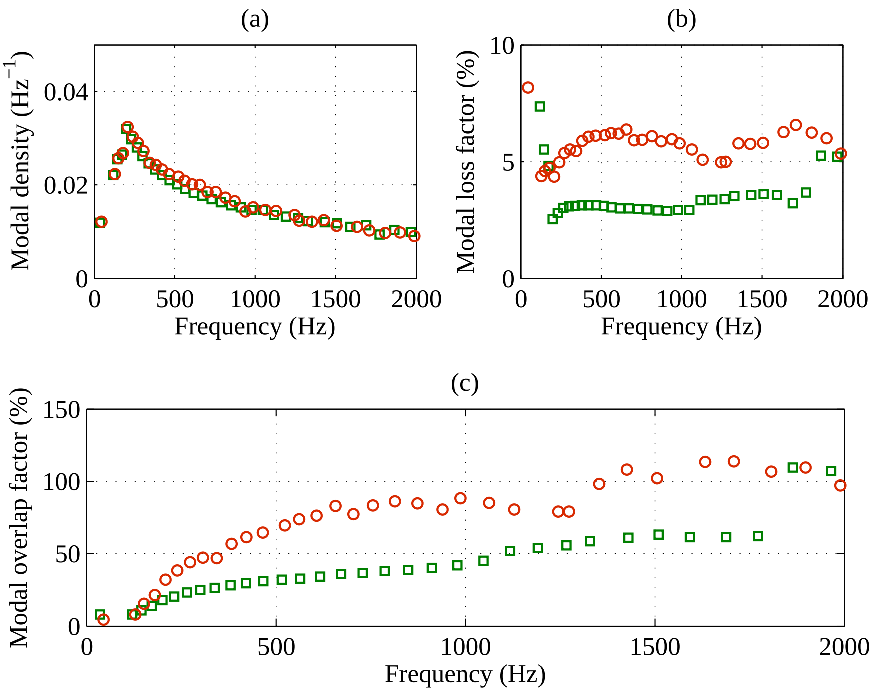


Figure 2: Estimated (a) modal density, (b) modal loss factor and (c) modal overlap factor of the uniform beam (□) and the acoustic black hole beam (○).

3 MODEL OF THE ACOUSTIC BLACK HOLE STRUCTURE

To give better understanding of the experimental results for the MOF, a model of flexural vibrations in non uniform thin plate is proposed. The governing equations of such a model are presented and solved using a finite difference scheme adapted to the ABH profile.

3.1 Governing equations

The equation of the free transverse vibration $W(x, y)$ of the structure in the case of an harmonic motion is [5] :

$$\nabla^2(D(x)\nabla^2 W(x, y)) - (1-\nu)\Big(\frac{\partial^2 D(x)}{\partial x^2}\frac{\partial^2 W(x,y)}{\partial y^2}\Big) - \rho(x)h(x)\omega^2 W(x, y) = 0 \tag{3}$$

where $\rho(x)$ is the mass density, $h(x)$ is the thickness, ω is the angular frequency $D(x)$ is the local complex flexural rigidity and ν the Poisson coefficient. These are geometrical and material properties of a structure that is equivalent to the plate and the damping layer if it is present.

The model uses a structural damping, represented with a loss factor included in the imaginary part of the complex flexural rigidity $D(x)$ in Eq. 3. As it was done in previous works, the effect of the layer is represented with equivalent mechanical properties that modify the flexural rigidity, the mass density and the thickness, following the model of Ross-Ungar-Kerwin [6] as used in the current context by Georgiev [2]. The complex flexural rigidity is thus given by :

$$D(x) = \begin{cases} D_p(x) = \dfrac{Eh_p(x)^3}{12(1-\nu^2)}, & \forall x > x_l \\ D_p(x)\Big[(1+j\eta_p) + \frac{E_l}{E_p}\left(\dfrac{h_l}{h_p(x)}\right)^3(1+j\eta_l) & \\ +\dfrac{3\left(1+\frac{h_l}{h_p(x)}\right)^2 + \frac{E_l h_l}{Eh_p(x)}(1-\eta_p\eta_l + j(\eta_p+\eta_l))}{1+\frac{E_l h_l}{E_p h_p(x)}(1+j\eta_l)}\Big], & \forall x \leq x_l \end{cases} \tag{4}$$

assuming the damping layer extends from 0 to x_l. Here η_p is the loss factor of the material of the plate, $D_p(x)$, E_p and $h_p(x)$ are the flexural rigidity, the Young modulus and the thickness of the plate alone, respectively , E_l, h_l and η_l are the Young modulus, the thickness and the loss factor of the damping layer, respectively. One must also take into account the effect of the mass of the layer and its thickness using an equivalent mass density and a total height of the equivalent plate. The thickness of the plate is, in the ABH case :

$$h_p(x) = \begin{cases} h_0, & \forall x > x_{ABH} \\ h_0\dfrac{(x+x_{ABH})^m}{(x_0+x_{ABH})^m}, & \forall x \leq x_{ABH}. \end{cases} \tag{5}$$

with x_{ABH} the abscissa delimiting the ABH extremity and x_0 the distance to zero of the point where the thickness reaches zero.

Free boundary conditions are considered but are not written here for the sake of conciseness.

3.2 Numerical resolution

The problem described in the previous section is numerically solved by using a second order finite difference scheme.

In classical cases for which the thickness of the plate is constant, such a second-order finite difference scheme is relatively straightforward. The present case is more difficult since the thickness is locally reduced, leading to an important numerical dispersion or an important computation cost. Indeed, the wavelength in the ABH structure dramatically decreases along the ABH profile, since it depends on the local thickness for a given frequency. To well represent very small wavelength and avoid numerical dispersion, the space mesh grid can be adapted, and thus non-uniform.

A solution is to use a coordinate change allowing to switch from a non-uniform "physical" mesh grid (coordinate x) to a transformed uniform mesh grid (coordinate $\tilde{x}$) where a transformed equation is solved. The interest is to avoid a tedious resolution on a non-uniform grid. The coordinate change is chosen in order to match the change in wavelength.

After discretizing the equation of motion and the boundary conditions in transformed coordinates, it is then possible to write the following numerical eigenvalue problem under a matrix formulation :

$$(K^{*} - \alpha^{2} M)\phi = 0 \tag{6}$$

where K^* and M represent the complex stiffness matrix and the mass matrix of the structure, α is an eigenvalue and ϕ is an eigenvector.

4 NUMERICAL RESULTS

4.1 Eigenvalues and modal loss factors

Most of the results of the model can be found observing the eigenvalues of the system, plotted on Figure 3. As expected, the eigenvalues of the uniform beam, with a homogeneous material loss factor, lay on a straight line in the complex plan; the angle of this line with the imaginary axis is the loss angle from which the loss factor is deduced. Comparing with the ABH case, it can be seen that, on the one hand the eigenvalues are slightly shifted towards the bottom, leading to a potential but small increase in modal density, and on the other hand the eigenvalues are shifted towards higher values on the real axis; this means a higher loss angle and a higher loss factor.

Another phenomenon can be observed in the ABH case. The majority of the eigenvalues lay around a straight line, that is, the modal loss factor is close to constant : this is typical of mode 15, indicated on Figure 3. Some values lay outside and have a very high real part (for example mode 22). The modes associated with these can be called 'hyper-damped' modes. They are not seen in the experiment.

4.2 Mode shapes and localization

A good understanding for the phenomena seen on Figure 3 is provided by an examination of the modal shapes. Shapes of modes 15 and 22 are represented on Figure 4, in the transformed coordinates used in the numerical resolution : this provides an adequate zoom on the tapered region of the structure. Additionally, an indicator I_c of localization of the energy in the ABH profile can be defined : this is simply the kinetic energy in the ABH profile over the total kinetic energy.

Mode 15 is representative of the majority of the modes in the ABH case. This is a rather typical beam mode, in the sense that it has a one dimensional flexural behaviour. The wavelength is smaller and the amplitude greater in the tapered region. This lead to a low to moderate I_c

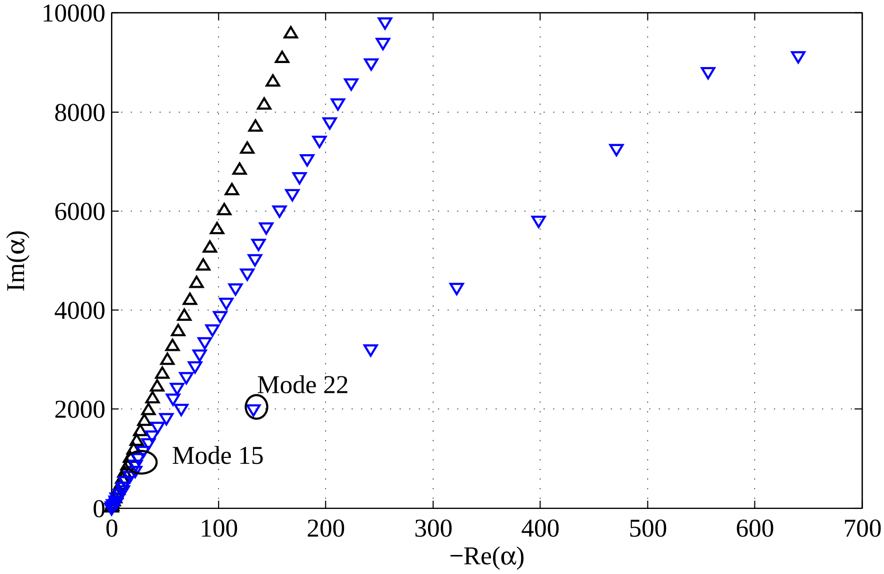


Figure 3: Eigenvalue spectrum in the complex plan of the simulated uniform beam (△) and acoustic black hole beam (▽).

(28%): a portion of the energy of the mode is concentrated in the ABH extremity that is more damped than the rest of the beam, allowing to obtain a rather high modal loss factor.

Mode 22 is one the 'hyper-damped' modes mentioned before. Visually it is localized in the ABH extremity. It also has a transverse behaviour. The localization is confirmed by its I_c of 86%. Whether or not it is related to the transverse behaviour, this is the origin of the high value of the loss factor for this 'family' of modes.

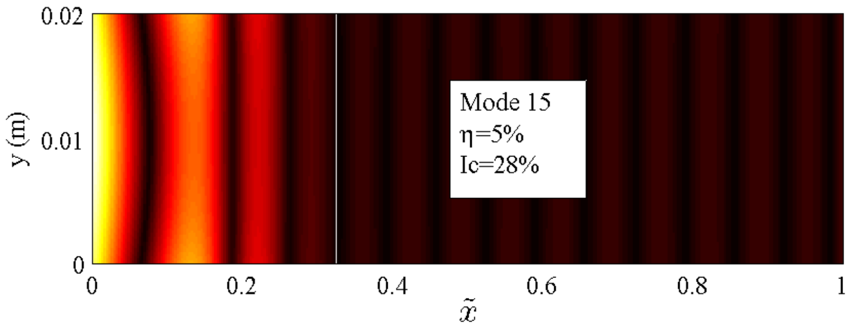
4.3 Simulated Modal Overlap Factor

A last result is the comparison of the experimental and modelled modal overlap factor : this is a global quantity that can provide a good insight of the adequateness of the model. Figure 5 shows this comparison. For the uniform beam, the experiments and the numerical model do not differ very much : the difference (lower MOF for the model) may be attributed to a poor estimation of the material parameters.

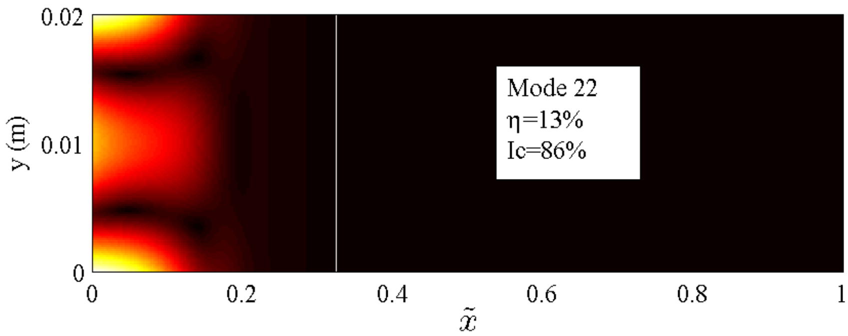
The case of the ABH is less satifying since while the model reproduces the augmentation of MOF, the latter is overestimated above 1000 Hz : a probable reason is that the theory fails to model the damping mechanisms above this frequency, assuming the estimated material parameters (especially of the damping layer) are correctly estimated. Nevertheless, Figure 5 shows that the ABH effect is a way to increase the modal overlap of the beam on which it is implemented.

5 CONCLUSIONS

A modal approach of the acoustic black hole effect has been presented in this paper. Experiments and numerical study both show that the ABH effect on a beam leads to a minor increase in modal density and a strong increase of the modal loss factor. This augmentation of the loss



(a)



(b)

Figure 4: Modal shape (modulus) of (a) mode 15 and (b) mode 22 of the acoustic black hole beam in the transformed coordinates. The tapered profile lays on the left of the white line. The loss factor η and the localization indicator I_c are indicated.

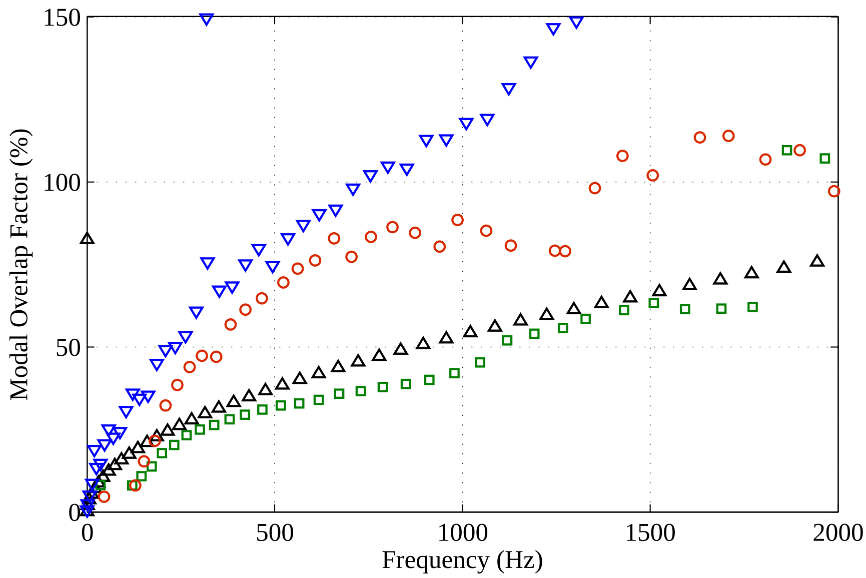


Figure 5: Modal overlap for the experimental uniform (□) and ABH beam (○) and the simulated uniform (△) and ABH beam (▽)

factor is due to high localization of the energy in a highly damped region. The model also shows the existence of a locally transverse behaviour, justifying the need for a plate model; this behaviour gives birth to 'hyper-damped' modes, not seen in the experiments.

While a direct comparison of the MOF of the experiment and the model gives mitigated results, it allows to assess a major tendency : the acoustic black hole effect is a good way to increase the modal overlap of a beam.

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